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On Trigonometric Fuzzy Information Measures

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ABSTRACT

In the literature of fuzzy information measures, there exist many well known parametric and non-parametric measures with their own merits and limitations. But our main emphasis is on applications of these measures to a variety of disciplines. It has been observed that trigonometric measures of fuzzy information have their own importance for application point of view particularly to geometry. In present communication, two sine and cosine trigonometric measures of fuzzy information are defined and characterized. These measures are generalized to obtain new measures of trigonometric fuzzy information. Some new trigonometric measures of fuzzy information are studied and their particular cases are also obtained.

Keywords: *Fuzzy set, Fuzzy entropy, Fuzzy Uncertainty, Cosine fuzzy information, Sine fuzzy information*

1. INTRODUCTION

When proposing fuzzy set, Zadeh [14] concerns were explicitly centred on their potential contribution in the domain of pattern classification, processing and communication of information, abstraction and summarization etc. Although the claims that fuzzy sets were relevant in these areas appeared unsustainable in the early sixties, however, the future development of information science and engineering proved that these intuitions were right. The specificity of fuzzy sets is to capture the idea of partial membership. The characteristic function of a fuzzy set is often called membership function and the role of that has well been explained by Singpurwalla and Booker [12] in probability measures of fuzzy sets.

A generalized theory of uncertainty has been well explained by Zadeh [16] where he remarked that uncertainty was an attribute of information. Before that the path breaking work of Shannon [10] had led to a universal acceptance of the theory that information was statistical in nature. A perception-based theory of probabilistic reasoning with imprecise probabilities had also been explained by Zadeh [15]. Some work related with uncertainty management for intelligence analysis was reported by Yager [13] whereas the generalized information theory, its aims, results and some open problems were discussed by Klir [7]. Chen [1] remarked that Shannon's [10] mathematical theory of information entropy was introduced to analyze the information carrying capacity of communication channels and that served as a measure of the degree of uncertainty or the extent of ignorance. Some work related with probabilistic differential entropy was done by Garbaczewski [3], while Nanda and Paul [17] had redefined Khinchin's version of entropy and discussed the properties of the aging classes based on their generalized entropy.

Taking into consideration the concept of fuzzy set, De Luca and Termini [2] suggested that corresponding to Shannon's [10] probabilistic entropy, the measure of fuzzy information could be defined as follow

$$H(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))\right], \tag{1}$$

where $\mu_A(x_i)$ are the membership values.

Bhandari and Pal [24] parametrically generalized (1) as given below:

$$H_\alpha(A) = \frac{1}{1-\alpha}\sum_{i=1}^{n}\log\left[\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right], \tag{2}$$

where $\alpha > 0, \alpha \neq 1$.

On the same lines many researchers have studied various generalized fuzzy information measures. Hooda [4], Hooda and Bajaj [6] and many more have studied various generalized additive and non-additive fuzzy information measures. Later on Hooda and Jain [5] and Hooda et al. [18] characterized sub additive trigonometric measure of fuzzy information corresponding to probabilistic entropy by Sharma and Taneja [11] and developed fuzzy mean code word lengths. It has been noted that trigonometric measures has its own importance in application point of view, particularly in geometry.

Mishra et al. [19] have developed new measures of weighted trigonometric fuzzy information, the findings of which have been applied to study the principle of maximum weighted fuzzy information.

In present communication, two sine and cosine trigonometric fuzzy information measures are introduced and characterized. Some generalized trigonometric fuzzy information measures are defined and their particular cases are studied in section 2. In section 3, a new cosine trigonometric fuzzy information measure is defined and characterized. A tangent inverse trigonometric fuzzy information measure is introduced and studied in section 4. In section 5, an application of fuzzy discrimination measure in strategic decision making is discussed followed by references.

2. SINE AND COSINE TRIGONOMETRIC FUZZY INFORMATION MEASURES

Most of fuzzy information measures have been defined analogous to probabilistic entropies. Here we introduce two trigonometric fuzzy information measures which have no analogous entropies and are given below:

$$H_1(A)=\sum_{i=1}^{n}\left[\sin\frac{\pi\mu_A(x_i)}{2}+\sin\frac{\pi(1-\mu_A(x_i))}{2}-1\right], \tag{3}$$

$$H_2(A)=\sum_{i=1}^{n}\left[\cos\frac{\pi\mu_A(x_i)}{2}+\cos\frac{\pi(1-\mu_A(x_i))}{2}-1\right]. \tag{4}$$

First of all we check the validity of the proposed measures (3) and (4).

Theorem 1:

The fuzzy information measure given by (3) is valid measure.

Proof:

To prove that the given measure is a valid measure, we shall show that (3) satisfies the four properties (P1) to (P4) mentioned in DeLuca and Termini [2].

(P1). $H_1(A)=0$ if and only if A is a crisp set.

Evidently,

$$H_1(A)=\sum_{i=1}^{n}\left[\sin\frac{\pi\mu_A(x_i)}{2}+\sin\frac{\pi(1-\mu_A(x_i))}{2}-1\right]=0,$$

if and only if either $\mu_A(x_i)=0$ or $1-\mu_A(x_i)=0$ for $i=1,2,\ldots\ldots,n$.

It implies $H_1(A)=0$ if and only if A is a crisp set.

(P2). $H_1(A)$ is maximum if and only if A is the fuzziest set $i.e.,\mu_A(x_i)=0.5$ for all $i=1,2,\ldots\ldots,n$. Differentiating $H_1(A)$ with respect to $\mu_A(x_i)$, we have

$$\frac{dH_1(A)}{d\mu_A(x_i)}=\frac{\pi}{2}\sum_{i=1}^{n}\left[\cos\frac{\pi\mu_A(x_i)}{2}-\cos\frac{\pi(1-\mu_A(x_i))}{2}\right], \tag{5}$$

which vanishes at $\mu_A(x_i)=0.5$.

Again differentiating (5) with respect to $\mu_A(x_i)$, we get

$$\frac{d^2H_1(A)}{d\mu_A(x_i)^2}=\frac{\pi^2}{4}\sum_{i=1}^{n}\left[-\sin\frac{\pi\mu_A(x_i)}{2}-\sin\frac{\pi(1-\mu_A(x_i))}{2}\right], \tag{6}$$

which is less than zero (<0) at $\mu_A(x_i)=0.5$. Hence $H_1(A)$ is maximum at $\mu_A(x_i)=0.5$ for $i=1,2,\ldots\ldots,n$.

Further from (2.3) we see that $H_1(A)$ is an increasing function of $\mu_A(x_i)$ in the region $0\le\mu_A(x_i)\le 0.5$ and $H_1(A)$ is a decreasing function of $\mu_A(x_i)$ in the region $0.5\le\mu_A(x_i)\le 1$.

(P3). Let A^* be sharpened version of A, which means that

if $0\le\mu_A(x_i)\le 0.5$, $\mu_{A^*}(x_i)\le\mu_A(x_i)$ for all i

and if $0.5\le\mu_A(x_i)\le 1$, $\mu_{A^*}(x_i)\ge\mu_A(x_i)$ for all i.

Since $H_1(A)$ is an increasing function of $\mu_A(x_i)$ in the region $0\le\mu_A(x_i)<0.5$ and $H_1(A)$ is a decreasing function of $\mu_A(x_i)$ in the region $0.5<\mu_A(x_i)\le 1$, therefore

$$\mu_{A^*}(x_i)\le\mu_A(x_i)\Rightarrow H_1(A^*)\le H_1(A)\quad\text{in } [0,0.5] \tag{7}$$

$$\mu_{A^*}(x_i)\ge\mu_A(x_i)\Rightarrow H_1(A^*)\le H_1(A)\quad\text{in } [0.5,1]. \tag{8}$$

Hence (7) and (8) together give

$$H_1(A^*)\le H_1(A).$$

(P4). From the definition It is evident that

$$H_1(A)=H_1(A^c),$$

Where A^c is complement of A obtained by replacing $\mu_A(x_i)$ by $1-\mu_A(x_i)$.

Hence $H_1(A)$ satisfies all the essential four properties of fuzzy information measures. Thus it is a valid measure of fuzzy information.

On the same line it can be proved that $H_2(A)$ is a valid fuzzy information measure.
Let us consider the function $\sin\pi x$, which is a concave function $\forall\, x\in[0,1]$. Then we consider the following fuzzy information measure and prove its validity:

$$H_3(A)=\sum_{i=1}^{n}\left[\sin\pi\mu_A(x_i)+\sin\pi(1-\mu_A(x_i))\right], \tag{9}$$

Theorem 2:

The fuzzy information measure given by (9) is valid measure.

Proof:

To prove that the given measure is a valid measure, we shall show that (9) satisfies the four properties (P1) to (P4).

(P1). $H_3(A) = 0$ if and only if A is a crisp set.
Evidently,

$$H_3(A) = \sum_{i=1}^{n}\left[\sin \pi\mu_A(x_i) + \sin \pi(1-\mu_A(x_i))\right] = 0,$$

if and only if either $\mu_A(x_i) = 0$ or $1 - \mu_A(x_i) = 0$ for $i = 1, 2, \ldots\ldots, n$.

It implies $H_3(A) = 0$ if and only if A is a crisp set.

(P2). $H_3(A)$ is maximum if and only if A is the fuzziest set $i.e., \mu_A(x_i) = 0.5$ for all $i = 1, 2, \ldots\ldots, n$.

Differentiating $H_3(A)$ with respect to $\mu_A(x_i)$, we have

$$\frac{dH_3(A)}{d\mu_A(x_i)} = \pi\sum_{i=1}^{n}\left[\cos \pi\mu_A(x_i) - \cos \pi(1-\mu_A(x_i))\right], \quad (10)$$

which vanishes at $\mu_A(x_i) = 0.5$.

Again differentiating (10) with respect to $\mu_A(x_i)$, we get

$$\frac{d^2H_3(A)}{d\mu_A(x_i)^2} = -\pi^2\sum_{i=1}^{n}\left[\sin \pi\mu_A(x_i) + \sin \pi(1-\mu_A(x_i))\right],$$

which is less than zero (< 0) at $\mu_A(x_i) = 0.5$.

Hence $H_3(A)$ is maximum at $\mu_A(x_i) = 0.5$ for $i = 1, 2, \ldots\ldots, n$.

$H_3(A)$ is an increasing function of $\mu_A(x_i)$ in the region $0 \le \mu_A(x_i) \le 0.5$ and $H_3(A)$ is a decreasing function of $\mu_A(x_i)$ in the region $0.5 \le \mu_A(x_i) \le 1$.

(P3). Let A^* be sharpened version of A, which means that

if $0 \le \mu_A(x_i) < 0.5$, $\mu_{A^*}(x_i) \le \mu_A(x_i)$ for all i

and if $0.5 < \mu_A(x_i) \le 1$, $\mu_{A^*}(x_i) \ge \mu_A(x_i)$ for all i.

Since $H_3(A)$ is an increasing function of $\mu_A(x_i)$ in the region $0 \le \mu_A(x_i) \le 0.5$ and $H_3(A)$ is a decreasing function of $\mu_A(x_i)$ in the region $0.5 \le \mu_A(x_i) \le 1$. Therefore,

$$\mu_{A^*}(x_i) \le \mu_A(x_i) \Rightarrow H_3(A^*) \le H_3(A) \quad \text{in } [0, 0.5] \quad (11)$$

$$\mu_{A^*}(x_i) \ge \mu_A(x_i) \Rightarrow H_3(A^*) \le H_3(A) \quad \text{in } [0.5, 1]. \quad (12)$$

Hence (11) and (12) together give

$$H_3(A^*) \le H_3(A).$$

(P4). It is evident from the definition that

$$H_3(A) = H_3(A^c),$$

where A^c is complement of A obtained by replacing $\mu_A(x_i)$ by $1 - \mu_A(x_i)$.

Hence $H_3(A)$ satisfies all the essential four properties of fuzzy information measures. Thus it is a valid measure of fuzzy information.

Next, we define the generalized sine trigonometric fuzzy information measure as given below:

$$H_4(A) = \sum_{i=1}^{n}\left[\sin \beta\mu_A(x_i) + \sin \beta(1-\mu_A(x_i))\right] - \sin\beta. \quad (13)$$

It may be noted that (13) reduces to (9) when $\beta = \pi$ and reduces to (3) when $\beta = \frac{\pi}{2}$.

Another generalized sine trigonometric measure of fuzzy information is

$$H_5(A) = \sum_{i=1}^{n}\left[\sin\left(\beta\mu_A(x_i) + \alpha\right) + \sin\left(\beta(1-\mu_A(x_i)) + \alpha\right) - \sin(\alpha+\beta)\right]. \quad (14)$$

In particular when $\alpha = 0$, (14) reduces to (13) and reduces to (9) when $\alpha = 0$ and $\beta = \pi$ and generalized cosine trigonometric measure of fuzzy information is

$$H_6(A) = \sum_{i=1}^{n}\left[\cos \beta\mu_A(x_i) + \cos \beta(1-\mu_A(x_i)) - (1+\cos\beta)\right], \quad (15)$$

which is a new generalized cosine trigonometric measure of fuzzy information and it reduces to the following fuzzy information measure when $\beta = \frac{\pi}{2}$

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$$H_7(A) = \sum_{i=1}^{n}\left[\cos\frac{\pi\mu_A(x_i)}{2} + \cos\frac{\pi(1-\mu_A(x_i))}{2} - 1\right] = H_2(A). \quad (16)$$

It can be easily verified that these fuzzy information measure satisfy the essential properties of validity.

3. A NEW COSINE TRIGONOMETRIC FUZZY INFORMATION MEASURE

Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be a set of universe and A be a fuzzy set in X having membership function $\mu_A(x_i)$ defined on X. Then cosine fuzzy information measure for fuzzy set A defined as

$$H_8(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\cos\left(\frac{|2\mu_A(x_i) - 1|}{2}\right)\pi\right] \quad (17)$$

Theorem 3:

$H_8(A)$ is a valid fuzzy information measure.

Proof:

For validity of the measure we shall prove that the four essential properties (P1) to (P4) are satisfied.

(P1). $H_8(A) = 0$ if and only if A is a crisp set.
Evidently, $H_8(A) = 0$ if and only if either $\mu_A(x_i) = 0$ or $1 - \mu_A(x_i) = 0$ for $i = 1, 2, \ldots\ldots, n$.

(P2). $H_8(A)$ is maximum iff $\mu_A(x_i) = 0.5$ for all $i = 1, 2, \ldots\ldots, n$.
Differentiating $H_8(A)$ with respect to $\mu_A(x_i)$, we have

$$\frac{dH_8(A)}{d\mu_A(x_i)} = -\frac{\pi}{n}\sum_{i=1}^{n}\left[\sin\left(\frac{|2\mu_A(x_i)-1|}{2}\right)\pi\right], \quad (18)$$

equation (3.2) reduces to zero when $\mu_A(x_i) = 0.5$ for all $i = 1, 2, \ldots\ldots, n$.

Differentiating again, we get

$$\frac{d^2H_8(A)}{d\mu_A(x_i)^2} = -\frac{\pi^2}{n}\sum_{i=1}^{n}\left[\cos\left(\frac{|2\mu_A(x_i)-1|}{2}\right)\pi\right], \quad (19)$$

equation (19) is less than zero when $\mu_A(x_i) = 0.5$ for all i.

Hence $H_8(A)$ is maximum when $\mu_A(x_i) = 0.5$ for all $i = 1, 2, \ldots\ldots, n$ or A is the fuzziest set.

Further it may be noted that $H_8(A)$ is an increasing function of $\mu_A(x_i)$ in the region $0 \le \mu_A(x_i) \le 0.5$ and $H_8(A)$ is a decreasing function of $\mu_A(x_i)$ in the region $0.5 \le \mu_A(x_i) \le 1$.

(P3). Let A^* be sharpened version of A, then

$\mu_{A^*}(x_i) \le \mu_A(x_i) \Rightarrow H_8(A^*) \le H_8(A)$ in $[0, 0.5]$
(20) $\mu_{A^*}(x_i) \ge \mu_A(x_i) \Rightarrow H_8(A^*) \le H_8(A)$ in $[0.5, 1]$.

(21)

Hence from (20) and (21) we can conclude that

$$H_8(A^*) \le H_8(A).$$

(P4). It is evident that if $\mu_A(x_i)$ replacing by $1 - \mu_A(x_i)$ for all $i = 1, 2, \ldots\ldots, n$, then

$$H_8(A^c) = H_8(A),$$

where A^c is complement of A.

Theorem 4:

Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be the universe of discourse. Let A and B be two fuzzy set in a fixed universe of discourse X. Let $A(x_i) = \mu_A(x_i)$ and $B(x_i) = \mu_B(x_i)$ satisfying either $A \subseteq B$ or $B \subseteq A$, then the following holds:

$$H_8(A \cup B) + H_8(A \cap B) = H_8(A) + H_8(B). \quad (22)$$

Proof:

Let us separate X into two parts X_1 and X_2, where

$$X_1 = \{x_1 \in X : A(x) \subseteq B(x)\}$$

and

$$X_2 = \{x_2 \in X : B(x) \subseteq A(x)\}.$$

It implies that for all $x_i \in X_1$, $\mu_A(x_i) \le \mu_B(x_i)$ and for all $x_i \in X_2$, $\mu_A(x_i) \ge \mu_B(x_i)$. From (17), we have

$$H_8(A \cup B) = \frac{1}{n}\sum_{i=1}^{n}\left[\cos\left(\frac{|2\mu_{A\cup B}(x_i)-1|}{2}\right)\pi\right]$$

$$= \frac{1}{n}\left[\sum_{x_i\in X_1}\cos\left(\frac{|2\mu_B(x_i)-1|}{2}\right)\pi + \sum_{x_i\in X_2}\cos\left(\frac{|2\mu_A(x_i)-1|}{2}\right)\pi\right]. \quad (23)$$

Again from definition (17), we have

$$H_8(A \cap B) = \frac{1}{n}\sum_{i=1}^{n}\left[\cos\left(\frac{|2\mu_{A\cap B}(x_i)-1|}{2}\right)\pi\right]$$

$$= \frac{1}{n}\left[\sum_{x_i\in X_1}\cos\left(\frac{|2\mu_A(x_i)-1|}{2}\right)\pi + \sum_{x_i\in X_2}\cos\left(\frac{|2\mu_B(x_i)-1|}{2}\right)\pi\right]. \quad (24)$$

Adding (23) and (24), we get

$H_8(A \cup B) + H_8(A \cap B) = H_8(A) + H_8(B)$.

Hence this theorem is proved.

Corollary 1:

Let A be a fuzzy set on X and A^c be its complement, then

$$H_8(A) = H_8(A^c), \quad (25)$$

and

$$H_8(A \cup A^c) = H_8(A \cap A^c). \quad (26)$$

4. TANGENT INVERSE TRIGONOMETRIC FUZZY INFORMATION MEASURE

Prakash and Gandhi [9] proposed the following tangent fuzzy information measure:

$$H_9(A) = -\sum_{i=1}^{n}\left[\tan\frac{\pi}{2n\mu_A(x_i)} + \tan\frac{\pi}{2n(1-\mu_A(x_i))} - n^2\tan\frac{\pi}{2n}\right], \; n > 3. \quad (27)$$

They proved that (4.1) was a valid fuzzy measure and applied to the geometry to estimate the perimeter and area of Polygon.

In the present section, we define tangent inverse trigonometric fuzzy information measure and prove its validity.

Definition 4.1

Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$ be a set of universe and A be a fuzzy set in X having membership function $\mu_A(x_i)$ defined on X. Then tangent inverse trigonometric fuzzy information measure for fuzzy set A is defined as

$$H_\alpha(A) = \frac{2}{1-\alpha}\sum_{i=1}^{n}\left[\tan^{-1}\left\{\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right\} - \frac{\pi}{4}\right]; \quad (28)$$

where $\alpha > 0, \; \alpha \neq 1$.

Theorem 5:

The measure in (28) is a valid fuzzy information measure.

Proof:

To prove that the measure (28) is a valid measure, we shall show that it is satisfying the four properties (P1) to (P4).

(P1). $H_\alpha(A) = 0$ if and only if A is a crisp set. If $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ then $H_\alpha(A) = 0$.
Hence $H_\alpha(A) = 0$ if A is crisp set, $i.e.$ $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ for $i = 1, 2, \ldots\ldots, n$.
Conversely, if $H_\alpha(A) = 0$, then

$$\frac{2}{1-\alpha}\sum_{i=1}^{n}\left[\tan^{-1}\left\{\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right\} - \frac{\pi}{4}\right] =$$

it implies

$$\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha = 1. \quad (29)$$

The equality (4.3) holds if either $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ $i.e.$ A is a crisp set.

Thus $H_\alpha(A) = 0$ if and only if A is a crisp set, $i.e.$ $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ for $i = 1, 2, \ldots\ldots, n$.

(P2). $H_\alpha(A)$ is maximum if and only if A is the fuzziest set, $i.e. \mu_A(x_i) = 0.5$ for $i = 1, 2, \ldots\ldots, n$.

Differentiating (28) with respect to $\mu_A(x_i)$, we have

$$\frac{dH_\alpha(A)}{d\mu_A(x_i)} = \frac{2}{1-\alpha}\sum_{i=1}^{n}\left[\frac{\alpha\mu_A^{\alpha-1}(x_i) - \alpha(1-\mu_A(x_i))^{\alpha-1}}{1+\left\{\mu_A^\alpha(x_i) + (1-\mu_A(x_i))^\alpha\right\}^2}\right]. \quad (30)$$

Again differentiating (30) with respect to $\mu_A(x_i)$, we have

$$\frac{d^2H_\alpha(A)}{d\mu_A(x_i)^2} = \frac{2\alpha}{1-\alpha}\sum_{i=1}^{n}\left[\frac{\begin{array}{l}\left[1+\left\{\mu_A^\alpha(x_i)+(1-\mu_A(x_i))^\alpha\right\}^2\right]\left[(\alpha-1)\left\{\mu_A^{\alpha-2}(x_i)+(1-\mu_A(x_i))^{\alpha-2}\right\}\right] \\ -\left\{\mu_A^{\alpha-1}(x_i)-(1-\mu_A(x_i))^{\alpha-1}\right\}\left[2\left\{\mu_A^\alpha(x_i)+(1-\mu_A(x_i))^\alpha\right\}\right] \\ \times\left\{\alpha\mu_A^{\alpha-1}(x_i)-\alpha(1-\mu_A(x_i))^{\alpha-1}\right\}\end{array}}{\left[1+\left\{\mu_A^\alpha(x_i)+(1-\mu_A(x_i))^\alpha\right\}^2\right]^2}\right]$$

For maxima, we have

$$\frac{dH_\alpha(A)}{d\mu_A(x_i)} = 0.$$

It implies

$$\mu_A^{\alpha-1}(x_i) - (1-\mu_A(x_i))^{\alpha-1} = 0$$

or

$$\mu_A(x_i) - \left(1-\mu_A(x_i)\right) = 0$$

or

$$\mu_A(x_i) = 0.5.$$

Here two cases arise:

Case 1. When $0 < \alpha < 1$, we get

$$\frac{d^2H_\alpha(A)}{d\mu_A(x_i)^2} < 0 \text{ (-ve)} \quad (31)$$

Case 2. When $\alpha > 1$, we have

$$\frac{d^2H_\alpha(A)}{d\mu_A(x_i)^2} < 0 \text{ (-ve)} \quad (32)$$

(31) and (32) together give

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$$\frac{d^2H_\alpha(A)}{d\mu_A(x_i)^2} < 0 \text{ (-ve)} \quad (33)$$

Hence $H_\alpha(A)$ is maximum at $\mu_A(x_i) = 0.5$ for all i.

(P3). From (30), we have $\frac{dH_\alpha(A)}{d\mu_A(x_i)} > 0$ (+ve) in the region $0 \le \mu_A(x_i) \le 0.5$. Hence $H_\alpha(A)$ is an increasing function in the region $0 \le \mu_A(x_i) \le 0.5$.

Similarly, from (30), we have $\frac{dH_\alpha(A)}{d\mu_A(x_i)} < 0$ (-ve) in the region $0.5 \le \mu_A(x_i) \le 1$. Hence $H_\alpha(A)$ is decreasing function in the region $0.5 \le \mu_A(x_i) \le 1$.
Let A^* be the sharpend version of A, then

$$\mu_{A^*}(x_i) \le \mu_A(x_i) \Rightarrow H_\alpha(A^*) \le H_\alpha(A) \quad \text{in } [0, 0.5] \quad (34)$$

$$\mu_{A^*}(x_i) \ge \mu_A(x_i) \Rightarrow H_\alpha(A^*) \le H_\alpha(A) \quad \text{in } [0.5, 1]. \quad (35)$$

Thus from (34) and (35) together give

$$H_\alpha(A^*) \le H_\alpha(A).$$

(P4). Let A^c be a complement of A. Then, from the definition, we have

$$H_\alpha(A) = H_\alpha(A^c).$$

Since $H_\alpha(A)$ satisfies all the properties of fuzzy information measure, therefore, it is a valid measure of fuzzy information.

Particular Case

When $\alpha \to 1$, (4.2) reduces to De Luca and Termini [2] fuzzy information measure.

Definition 4.2

The fuzzy discrimination measure is said to be valid if it satisfies

(i) Non negativity *i.e.* $I(A,B) \ge 0$,
(ii) $I(A,B) = 0$, if $A = B$,
(iii) $I(A,B) \ge 0$ is convex function in $]0,1[$,
(iv) $I(A,B) \ge 0$ should not change, when $\mu_A(x_i)$ is changed to $1-\mu_A(x_i)$ and $\mu_B(x_i)$ is changed to $1-\mu_B(x_i)$.

Based on (1), Bhandari and Pal [25] have suggested the simplest measure of fuzzy discrimination as follows:

$$I_{BP}(A,B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} + (1-\mu_A(x_i))\log\frac{(1-\mu_A(x_i))}{(1-\mu_B(x_i))}\right], \quad (36)$$

Corresponding to (28) we define a new inverse tangent fuzzy discrimination measure as follows:

Theorem 6:

Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$, $A, B \in FS(X)$, then

$$I_\alpha(A,B) = \frac{2}{\alpha-1}\sum_{i=1}^{n}\left[\tan^{-1}\left\{\begin{array}{l}\mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i) + \\ (1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha}\end{array}\right\} - \frac{\pi}{4}\right], \quad (37)$$

where $\alpha > 0, \alpha \ne 1$ is a valid fuzzy discrimination measure.

Proof:

It can be easily verified that $I_\alpha(A,B)$ is valid fuzzy measure of discrimination.

5. AN APPLICATION OF FUZZY DISCRIMINATION IN STRATEGIC DECISION MAKING

In current scenario, the applications of the fuzzy discrimination measure have discussed in various fields, in bio-informatics by Poletti et al. [20], Fan et al. [21], and Bhatia and Singh [23] discussed in image thresholding, Ghosh et al. [22] applied it in automated leukocyte recognition. In the present communication, the application of the above fuzzy discrimination measure in strategic decision making is proposed.

Decision making problem is the process of finding the optimal solution from all the existing feasible alternatives. It is assumed that a firm Y desire to apply m strategies $\{S_1, S_2, \ldots\ldots, S_m\}$ to meet its goal. Let each strategy has different degree of effectiveness. If it has different input associated with it and let it be $\{l_1, l_2, \ldots\ldots, l_n\}$. The fuzzy set Y denotes the effectiveness of a particular strategy with uniform input. Then,

$$Y = \{(S_i, \mu_S(S_i)) : i = 1, 2, \ldots, m\} \quad (38)$$

Further, let I denotes the degree of effectiveness of a strategy when it is implemented with input l_j, then

$$I = \left\{ \left(Y, \mu_I \left(S_i \right) \right) : i = 1, 2, ..., m \right\}, \qquad (39)$$

where $j = 1, 2, ..., n$.

Substituting $A = Y$ and $B = I$ in the fuzzy discrimination measure given by (4.9), and we calculate $I_\alpha(Y, I)$. Then, the most effective l_j is determined by

$$I_t = \left[\min_{1 \le j \le n} \left\{ I_\alpha (Y, I) \right\} \right]_{0 < \alpha < 1} . \qquad (40)$$

It is assumed that $I_t (1 \le t \le n)$ determines the minimum value of $\left\{ I_\alpha (Y, I) \right\}_{0<\alpha<1}$. With this I_t find $\max_{1 \le i \le m} \left\{ \mu_{I_t} (S_i) \right\}$, let it correspond to $S_p \left(1 \le p \le m \right)$. Hence, if the strategy S_p is implemented with input budget of I_t the firm will meet its goal in the most input effective manner.

5.1 Numerical Illustration

Let $m = n = 5$ in the above model. Table 1 shows the efficiency of different strategies at uniform inputs. Table 2 illustrates the efficiency of different strategies at particular inputs and Table 3 the numerical values of discrimination measure

$$\left[\left\{ I_\alpha (Y, I) \right\}_{1 \le j \le n} \right]_{0<\alpha<1} .$$

Table 1: Efficiency of different strategies at uniform inputs

$\mu_Y(S_1)$	$\mu_Y(S_2)$	$\mu_Y(S_3)$	$\mu_Y(S_4)$	$\mu_Y(S_5)$
0.3	0.5	0.4	0.6	0.2

Table 2: Efficiency of different strategies at particular inputs

	$\mu_I(S_1)$	$\mu_I(S_2)$	$\mu_I(S_3)$	$\mu_I(S_4)$	$\mu_I(S_5)$
l_1	0.3	0.6	0.4	**0.7**	0.2
l_2	0.5	0.3	0.8	0.4	0.7
l_3	0.6	0.7	0.6	0.9	0.4
l_4	0.5	0.6	0.3	0.8	0.9
l_5	0.4	0.5	0.4	0.2	0.6

Table 3: Numerical values of discrimination measure

$$\left[\left\{ I_\alpha (Y, I) \right\}_{1 \le j \le n} \right]_{0<\alpha<1}$$

	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$
l_1	**0.0041885**	**0.0126504**	**0.0151538**	**0.0128018**
l_2	0.1180805	0.3591652	0.4293109	0.3587822
l_3	0.0694641	0.2131733	0.2591029	0.2216869
l_4	0.1421777	0.4548828	0.5662826	0.4871129
l_5	0.0745879	0.2263425	0.2704184	0.2262243

The calculated numerical data of the proposed fuzzy discrimination measure indicates that input l_1 is more suitable. The assessment of the results existing in Table 2 and Table 3 points out that strategy S_4 is most effectual. Thus a firm will achieve its goal most efficiently if the strategy S_4 is applied with an input l_1.

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