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Spline Interpolations besides Widely used Growth Models for the Fork Length, Weight and Fork Length-Weight Relationships of Capoeta Capoeta Angorae Hanko 1924 from the Upper Water Systems of the River Ceyhan, Turkey

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ABSTRACT

In this study, the fork length (FL), weight (W) growth and the fork length-weight relationships of *Capoeta capoeta angorae Hanko* 1924 from the upper water systems of the River Ceyhan, Turkey were investigated. For this purpose, for growth curves of the fork length and weight, spline interpolation functions, alternative modeling passing to exactly all data points with respect to widely used models such as logistic and Gompertz models, applied to growth data set were be submitted and then for the fork length-weight relationships**,** spline interpolation functions with respect to the described model $w=m(FL)^n$ applied to fork length-weight data set were be submitted. These interpolation functions and growth models used were examined separately for female and male. These interpolation models used are linear spline, quadratic spline and cubic spline functions. The growth curves of the fork length and the weight of spline interpolation functions with widely used logistic and Gompertz models were shown on the same graph, respectively and then the curves of the fork length-weight relationships of spline interpolation functions and the models, $w=m(FL)^n$, were shown on the same graph. Thus, the differences in these models have been observed. The estimates for some intermediate values were done by using spline interpolations, logistic and Gompertz models. By using spline interpolations, the investigator is shown to obtain new ideas and interpretations in addition to the information of the well-known classical analysis.

Keywords: *Capoeta capoeta angorae Hanko, Spline Interpolations, Logistic model, Gompertz model*

1. INTRODUCTION

Growth parameters of fish populations are important data for fisheries management and vary among populations [1]. In this study, the fork length and weight growth of Capoeta capoeta angorae from the upper parts of the River Ceyhan from May 2000 to February 2001 [2] were investigated. For that reason, some growth models were used.

The data given such as (t_0,y_0), (t_1,y_1), …, (t_n,y_n) can be interpolated by a polynomial $P_n(t)$ of degree n or less so that the curve of $P_n(t)$ passes through these n+1 points. If n is a large, there may be trouble: $P_n(t)$ may tend to oscillate for t between the nodes $t_0,t_1,\ldots,t_n$. Hence we must be prepared for numeric instability [3]. Although polynomial interpolation is valid on non-equally spaced discrete grids, it may develop a polynomial wiggle. There exists an alternative method to overcome the limitations of polynomial interpolation. If the entire interpolation interval is decomposed into smaller intervals connected at the given data points, the degree of interpolating polynomials can be reduced to avoid the polynomial wiggle. This idea leads to the spline interpolation [4]. For that reason, **w**e can find a polynomial between each consecutive two data points. So we will get piecewise polynomials for all data set. As we know that if the lower degree polynomials are independent of each other, a piecewise approximation is obtained. An alternate approach is to fit a lower degree polynomial to connect each pair of data points and to require the set of lower degree polynomials to be consistent with each other in some sense. This type of polynomial is called a spline interpolation function or simply a spline [5].

Spline interpolation is preferred over polynomial interpolation because the interpolation error can be made small even when using low degree polynomials for the spline.

Splines can be of any degree. Linear splines straight line segments connected each pair of data points. Linear splines are independent of each other from interval to interval. Linear splines yield first order approximating polynomials. The slopes (i.e., first derivatives) and curvature (i.e., second derivatives) are discontinuous at every data point. Quadratic splines yield second order approximating polynomials. The slopes of the quadratic splines can be forced to be continuous at each data point, but the curvatures are still discontinuous. A cubic spline yields a third degree polynomial connecting each pair of data points. The slopes and curvatures of the cubic splines can be forced to be continuous at each data point. In fact, these requirements are necessary to obtain the additional conditions required to fit a cubic polynomial to two data points. Higher degree splines can be defined in a similar manner. However, cubic splines have proven to be a good comprise between accuracy and complexity [5]. Cubic spline functions are the most popular spline functions, for a variety of reasons. They are a smooth function with which to fit data; and when used for interpolation, they do not have the oscillatory behavior that characterizes high

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degree polynomial interpolation. Splines are also fairly simple to calculate and use [6]. Consequently, cubic spline functions with linear and quadratic spline functions are given in this manuscript.

Instead of applying one polynomial to all data set, in order to minimize errors, this model is based on applying many polynomials (linear, quadratic or cubic) to the data in corresponding intervals [7]

By using these spline interpolation functions, we could estimate for the intermediate values between 1+ year and 7+ years. Actually, cubic spline gives the best estimate value among the spline interpolation functions.

Some authors used cubic spline interpolation by using certain knot points in their study about lactation. These authors determined several knot points instead of all lactation days and then they used cubic spline function [8]. But that cubic spline function did not pass the observed average daily milk yield for the lactation days**.** Nowadays, it is not difficult to construct spline functions which pass through all the data set, since it is easy to make calculations using computers. Therefore, in this study, we worked on the spline functions passing through all the data set. Korkmaz [9] was used Lagrange and Newton interpolation functions passing through all the data set in his study. But, in this study, spline interpolation functions were used for growth models.

2. MATERIALS AND METHODS

2.1 Materials

In this study, the mean fork length and weight of the growth features of Capoeta capoeta angorae from the upper parts of the River Ceyhan from May 2000 to February 2001 were used [1]. For the presentation of the spline interpolation functions, the values of the fork lengths (cm) and weight (g) in the age groups with female and male of Capoeta capoeta angorae from the upper parts of the River Ceyhan from May 2000 to February 2001 in Table 1 in their articles were used in Table 1 in this study.

Table 1: The mean values of fork length (FL) (cm) and weight (W) (g) of Capoeta capoeta angorae from the upper parts of the River Ceyhan (M: Male, F: Female)

Age	**1+**		**2+**		**3+**		**4+**		**5+**		**6+**		**7+**	
Sex	M	F	M	F	M	F	M	F	M	F	M	F	M	F
FL	10.3	10.6	14.9	14.7	17.3	17.9	22.2	22.7	25.0	25.6	27.8	28.6	31.5	34.8
W	17.4	20.1	50.8	48.7	81.5	90.2	132.3	137.2	201.6	206.3	311.3	292.9	401.0	491.5

2.2 Methods

In this study, some classical growth models used are given in Table 2. The conditions for linear, quadratic and cubic spline interpolation fit are that a set of linear, quadratic and cubic spline interpolation through the data points was passed by using new linear, quadratic and cubic spline interpolation in each interval, respectively [10].

Table 2: Models used in the growth curves

Models	Equation
Logistic	y=a/(1+exp(b-ct))
Gompertz	y=a.exp(-exp(b-ct))
Relationship between Fork length and weight	$W=m(FL)^n$
where: y = the fork length (FL) (cm) or the weight (W) (g), t= time (year), exp(1)= e (the base of natural logarithm), a = the average asymptotic fork length or weight, m, n, b and c = the coefficients about growth	

Linear, quadratic and cubic spline interpolations are to be constructed to interpolate the equation (1). On each interval $[t_0,t_1],[t_1,t_2],\ldots,[t_{n-1},t_n]$, these spline interpolations are given by different linear, quadratic and cubic polynomials, respectively.

$$S(t)=\begin{cases} S_0(t) & t\in[t_0,t_1] \\ S_1(t) & t\in[t_1,t_2] \\ \ldots\ldots\ldots\ldots & \\ \ldots\ldots\ldots\ldots & \\ \ldots\ldots\ldots\ldots & \\ S_{n-1}(t) & t\in[t_{n-1},t_n] \end{cases} \qquad (1)$$

The polynomials S_{i-1} and S_i interpolate the same value at the point t_i and therefore the equation (2) is given

$$S_{i-1}(t_i)=y_i=S_i(t_i) \qquad (1\le i\le n-1) \qquad (2)$$

Hence, S is automatically continuous [11].

3. RESULTS

The functions of linear, quadratic and cubic splines in addition to logistic and Gompertz models were tested for the fork length and weight growth and their relationship of *Capoeta capoeta angorae Hanko* 1924 from the upper water systems of the River Ceyhan, Turkey. For that reason, by using Table 1, according to fork length, weight growth and their relationship, linear,

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quadratic and cubic splines for female and male were given in Tables 3, 4, 5, 6, 7 and 8, respectively and according to the fork length and weight growth, logistic and Gompertz models were given in Table 9 and then for the relationship between the fork length and weight growth, the functions were given in Table 10. It is known that the functions of linear, quadratic and cubic splines are piecewise polynomials. So the functions of linear, quadratic and cubic splines can be separately investigated for each interval. Furthermore, how to change these functions on each interval can be separately observed. But, the functions of logistic and Gompertz models do not give us separate results on each interval. These models give only general result. Furthermore, for each interval rate and turning point could be found by using the first and second derivatives of the spline interpolation functions, respectively. But, on each interval rate and turning point could not be found for the functions, logistic and Gompertz models. They only gives rate and turning point for all data set.

Table 3: Linear splines of the fork length and weight growth and fork length-weight relationship for female

Splines	$S_{1ff}(t)$ for fork length	$S_{1wf}(t)$ for weight	$S_{1rf}(FL)$ for fork length-weight relationship
Linear	$6.5+4.1t,\ 1\le t\le 2$	$-8.5+28.6t,\ 1\le t\le 2$	$-53.84+6.98FL,\ 10.6\le FL\le 14.7$
	$8.3+3.2t,\ 2\le t\le 3$	$-34.3+41.5t,\ 2\le t\le 3$	$-141.94+12.97FL,\ 14.7\le FL\le 17.9$
	$3.5+4.8t,\ 3\le t\le 4$	$-50.8+47.0t,\ 3\le t\le 4$	$-85.07+9.79FL,\ 17.9\le FL\le 22.7$
	$11.1+2.9t,\ 4\le t\le 5$	$-139.2+69.1t,\ 4\le t\le 5$	$-403.69+23.83FL,\ 22.7\le FL\le 25.6$
	$10.6+3.0t,\ 5\le t\le 6$	$-226.7+86.6t,\ 5\le t\le 6$	$-532.69+28.87FL,\ 25.6\le FL\le 28.6$
	$-8.6+3.0t,\ 6\le t\le 7$	$-898.7+198.6t,\ 6\le t\le 7$	$-623.22+32.03FL,\ 28.6\le FL\le 34.8$

where t is the age, $S_{1ff}(t)$, $S_{1wf}(t)$ and $S_{1rf}(FL)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of linear spline for female

Table 4: Linear splines of the fork length and weight growth and fork length-weight relationship for male

Splines	$S_{1fm}(t)$ for fork length	$S_{1wm}(t)$ for weight	$S_{1rm}(FL)$ for fork length-weight relationship
Linear	$5.7+4.6t,\ 1\le t\le 2$	$-16.0+33.4t,\ 1\le t\le 2$	$-57.39+7.26FL,\ 10.3\le FL\le 14.9$
	$10.1+2.4t,\ 2\le t\le$	$-10.6+30.7t,\ 2\le t\le 3$	$-139.80+12.79FL,\ 14.9\le FL\le 17.3$
	$2.6+4.9t,\ 3\le t\le$	$-70.9+50.8t,\ 3\le t\le 4$	$-97.86+10.37FL,\ 17.3\le FL\le 22.2$
	$11.0+2.8t,\ 4\le t\le$	$-144.9+69.3t,\ 4\le t\le 5$	$-417.15+24.75FL,\ 22.2\le FL\le 25.0$
	$11.0+2.8t,\ 5\le t\le$	$-346.9+109.7t,\ 5\le t\le 6$	$-777.86+39.18FL,\ 25.0\le FL\le 27.8$
	$5.6+3.7t,\ 6\le t\le$	$-226.9+89.7t,\ 6\le t\le 7$	$-362.66+24.24FL,\ 27.8\le FL\le 31.5$

where t is the age, $S_{1fm}(t)$, $S_{1wm}(t)$ and $S_{1rm}(FL)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of linear spline for male

Table 5: Quadratic splines of the fork length and weight growth and fork length-weight relationship for female

Splines	Quadratic
$S_{2ff}(t)$ for fork length	$\begin{cases} 10.60+6.10(t-1)^2, & 1\le t\le 1.5 \\ 6.28+4.21t-1.89(t-2)^2, & 1.5\le t\le 2.5 \\ 6.05+3.95t+1.63(t-3)^2, & 2.5\le t\le 3.5 \\ 6.33+4.09t-1.49(t-4)^2, & 3.5\le t\le 4.5 \\ 14.14+2.29t-0.31(t-5)^2, & 4.5\le t\le 5.5 \\ -5.91+5.75t+3.77(t-6)^2, & 5.5\le t\le 6.5 \\ 34.80-9.52(t-7)^2, & 6.5\le t\le 7.0 \end{cases}$
$S_{2wf}(t)$ for weight	$\begin{cases} 20.10+37.37(t-1)^2, & 1\le t\le 1.5 \\ -30.62+39.66t+2.29(t-2)^2, & 1.5\le t\le 2.5 \\ -37.14+42.45t+0.50(t-3)^2, & 2.5\le t\le 3.5 \\ -101.47+59.67t+16.73(t-4)^2, & 3.5\le t\le 4.5 \\ -113.41+63.94t-12.45(t-5)^2, & 4.5\le t\le 5.5 \\ -783.96+179.48t+127.99(t-6)^2, & 5.5\le t\le 6.5 \\ 491.50-307.46(t-7)^2, & 6.5\le t\le 7.0 \end{cases}$
$S_{2rf}(FL)$ for fork length-weight relationship	$\begin{cases} 20.10+1.97(FL-10.6)^2, & 10.6\le FL\le 12.7 \\ -124.09+11.75t+0.90(FL-14.7)^2, & 12.7\le FL\le 16.3 \\ -104.30+10.87t-1.18(FL-17.9)^2, & 16.3\le FL\le 20.3 \\ -268.07+17.85t+2.63(FL-22.7)^2, & 20.3\le FL\le 24.2 \\ -471.84+26.49t+0.35(FL-25.6)^2, & 24.2\le FL\le 27.1 \\ -677.06+33.91t+2.13(FL-28.6)^2, & 27.1\le FL\le 31.7 \\ 491.50-7.60(FL-34.8)^2, & 31.7\le FL\le 34.8 \end{cases}$
where t is the age, $S_{2ff}(t)$, $S_{2wf}(t)$ and $S_{2rf}(t)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of quadratic spline for female	

Table 6: Quadratic splines of the fork length and weight growth and fork length-weight relationship for male

Splines	Quadratic
$S_{2fm}(t)$ for fork length	$\begin{cases} 10.30+7.16(t-1)^2, & 1\le t\le 1.5 \\ 6.73+4.08t-3.07(t-2)^2, & 1.5\le t\le 2.5 \\ 6.82+3.49t+2.48(t-3)^2, & 2.5\le t\le 3.5 \\ 5.59+4.15t-1.82(t-4)^2, & 3.5\le t\le 4.5 \\ 13.07+2.39t+0.05(t-5)^2, & 4.5\le t\le 5.5 \\ 4.19+3.94t+1.50(t-6)^2, & 5.5\le t\le 6.5 \\ 31.50-5.43(t-7)^2, & 6.5\le t\le 7.0 \end{cases}$

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$S_{2wm}(t)$ for weight	$\begin{cases} 17.40+48.64(t-1)^2, & 1\le t\le 1.5 \\ -21.83+36.32t-12.33(t-2)^2, & 1.5\le t\le 2.5 \\ -34.01+38.50t+14.51(t-3)^2, & 2.5\le t\le 3.5 \\ -102.37+58.67t+5.65(t-4)^2, & 3.5\le t\le 4.5 \\ -247.89+89.90t+25.58(t-5)^2, & 4.5\le t\le 5.5 \\ -396.40+117.95t+2.47(t-6)^2, & 5.5\le t\le 6.5 \\ 401.0-120.42(t-7)^2, & 6.5\le t\le 7.0 \end{cases}$
$S_{2rm}(FL)$ for fork length-weight relationship	$\begin{cases} 17.40+1.85(FL-10.3)^2, & 0.3\le FL\le 12.6 \\ -128.52+12.04FL+0.77(FL-14.9)^2, & 12.6\le FL\le 16.1 \\ -115.33+11.38FL-1.04(FL-17.3)^2, & 16.1\le FL\le 19.8 \\ -257.23+17.55FL+2.30(FL-22.2)^2, & 19.8\le FL\le 23.6 \\ -635.32+33.48FL+3.39(FL-25.0)^2, & 23.6\le FL\le 26.4 \\ -725.86+37.31FL-2.02(FL-27.8)^2, & 26.4\le FL\le 29.7 \\ 401.0-8.06(FL-31.5)^2, & 29.7\le FL\le 31.5 \end{cases}$
where t is the age, $S_{2fm}(t)$, $S_{2wm}(t)$ and $S_{2rm}(t)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of quadratic spline for male	

Table 7: Cubic splines of the fork length and weight growth and fork length-weight relationship for female

Splines	Cubic
$S_{3ff}(t)$ for fork length	$\begin{cases} 6.11+4.49t-0.39(t-1)^3, & 1\le t\le 2 \\ 8.06+3.32t-1.17(t-2)^2+1.04(t-2)^3, & 2\le t\le 3 \\ 5.53+4.12t+1.97(t-3)^2-1.29(t-3)^3, & 3\le t\le 4 \\ 5.95+4.19t-1.90(t-4)^2+0.62(t-4)^3, & 4\le t\le 5 \\ 14.45+2.23t-0.05(t-5)^2+0.82(t-5)^3, & 5\le t\le 6 \\ 1.05+4.59t+2.41(t-6)^2-0.80(t-6)^3, & 6\le t\le 7 \end{cases}$
$S_{3wf}(t)$ for weight	$\begin{cases} -4.96+25.06t+3.54(t-1)^3, & 1\le t\le 2 \\ -22.66+35.68t+10.62(t-2)^2-4.80(t-2)^3, & 2\le t\le 3 \\ -37.35+42.52t-3.79(t-3)^2+8.27(t-3)^3, & 3\le t\le 4 \\ -101.81+59.75t+21.02(t-4)^2-11.68(t-4)^3, & 4\le t\le 5 \\ -127.55+66.77t-14.01(t-5)^2+33.84(t-5)^3, & 5\le t\le 6 \\ -548.69+140.27t+87.50(t-6)^2-29.17(t-6)^3, & 6\le t\le 7 \end{cases}$
$S_{3rf}(FL)$ for fork length-weight relationship	$\begin{cases} -29.96+4.72FL+0.13(FL-10.6)^3, & 10.6\le FL\le 14.7 \\ -120.08+11.48FL+1.65(FL-14.7)^2-0.37(FL-14.7)^3, & 14.7\le FL\le 17.9 \\ -100.75+10.67FL-1.90(FL-17.9)^2+0.36(FL-17.9)^3, & 17.9\le FL\le 22.7 \\ -252.68+17.18FL+3.26(FL-22.7)^2-0.33(FL-22.7)^3, & 22.7\le FL\le 25.6 \\ -502.35+27.68FL+0.36(FL-25.6)^2+0.01(FL-25.6)^3, & 25.6\le FL\le 28.6 \\ -569.22+30.14FL+0.46(FL-28.6)^2-0.02(FL-28.6)^3, & 28.6\le FL\le 34.8 \end{cases}$
where t is the age, $S_{3ff}(t)$, $S_{3wf}(t)$ and $S_{3rf}(t)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of cubic spline for female	

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Table 8: Cubic splines of the fork length and weight growth and fork length-weight relationship for male

Splines	Cubic
$S_{3fm}(t)$ for fork length	$\begin{cases} 4.89+5.41t-0.81(t-1)^3, & 1\le t\le 2 \\ 8.93+2.98t-2.42(t-2)^2+1.84(t-2)^3, & 2\le t\le 3 \\ 6.33+3.66t+3.10(t-3)^2-1.86(t-3)^3, & 3\le t\le 4 \\ 5.05+4.29t-2.47(t-4)^2+0.98(t-4)^3, & 4\le t\le 5 \\ 13.52+2.30t+0.48(t-5)^2+0.03(t-5)^3, & 5\le t\le 6 \\ 7.82+3.33t+0.56(t-6)^2-0.19(t-6)^3, & 6\le t\le 7 \end{cases}$
$S_{3wm}(t)$ for weight	$\begin{cases} -18.04+35.44t-2.04(t-1)^3, & 1\le t\le 2 \\ -7.83+29.31t-6.13(t-2)^2+7.52(t-2)^3, & 2\le t\le 3 \\ -37.32+39.61t+16.42(t-3)^2-5.23(t-3)^3, & 3\le t\le 4 \\ -94.77+56.77t+0.74(t-4)^2+11.79(t-4)^3, & 4\le t\le 5 \\ -266.54+93.63t+36.12(t-5)^2-20.05(t-5)^3, & 5\le t\le 6 \\ -323.02+105.72t-24.03(t-6)^2+8.01(t-6)^3, & 6\le t\le 7 \end{cases}$
$S_{3rm}(FL)$ for fork length-weight relationship	$\begin{cases} -34.38+5.03FL+0.11(FL-10.3)^3, & 10.3\le FL\le 14.9 \\ -123.94+11.73FL+1.46(FL-14.9)^2-0.42(FL-14.9)^3, & 14.9\le FL\le 17.3 \\ -116.14+11.42FL-1.58(FL-17.3)^2+0.28(FL-17.3)^3, & 17.3\le FL\le 22.2 \\ -223.08+16.01FL+2.52(FL-22.2)^2+0.22(FL-22.2)^3, & 22.2\le FL\le 25.0 \\ -677.98+35.18FL+4.33(FL-25.0)^2-1.04(FL-25.0)^3, & 25.0\le FL\le 27.8 \\ -662.96+35.05FL-4.38(FL-27.8)^2+0.39(FL-27.8)^3, & 27.8\le FL\le 31.5 \end{cases}$
where t is the age, $S_{3fm}(t)$, $S_{3wm}(t)$ and $S_{3rm}(t)$ are the values of the fork length (FL) (cm), weight (W) (cm) and fork length-weight relationship of cubic spline for male	

Table 9: Logistic and Gompertz models of the fork length (FL) and weight (W) growth

Models	Sex	Growth	Functions (y) and Their parameters (a,b,c)
Logistic	Female	FL	$53.91/(1+\exp(1.64-0.31t))$
		W	$7978.53/(1+\exp(5.97-0.46t))$
	Male	FL	$38.65/(1+\exp(1.34-0.39t))$
		W	$644.15/(1+\exp(3.84-0.62t))$
Gompertz	Female	FL	$81.47\exp(-\exp(0.83-0.14t))$
		W	$7316.69\exp(-\exp(1.91-0.13t))$
	Male	FL	$46.16\exp(-\exp(0.60-0.22t))$
		W	$1825.25\exp(-\exp(1.68-0.18t))$
where: exp(1)=e (the base of natural logarithm), t=the age and y=the value of fork length (cm) or weight (g)			

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Table 10: Fork length-weight relationship (FL-W)

Models	Sex	Weight function (W) with recpect to fork length (FL)
Relationship between fork length and weight	Female	$0.03(FL)^{2.76}$
	Male	$0.02(FL)^{2.87}$

By using Table 1, the estimates of linear spline, quadratic spline, cubic spline and logistic, Gompertz models for the fork length and weight growth were given in Table 11. The estimates of the average fork lengths of linear spline, quadratic spline and cubic spline are exactly same with the mean fork length and weight observed. So, the error sum of squares (SSE) of the spline interpolation functions for female and male are equal to zero. While logistic and Gompertz models for the fork length growth are slightly different from the mean data values, logistic and Gompertz models for the weight growth are abnormally different from the mean data values. For that reason, in this study the use of these models for weight growth are not recommended. Normally the curves of the functions or the models provide the best fit to the data in the sense that the sum of squares of the deviations is the smallest [12]. So this shows that spline interpolation functions are important to use in regression. Actually, we could say that the cubic spline interpolation function gave the best estimate value among the spline interpolation functions, logistic and Gompertz models.

Table 11: The observed and estimated fork length (FL) (cm) and weight (W) (g) according to linear spline, quadratic spline, cubic spline and logistic and Gompertz models for female (F), and male (M) with their Error Sum of Squares (SSE)

Models	Growth	Sex	1	2	3	4	5	6	7	SSE
Observed	FL	F	10.6	14.7	17.9	22.7	25.6	28.6	34.8	
		M	10.3	14.9	17.3	22.2	25.0	27.8	31.5	
	W	F	20.1	48.7	90.2	137.2	206.3	292.9	491.5	
		M	17.4	50.8	81.5	132.3	201.6	311.3	401.0	
Linear spline	FL	F	10.6	14.7	17.9	22.7	25.6	28.6	34.8	0
		M	10.3	14.9	17.3	22.2	25.0	27.8	31.5	0
	W	F	20.1	48.7	90.2	137.2	206.3	292.9	491.5	0
		M	17.4	50.8	81.5	132.3	201.6	311.3	401.0	0
Quadratic spline	FL	F	10.6	14.7	17.9	22.7	25.6	28.6	34.8	0
		M	10.3	14.9	17.3	22.2	25.0	27.8	31.5	0
	W	F	20.1	48.7	90.2	137.2	206.3	292.9	491.5	0
		M	17.4	50.8	81.5	132.3	201.6	311.3	401.0	0
Cubic spline	FL	F	10.6	14.7	17.9	22.7	25.6	28.6	34.8	0
		M	10.3	14.9	17.3	22.2	25.0	27.8	31.5	0
	W	F	20.1	48.7	90.2	137.2	206.3	292.9	491.5	0
		M	17.4	50.8	81.5	132.3	201.6	311.3	401.0	0
Logistic	FL	F	11.3	14.3	17.8	21.7	25.8	30.0	34.0	4.2
		M	10.8	14.1	17.8	21.6	25.2	28.4	31.1	2.1
	W	F	32.3	51.1	80.7	127.2	199.9	312.3	484.0	817.0
		M	24.7	44.6	78.4	132.3	209.3	304.5	403.0	210.3
Gompertz	FL	F	11.1	14.3	17.9	21.8	25.8	29.9	34.0	3.6
		M	10.6	14.2	17.9	21.6	25.1	28.3	31.2	1.7
	W	F	19.8	40.2	75.2	130.4	211.6	323.9	471.2	1746.3
		M	20.7	43.5	81.0	135.9	209.3	300.0	404.9	280.3

The fork length curves of spline interpolation functions and logistic and Gompertz models for female and male were given in the same graph Figures 1 and 2, respectively.

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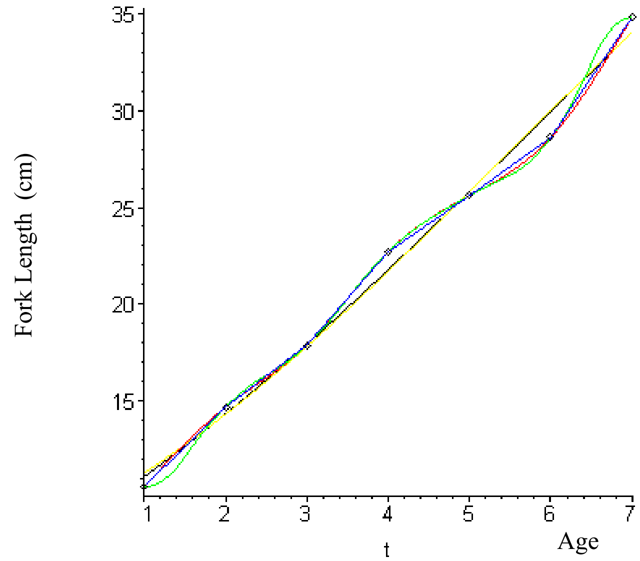
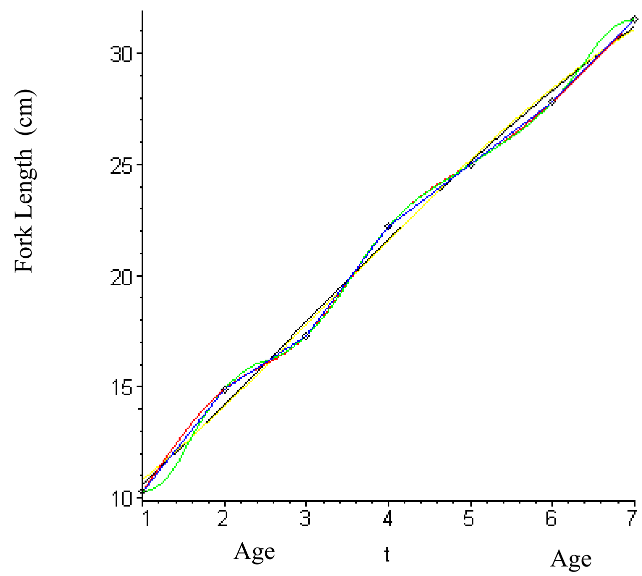


Colors of Linear, Quadratic, Cubic splines, logistic and Gompertz models for female: blue, green, red, yellow and black

Fig 1: Fork length of *Capoeta capoeta angorae Hanko* population for female according to the age



Colors of Linear, Quadratic, Cubic splines, logistic and Gompertz models for female: blue, green, red, yellow and black

Fig 2: Fork length of *Capoeta capoeta angorae Hanko* population for male according to the age

The weight curves of spline interpolation functions and logistic and Gompertz models for female and male were given in the same graph Figures 3 and 4, respectively.

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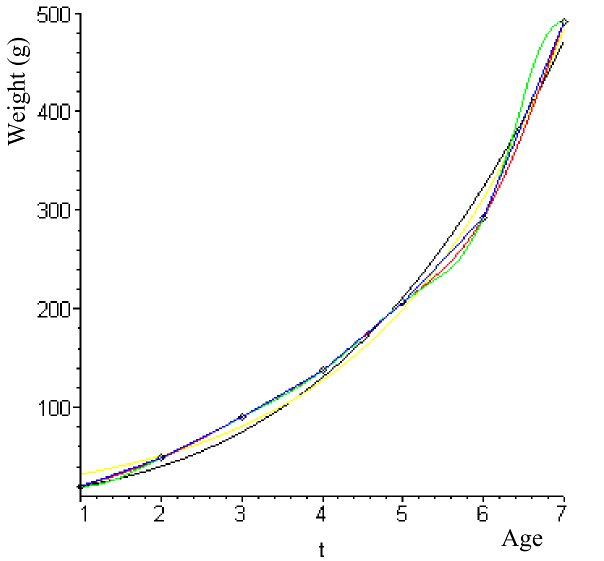
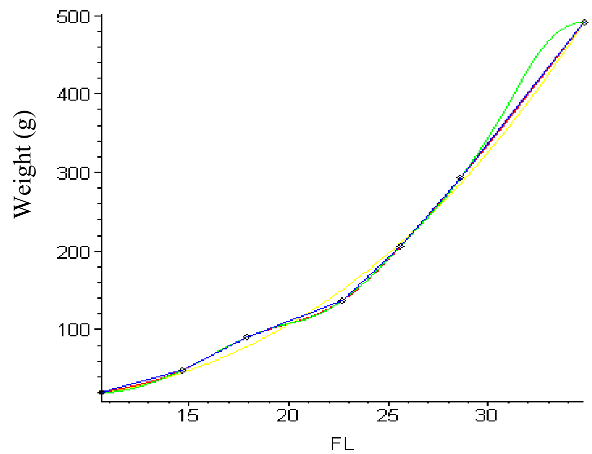


Colors of Linear, Quadratic, Cubic splines, logistic and Gompertz models for female: blue, green, red, yellow and black

Fig 3: Weight of *Capoeta capoeta angorae Hanko* population for female according to the age

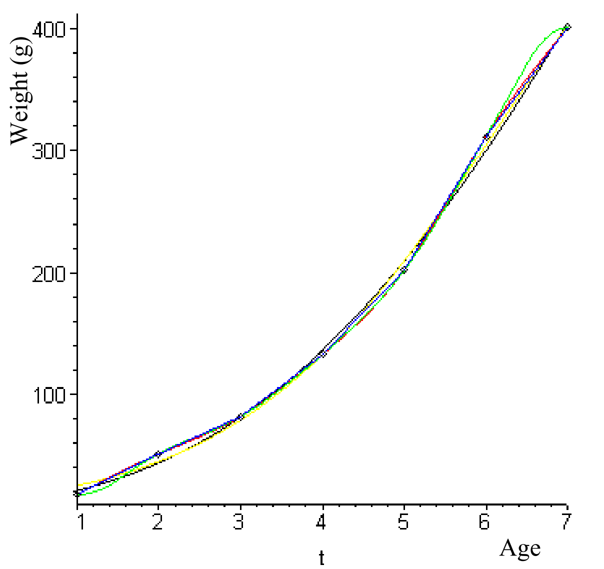
The curves of spline interpolation functions and the function about the fork length-weight relationship (FL-W) for female and male were given in the same graph Figures 5 and 6, respectively.



Colors of Linear, Quadratic, Cubic splines models and the function about the fork length-weight relationship for female: blue, green, red and yellow

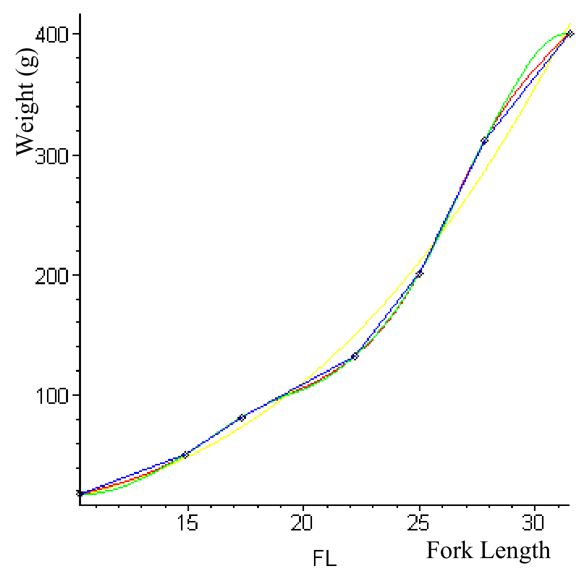
Fork Length

Fig 5: Weight of *Capoeta capoeta angorae Hanko* population for female according to the fork length



Colors of Linear, Quadratic, Cubic splines, logistic and Gompertz models for female: blue, green, red, yellow and black

Fig 4: Weight of *Capoeta capoeta angorae Hanko* population for male according to the age



Colors of Linear, Quadratic, Cubic splines models and the function about the fork length-weight relationship for female: blue, green, red and yellow

Fig 6: Weight of *Capoeta capoeta angorae Hanko* population for male according to the fork length

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By using the spline interpolation functions and logistic and Gompertz models, the estimates of fork length and weight for some intermediate ages were found in Table 12. Although we do not have a chance to check whether the values in Table 12 are correct or not, it could be said that the spline interpolations especially the cubic spline gave the better results for estimating the values of intermediate ages.

Table 12: Some estimates of fork length (FL) (cm) and weight (W) (g) with spline interpolation functions and logistic and Gompertz models for female (F) and male (M)

Models	Growth	Sex	Intermediate Age					
			1.5	2.5	3.5	4.5	5.5	6.5
Linear spline	FL	F	12.7	16.3	20.3	24.2	27.1	31.7
		M	12.6	16.1	19.8	23.6	26.4	29.7
	W	F	34.4	69.5	113.7	171.8	249.6	392.2
		M	34.1	66.2	106.9	167.0	256.5	356.2
Quadratic spline	FL	F	12.1	16.3	20.3	24.4	26.7	32.4
		M	12.1	16.2	19.7	23.8	26.2	30.1
	W	F	29.4	69.1	111.5	171.2	235.2	414.6
		M	29.6	65.9	104.4	163.0	252.9	370.9
Cubic spline	FL	F	12.8	16.2	20.3	24.4	26.8	31.4
		M	12.9	16.0	19.7	23.8	26.3	29.6
	W	F	33.1	68.6	111.5	170.9	240.4	381.3
		M	34.9	64.9	104.8	162.3	254.9	359.2
Logistic	FL	F	12.8	16.0	19.7	23.7	27.9	32.0
		M	12.4	15.9	19.7	23.4	26.9	29.8
	W	F	40.6	64.2	101.4	159.5	250.0	389.2
		M	33.3	59.4	102.5	167.9	255.4	354.5
Gompertz	FL	F	12.7	16.1	19.8	23.8	27.9	32.0
		M	12.4	16.1	19.8	23.4	26.7	29.8
	W	F	28.5	55.5	99.9	167.4	263.6	393.0
		M	30.5	60.2	106.1	170.3	252.6	350.9

4. DISCUSSION

Wypij *et al.* [13] reported that regression splines were used to describe the change in intercept and slope with age. Aggrey [14] compared three non-linear models (Richards, Gompertz and logistic) and the spline linear regression model by using an unselected chicken population. Kong and Yan [15] proposed using cubic smoothing splines to describe tumor growth for each treatment group and for each subject, respectively. They expressed that the proposed smoothing splines are quite flexible in modeling different growth patterns.

This study is similar to the study of Liu and Rousseau [16]. They reported that their paper aims to propose the use of spline functions for the description and visualization of discrete informetric data. They also expressed that interpolating cubic splines: are interpolating functions which pass through the given data points; have first and second derivatives in the data points, implying that they connect data points in a smooth way; satisfy a best-approximation property which tends to reduce curvature. They also reported that these properties are illustrated in the paper using real citation data. They explained that findings - the paper reveals that calculating splines yields a differentiable function that still captures small but real changes and then it offers a middle way between connecting discrete data by line segments and providing an overall best-fitting curve.

Since in our study different spline interpolation functions were used for each consecutive two data points, the possible measurement error will not affect the entire data set. Since according to the study of literature, spline interpolation functions as in this study were not used, a better fit to all data set could be said in this study.

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