Vol. 4, No. 6 June 2014 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

Light Speed Anisotropy Constraints via Measurement of Relativistic Light Aberration

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ABSTRACT

The Mansouri-Sexl theory is a well known test theory of relativity. In the current paper we will derive the Mansouri-Sexl formalism for the light aberration and we will show how to improve on the theoretical and experimental basis by constraining the Mansouri-Sexl parameter "α". In the process of constraining the Mansouri parameter we devise a novel experiment for measuring and constraining light speed anisotropy as well. An overwhelming number of experiments dealing with light speed anisotropy are laboratory-bound, we take the approach of setting up an astronomical experiment instead of a lab-based one as it befits the relativistic aberration effect. Our paper is organized as follows: in the first section we give a new and more complete derivation of the Mansouri-Sexl aberration effect. In the second part, we apply the newly expanded Mansouri-Sexl aberration formalism in order to devise an astronomical experiment used for constraining the parameter "α". This turns the Mansouri-Sexl aberration experiment into a very powerful tool for constraining light speed anisotropy.

Keywords: *Mansouri-Sexl Test Theory, Relativistic Aberration, Light Speed Anisotropy*

1. THE ROBERTSON-MANSOURI-SEXL TEST THEORY

The test theories of special relativity differ in their assumptions about the form of the Lorentz transforms. The main test theories of special relativity (SR) are named after their authors, Robertson[1], Mansouri and Sexl[2,3,4] (RMS) .. These test theories can also be used to examine potential alternate theories to SR - such alternate theories predict particular values of the parameters of the test theory, which can easily be compared to values determined by experiments analyzed with the test theory. The existing experiments put rather strong experimental constraints on any alternative theory.
RMS starts by admitting by reduction to absurd that there is a preferential inertial frame □ in which the light propagates isotropically with the speed c_0. All other frames in motion with respect to □ are considered non-preferential and the light speed is anisotropic. The light speed in the non-preferential frames can be deduced via simple calculations described in[3]. We start with the Mansouri-Sexl transforms:

$$
\begin{aligned}
x &= b(v)(X - vT) \\
y &= d(v)Y \\
z &= d(v)Z \\
t &= a(v)T + \varepsilon(v)x = (a - b\varepsilon v)T + b\varepsilon X
\end{aligned}
\tag{1.1}
$$

where v is the relative speed between S and □ and (x,t) are the coordinates in S and (X,T) represent their correspondents in □. Inverting, we obtain:

=

$$
\begin{aligned}
X &= \left(\frac{1}{b} - \frac{v\varepsilon}{a}\right)x + \frac{v}{a}t \\
T &= \frac{t - \varepsilon x}{a} = -\frac{\varepsilon}{a}x + \frac{1}{a}t
\end{aligned}
\tag{1.2}
$$

Exactly as in the original Mansouri-Sexl paper we start with the line element that allows us to calculate the light speed in frame S as a function of the isotropic light speed in frame □:

$$0 = X^2 - c_o{}^2T^2 = \left(\left(\frac{1}{b} - \frac{v\varepsilon}{a}\right)^2 - \frac{c_0{}^2\varepsilon^2}{a^2}\right)x^2 + 2xt\left(\frac{v}{a}\left(\frac{1}{b} - \frac{v\varepsilon}{a}\right) + \frac{c_0{}^2\varepsilon}{a}\right) - t^2\frac{c_0{}^2 - v^2}{a^2} \tag{1.3}$$

$$\left(\left(\frac{1}{b} - \frac{v\varepsilon}{a}\right)^2 - \frac{c_0{}^2\varepsilon^2}{a^2}\right)\frac{x^2}{t^2} + 2\frac{x}{t}\left(\frac{v}{a}\left(\frac{1}{b} - \frac{v\varepsilon}{a}\right) + \frac{c_0{}^2\varepsilon}{a}\right) - \frac{c_0{}^2 - v^2}{a^2} = 0 \tag{1.4}$$

The anisotropic speed of light propagating along the x axis in S is:

$$c_{\pm} = \frac{-(v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon) \pm \sqrt{(v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon)^2 + (c_0{}^2 - v^2)((\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2)}}{(\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2} \qquad (1.5)$$

The two separate solutions exist for

$$v(\frac{a}{b} - v\varepsilon) + ac_0{}^2\varepsilon)^2 + (c_0{}^2 - v^2)((\frac{a}{b} - v\varepsilon)^2 - c_0{}^2\varepsilon^2 > 0 \qquad (1.6)$$

In SR $a(v) = \frac{1}{\gamma(v)}, b = \gamma(v), \varepsilon = -\frac{v}{c^2}$

so $c_+ = c_- = c_0$.

Exactly like in the original Mansouri-Sexl paper[4] by transforming the light cone $X^2 - c_o{}^2T^2 = 0$ into S we obtain

$$\frac{c(\theta)}{c_0} = 1 - \frac{v}{c_0}(1 + 2\alpha)\cos\theta \qquad (1.7)$$

Expression (1.7) is valid if slow clock transport synchronization has been used. According to Mansouri, the one-way light speed is a measurable quantity in this case and it is direction dependent for $\alpha \neq -0.5$. We will exploit this property in the Mansouri-Sexl theory of the aberration experiment constructed later in our paper.

2. THE RMS THEORY FOR ABERRATION

In his 1905 paper, "On the Electrodynamics of Moving Bodies", Einstein produces an interesting blueprint for deriving the general formula for the Doppler effect. He starts by considering a generic electromagnetic wave of phase □, frequency $\nu = \omega/2\pi$, and of wave-vector (l, m, n) propagating with speed c towards the origin O of a frame K. From the perspective of frame K , of coordinates (x,y,z,t) the phase is:

$$\Phi = \omega(t - \frac{lx + my + nz}{c}) : \qquad (2.1)$$

Let k be a system moving with the speed v along the positive x axis of frame K (see fig 1). We want to determine the form of the phase from the perspective of k, departing from the light source.

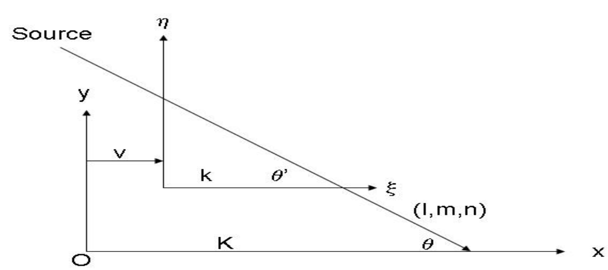

Fig 1: Reference frames K and k

Since K and k are in a translation motion along the x axis with respect to each other we replace the Lorentz transformations in Einstein derivation:

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$$\xi = \gamma(x - vt)$$
$$\psi = y$$
$$\zeta = z$$
$$\tau = \gamma(t - \frac{vx}{c^2})$$
$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

(2.2)

with the corresponding Mansouri-Sexl transforms described by (1.1). In (1.1) (X,Y,Z,T) represent the coordinates in the preferential frame □ while (x',y',z',t') are the corresponding coordinates in the lab frame S'. In the preferential frame □

$$\Phi = \Omega(T - \frac{lX + mY + nZ}{c})$$

(2.3)

In the lab frame S', where c' is assumed to be the (anisotropic) light speed, the phase ϕ' (all variables in frame S' are lowercase by convention) must have a form similar to the form (2.3) described in the preferential frame □:

$$\phi' = \omega'(a - b\varepsilon v + \frac{bvl'}{c'})(T - cb\frac{\frac{l'}{c'} - \varepsilon}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{X}{c} - \frac{dm'}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{Y}{c} - \frac{dn'}{a - b\varepsilon v + \frac{bvl'}{c'}}\frac{Z}{c}) \qquad (2.4)$$

In the lab frame, by comparing (2.4) with (2.3) we obtain the Mansouri-Sexl aberration formula and the Doppler shifted frequency:

$$l' = \frac{(a - b\varepsilon v)l + b\varepsilon c}{b(\frac{c}{c'} - \frac{vl}{c'})} = \frac{c'}{c}\frac{(a - b\varepsilon v)\cos\theta + b\varepsilon c}{b(1 - \frac{v}{c}\cos\theta)}$$

(2.5)

A quick sanity check shows that in SR $a(v) = \frac{1}{\gamma(v)}, b = \gamma(v), \varepsilon = -\frac{v}{c^2}, c' = c = c_0$, so

$$\cos\theta' = \frac{\cos\theta - \frac{v}{c_0}}{1 - \frac{v}{c_0}\cos\theta}$$

(2.6)

in perfect agreement with Einstein's derivation[16]. The above relationship is easily invertible, an effect that will come in handy later:

$$\cos\theta = \frac{\cos\theta' + \frac{v}{c_0}}{1 + \frac{v}{c_0}\cos\theta'}$$

(2.7)

3. THE EXPERIMENTAL DETERMINATION OF THE MANSOURI-SEXL PARAMETER "α"

The Mansouri-Sexl parameter ε encapsulates the clock synchronization convention. As such, ε can be expressed as a function of the other two Mansouri-Sexl parameters[2], "a(v)" and "b(v)":

$$\varepsilon_E = -\frac{\frac{v}{c^2}a(v)}{(1 - \frac{v^2}{c^2})b(v)} \qquad (3.1)$$

for "Einstein" clock synchronization and:

$$\varepsilon_T = \frac{1}{b(v)}\frac{da(v)}{dv} = \frac{2\alpha}{b}\frac{v}{c^2} \qquad (3.2)$$

for the case of slow clock "transport". It is known that the experimental results are invariant with respect to clock synchronization, so either method will produce the same result, in this particular experiment we will choose the slow clock transport scheme. The use of aberration measurement in determining light speed anisotropy has been hampered by the fact that the angle θ in (3.5-3.6) is not known. We can get around this difficulty by observing that what is really being measured is $\tan\theta' = \frac{v}{c(\theta)}$, (see fig. 2) thus, executing a set of experiments for varying positions of the Earth in its orbit around the Sun will produce the measurements necessary to constrict the Mansouri parameter.

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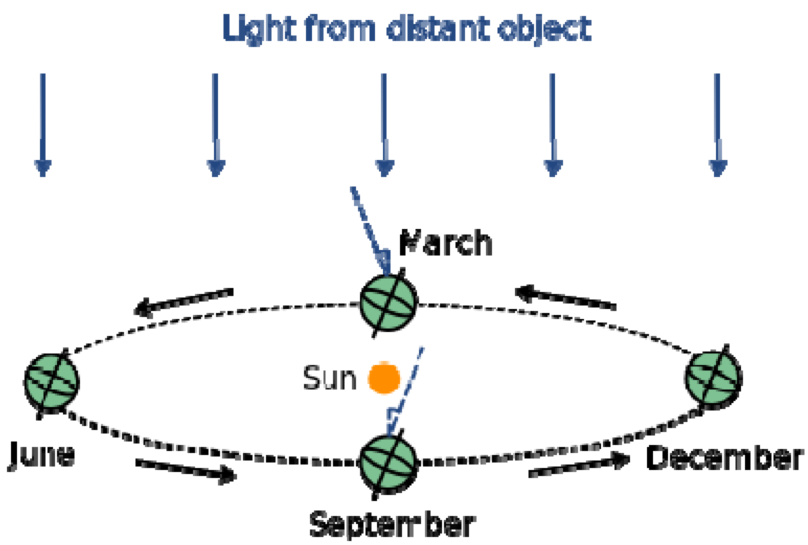

Fig 2: Aberration measurements during Earth yearly revolution

Indeed, from (2.7) we obtain that for such experiments:

$$\tan\theta' = \frac{\frac{v}{c_0}}{1-\frac{v}{c_0}(1+2\alpha)\cos\theta} \approx \frac{v}{c_0} + (\frac{v}{c_0})^2(1+2\alpha)\cos\theta \qquad (3.3)$$

Substituting (2.7) in (3.3) allows us to get rid of the unknown angle θ , leading to the expression:

$$\tan\theta' \approx \frac{v}{c_0} + (\frac{v}{c_0})^2(1+2\alpha)\frac{\cos\theta' + \frac{v}{c_0}}{1+\frac{v}{c_0}\cos\theta'} \approx \frac{v}{c_0} + (\frac{v}{c_0})^2(1+2\alpha)(1-\frac{v}{c_0}\cos\theta')(\cos\theta' + \frac{v}{c_0}) \qquad (3.4)$$

where θ' is the angle as measured in the lab frame S'. Note its presence on both sides of (3.4) as explained earlier. Dropping the higher order powers of $\frac{v}{c_0}$ allows us to further simplify the expression to:

$$\tan\theta' \approx \frac{v}{c_0} + (\frac{v}{c_0})^2(1+2\alpha)\cos\theta' \qquad (3.5)$$

Thus, measurements executed for the angle θ' throughout the year allow setting the boundaries for $1+2\alpha$. It is worth mentioning that the Earth revolution speed v varies according to the formula:

$$v(t) = v_s + v_e \sin[\Omega_y(t-t_0)]\cos\Phi_E + v_d \sin[\Omega_d(t+t_d)]\cos\Phi_A \qquad (3.6)$$

The laboratory velocity v(t) has contributions from the motion of the Sun with respect to frame Σ with a constant velocity $v_s = 377 km/s$, Earth's orbital motion around the Sun $v_e = 30 km/s$, and Earth's daily rotation v_d . For Berkeley, where the astronomy laboratory is located (latitude 37^{O} 52'18" N), $v_d = 0.355 km/s$. $\Phi_A \approx 8^o$ is the angle between the equatorial plane and the velocity of the Sun. $\Phi_E \approx 6^o$ is the declination between the plane of Earth's orbit and the velocity of the Sun, $2\pi/\Omega_y = 1yr$, $2\pi/\Omega_d = 1$ sidereal day.

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5. CONCLUSION

We derived the Mansouri-Sexl formalism for the light aberration and we demonstrated a means to improve on the theoretical and experimental basis such as to constrain the Mansouri-Sexl parameter "α" thus setting up a novel experiment for measuring and constraining light speed anisotropy. An overwhelming number of current experiments dealing with light speed anisotropy are laboratory-bound[21-24] , so we took the novel approach of setting up instead an astronomical experiment based on the relativistic aberration.

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