

Theoretical Prediction of Neutrino Mass

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ABSTRACT

In the Standard Model, all neutrinos are massless fermions. But from various recent experiments it is well-established that neutrinos undergo flavour oscillations, violating lepton flavour conservation and implying that neutrinos have non-zero mass. The measurement of the absolute neutrino masses is a fundamental unsolved problem in the particle physics till today. In this article, we calculate the mass of different neutrino flavor states which may be verified in future.

Keywords: *Neutrino oscillation; CP violation; neutrinoless double beta decay; models beyond the standard model*

1. INTRODUCTION

The successful discovery of Higgs boson [1,2] at the LHC brings a renewed perspective in particle physics. The Higgs boson is considered as the final particle of the Standard Model (SM) of particle physics. After the confirmation of Higgs boson [3], the SM appears to be the right theory at the electroweak scale with all its parameters and fields. The SM [4–6] of particle physics describes the electromagnetic, weak, and strong interactions of elementary particles. This model assumes three generations (or families) of quarks and three generations of leptons. Quarks are (top, bottom), (charm, strange) and (up, down). Leptons are (electron, electron-neutrino), (muon, muon-neutrino) and (tau, tau-neutrino). According to the SM, neutrinos are massless, color-neutral and charge-neutral spin-one-half (1/2) fermions [7–9]. They obey the Pauli's Exclusion Principle and Fermi-Dirac statistics. As per our knowledge, they only interact with charged fermions and massive gauge bosons through the weak interaction. All neutrinos are left-handed, and all antineutrinos are right-handed. The existence of non-zero neutrino mass was firmly established by the Super-Kamiokande Collaboration in 1998, and intensively studied subsequently. From various experiments on atmospheric, solar and terrestrial neutrinos it is now well-established that neutrinos undergo flavour oscillations, violating lepton flavour conservation and implying that neutrinos have non-zero mass. While a neutrino flavor state (say, a muon-type neutrino) travels a distance, it transforms into a different flavor state (say, an electron-type neutrino). This flavor-changing probability depends on the neutrino energy and the distance traversed between the source and the detector. The flavor states ν_e , ν_μ , and ν_τ are quantum mechanical mixtures of the mass eigenstates ν_1 , ν_2 , and ν_3 [10]. Experimental neutrino oscillation data indicate the possibility of CP violation in the lepton sector [11]. We still do not know how the neutrino masses are ordered: normal mass hierarchy (NMH) ($m_1 < m_2 < m_3$) or inverted mass hierarchy (IMH) ($m_3 < m_1 < m_2$). Again we do not know whether neutrinos are Majorana (particles are same with their antiparticles) or Dirac (particles are different from their antiparticles) particles. If neutrinos are

Majorana fermions, lepton number cannot be conserved. Conversely, lepton number violation indicates that massive neutrinos are Majorana fermions. The most sensitive probe of lepton number conservation is the pursuit of neutrinoless double beta decay $(A, Z) \rightarrow (A, Z + 2) + 2e^-$.

Neutrino oscillation experiments are sensitive to differences in m^2 , but do not measure the absolute neutrino masses. Till today we do not know the mechanism responsible for the generation of neutrino masses [11]. The origin of neutrino masses is one of the biggest puzzles in particle physics. In this article, we calculate the mass of different neutrino flavor states which may be verified theoretically as well as experimentally in future.

2. CALCULATION OF NEUTRINO MASS

The mass of neutrino can be calculated by using the formula [12,13]:

$$m_\nu = \frac{g_w^2}{2l_w c^2}, \quad (1)$$

where g_w is a kind of electron-neutrino 'charge' and l_w is a fundamental length for beta decay and defined as:

$$g_w^2 = G_F \left(\frac{m_e c}{\hbar} \right)^2, \quad (2)$$

$$l_w = \left(\frac{G_F}{\hbar c} \right)^{1/2} = 6 \times 10^{-17} \text{ cm}, \quad (3)$$

where m_e is the electron rest mass and G_F is the universal Fermi coupling constant, $G_F = 1.5 \times 10^{-49} \text{ erg.cm}^3$. Substituting the values of g_w and l_w from eqs. (2) and (3) in eq. (1), we get the electron-neutrino mass formula:

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$$m_{\nu_e} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_e^2. \quad (4)$$

Now we have the values of different parameters:

$$G_F = 1.5 \times 10^{-49} \text{ erg.cm}^3, \text{ The rest mass of electron } m_e = 0.51 \text{ MeV} = 0.51 \times 10^6 \times 1.6 \times 10^{-19} \times 10^7 \text{ erg}$$

$$= 8.16 \times 10^{-7} \text{ erg} = \frac{8.16 \times 10^{-7}}{9 \times 10^{20}} \text{ gm} \quad (\text{dividing by } c^2)$$

$$= 9.06 \times 10^{-28} \text{ gm and}$$

$$\hbar^3 = \frac{(6.63 \times 10^{-27})^3}{(2\pi)^3} \text{ erg}^3 \cdot \text{s}^3 = 1.175 \times 10^{-81} \text{ erg}^3 \cdot \text{s}^3$$

Using the above values in eq. (4), we get

$$m_{\nu_e} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_e^2$$

$$= \frac{1}{2} \sqrt{\frac{1.5 \times 10^{-49} \times 3 \times 10^{10}}{1.175 \times 10^{-81}}} \times (9.06 \times 10^{-28})^2 \text{ gm}$$

$$= 0.5 \times 1.956 \times 10^{21} \times 8.21 \times 10^{-55} \text{ gm} = 0.8 \times 10^{-33} \text{ gm}$$

$$= 0.8 \times 10^{-33} \times 9 \times 10^{20} \text{ erg} = \frac{0.8 \times 10^{-33} \times 9 \times 10^{20}}{1.6 \times 10^{-12}} \text{ ev} = 0.45 \text{ eV.} \quad (5)$$

Our calculated value of electron-neutrino mass consistent with the experimental upper limit [14] of $m_{\nu_e} < 2 \text{ eV}$ obtained from tritium beta decay at 95 % C. L.

Similarly, analogous to eq. (4) we can write the muon-neutrino mass formula as:

$$m_{\nu_\mu} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_\mu^2. \quad (6)$$

The muon mass $m_\mu = 105.4 \text{ MeV} = \frac{105.4 \times 1.6 \times 10^{-6}}{9 \times 10^{20}} \text{ gm} = 1.87 \times 10^{-25} \text{ gm}$. Now using the values of G_F , c , \hbar^3 and m_μ in eq. (6) we get,

$$m_{\nu_\mu} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_\mu^2$$

$$= \frac{1}{2} \sqrt{\frac{1.5 \times 10^{-49} \times 3 \times 10^{10}}{1.175 \times 10^{-81}}} \times (1.87 \times 10^{-25})^2$$

$$= 0.5 \times 1.956 \times 10^{21} \times 3.50 \times 10^{-50} \text{ gm}$$

$$= \frac{3.423 \times 10^{-29} \times 9 \times 10^{20}}{1.6 \times 10^{-6}} \text{ MeV}$$

$$= 0.1863 \text{ MeV.} \quad (7)$$

Our calculated value of muon-neutrino mass satisfies the upper limit [15] of $m_{\nu_\mu} < 2.2 \text{ MeV}$.

Similarly, analogous to eq. (4) we can write the tau-neutrino mass formula as:

$$m_{\nu_\tau} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_\tau^2 \quad (8)$$

The tau mass, $m_\tau = 1776.83 \text{ MeV}$

$$= \frac{1776.83 \times 1.6 \times 10^{-6}}{9 \times 10^{20}} = 3.16 \times 10^{-24} \text{ gm.}$$

Now using the values of G_F , c , \hbar^3 and m_τ in eq. (8) we get,

$$m_{\nu_\tau} = \frac{1}{2} \left(\frac{G_F c}{\hbar^3} \right)^{1/2} m_\tau^2$$

$$= \frac{1}{2} \sqrt{\frac{1.5 \times 10^{-49} \times 3 \times 10^{10}}{1.175 \times 10^{-81}}} \times (3.16 \times 10^{-24})^2$$

$$= 0.5 \times 1.956 \times 10^{21} \times 9.99 \times 10^{-48} \text{ gm}$$

$$= \frac{0.5 \times 1.956 \times 10^{21} \times 9.99 \times 10^{-48} \times 9 \times 10^{20}}{1.6 \times 10^{-6}} \text{ MeV}$$

$$= 5.48 \text{ MeV.} \quad (9)$$

CLEO [16] sets an upper limit on the mass of tau-neutrino $m_{\nu_\tau} < 30 \text{ MeV}$ at 95 % C. L. from the combined study of $\tau \rightarrow 5\pi\nu_\tau$ and $\tau \rightarrow 3\pi 2\pi^0\nu_\tau$. Our calculated value of tau-neutrino mass satisfies this upper limit. Our results [eqs. (5), (7) and (9)] may be verified theoretically as well as experimentally in future.

3. FURTHER DISCUSSIONS

Neutrino is an elementary particle. It has three flavors: electron-neutrino, muon-neutrino and tau-neutrino. All the three neutrinos undergo flavor oscillation, converts from one form to another, this

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oscillation is possible because neutrinos have non-zero mass. Neutrino has speed less than light implies that neutrino has mass, however tiny it may be. The existence of small neutrino mass is one of the signatures of physics beyond the SM. Till today, the absolute values of neutrino masses are not known [17,18]. In this article, we have calculated the mass of different neutrino flavor states. Our results [eqs. (5), (7) and (9)] may be verified theoretically as well as experimentally in future.

The fundamental nature of the neutrino is still not known, namely whether neutrino is its own antiparticle or not (Majorana or Dirac particle). This question can be answered by the “neutrinoless double beta decay”. The detection and study of neutrinoless double beta decay may clarify some problems of neutrino physics [19]: (i) lepton number non-conservation, (ii) neutrino nature: whether neutrino is a Dirac or Majorana particle, (iii) absolute mass scale of neutrino, (iv) the type of neutrino mass hierarchy (normal, inverted, or quasidegenerate), (v) CP violation in the lepton sector. There are several new neutrino experiments under construction and some are taking data. The aim of these experiments would be to study the neutrino oscillations, search for neutrinoless double beta decay, measurement of absolute neutrino mass, the nature of neutrinos and CP violation in the lepton sector. We hope the current generation of neutrino oscillation experiments, including Double Chooz, RENO, Daya Bay, T2K, and NOvA will resolve neutrino mass hierarchy problem and may provide a first signal for CP violation in the lepton sector.

Neutrinos with mass may be one of the candidates for the dark matter. Small neutrino masses are sensitive to new physics at scales ranging from a TeV up to grand unification and superstring scales. The very smallness of neutrino mass leads many theorists to believe that they provide a window on physics at much higher energies than our accelerator can reach. Neutrinos are important for the study of the sun, stars, core-collapse supernovae, origins of the cosmic rays, large-scale structure of the universe, big bang nucleosynthesis, and possibly baryogenesis. Solar neutrinos may tell us more about neutrino oscillations and other neutrino properties. Neutrinos from stellar core collapse can give information about astrophysical gravitational collapse. MeV geo-neutrinos from terrestrial radioactivity can be used to study about the earth. In the TeV and higher energy region, cosmic neutrinos dominate over atmospheric neutrinos [11]. These cosmic neutrinos can provide information about the origin of ultra-high-energy cosmic rays. Thus, the study of neutrino physics may connect many disciplines together, from particle physics to nuclear physics to astrophysics to cosmology.

What will happen to the Standard Model (new SM with nonzero neutrino masses, ν SM) if neutrinos do possess some mass? The answer is that we are not sure [11,20]. According to some scientists there are many different ways to modify the SM in order to accommodate neutrino masses. But which one is the correct ν SM? The

next-generation neutrino experiments may give the correct answer in future. Thus, neutrino physics continues to be a very exciting field and may also bring us new surprises in this 21st century.

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