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Shear Free Bianchi Type III String Cosmological Models with Bulk Viscosity and Time-Dependent Λ Term

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ABSTRACT

In this paper, we have examined non-shearing locally rotationally symmetric Bianchi Type - III String cosmological models in the presence of bulk viscosity and variable cosmological term Λ. To obtain a determine model of the universe, we have assumed that the coefficient of the viscosity ξ is inversely proportional to the expansion scalar θ and Λ is proportional to the Hubble parameter H. The physical and geometrical behaviors of the models are also discussed.

Keywords: *Bianchi type-III space time, bulk viscosity, cosmology.*

1. INTRODUCTION

Cosmology is a science developed in the beginning of twentieth century rapidly. The aim of cosmology is to determine the large scale structure of the physical universe. Cosmology is one of the greatest intellectual achievements of all time beginning from its origin. Cosmology, as a common man understands, is that branch of astronomy, which deals with the large scale structure of the universe. The present universe is both spatially homogeneous and isotropic. The basic problem in cosmology is to find the cosmological models of universe and to compare the resulting models with the present day universe using astronomical data. In most treatments of cosmology, cosmic fluid is considered as perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of an expanding universe.

In the last few years the study of cosmic strings has attracted considerable interest as they are believed to play an important role during early stages of the universe. The idea was that particles like the photon and the neutron could be regarded as waves on a string. The presence of strings in the early universe is a by product of Grant Unified Theories (GUT). Cosmic strings have stress energy and coupled in a simple way to the gravitational field. The general relativistic treatment of cosmic strings has been originally given by Letelier [1] and Stachel. [2]

It appears that after the 'Big-Bang' the universe may have experienced a number of phase transitions. These phase transitions can produce vacuum domain structures such as domain walls, cosmic strings and monopoles. Letelier and Verdaguer [3] studied a new model of cloud formed by massive strings in the context of general relativity. They have considered the Bianchi type-I model as they are supported to be reasonable representation of the early universe.

Krori et al. [4, 5] studied the problem of cosmic strings taking Bianchi types I, II, III, V, VI, VIII and IX space-times and observed that the universe was dominated by massive strings. Roy and Banerjee [6] have investigated some LRS Bianchi type II string cosmological models which represent geometrical and massive strings. Some cosmological solutions of massive strings for Bianchi type I space-time in presence and absence of magnetic field have investigated by Banerjee et al [7]. Super string cosmology with the cosmological implications of duality symmetries have discussed by Lidsey et al [8] . Bali et al. [9-13] have a investigated Bianchi Types I, V, IX string cosmological models in General Relativity. Wang [14-17] has investigated and discussed some cosmological models and their physical implications in some Bianchi Type space-times.

At the early stages of the evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation. Murphy [18] constructed isotropic homogeneous spatially-flat cosmological model with a fluid containing bulk viscosity alone because the shear viscosity cannot exist due to assumption of isotropy. He observed that the 'Big-Bang' singularity of finite past may be avoided by introduction of bulk viscosity.

Padamanabhan and Chitre [19] have shown that the presence of bulk viscosity leads to inflationary like solutions in general relativity. Mohanty and Pradhan [20] extended the work of Murphy [18] by considering the special law of variation for Hubble's parameter presented by Berman [21] and solved Einstein's field equations when the universe is filled with viscous fluid. Pradhan and Pandey [22] have investigated an LRS Bianchi type-I models with bulk viscosity in the cosmological theory based on Lyra's geometry. Bali and Dave[23] investigated the Bianchi type-III string cosmological model with bulk viscosity. Recently Bali and Pradhan [24] investigated the Bianchi type-III string cosmological model with time dependent bulk viscosity.

In 1917, Einstein introduced the cosmological constant into his field equations in order to obtain a static cosmological model since his equations without the cosmological constant admitted only non static solutions. In general relativistic quantum field theory, the cosmological constant is explained as the vacuum energy density obtained by Zel'dovich [25, 26]. Bergmann [27] has studied the cosmological constant in terms of the Higgs scalar field. Linde [28] proposed that the term Λ is a function of temperature and is related to the process of broken symmetries.

In modern cosmological theories the cosmological constant Λ remains a focal point of interest. The cosmological models without the cosmological constant are unable to explain satisfactorily problems like structure formation and the age of the universe is explained by Singh et al. [29]. Recent interest in the cosmological constant term Λ has received considerable attention among researchers for various concepts. Other researchers like Zeldovich [30], Bertolami [31,32], Ozer and Taha [33], Weinberg [34], Carroll et al. [35], Carlberg et al. [36], Friemann and Waga [37] and Pradhan et al. [38] investigated more significant cosmological models with cosmological constant Λ. Ratra and Peebles [39] discussed in detail the cosmological constant problem and cosmology with a time-varying cosmological constant.
In this paper, we have investigated non-shearing locally rotationally symmetric (LRS) Bianchi type - III string cosmological models in the presence of bulk viscosity. To obtain a determine model we assume that the coefficient of the viscosity is inversely proportional to the expansion scalar and for the cosmological term Λ, we assume that it is proportional to the Hubble parameter $\Lambda \propto H$.

2. FILED EQUATIONS OF STRING COSMOLOGICAL MODEL WITH BULK VISCOSITY

The Bianchi type-III space-time metric we consider here is

$$ds^2 = -dt^2 + A^2 dx^2 + B^2\left(e^{2x} dy^2 + dz^2\right), \qquad (1)$$

where A and B are functions of time t alone.

The energy-momentum tensor for a cloud of string with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta\left(u_i u_j + g_{ij}\right), \qquad (2)$$

where u_i and x_i satisfy the relations

$$u^i u_i = -x^i x_i = -1, \qquad u^i x_i = 0 \qquad (3)$$

Here, $\theta = u^i{}_{;i}$ is the scalar of expansion, ξ is the coefficient of bulk viscosity, λ is the rest energy density of particles, Letelier[1] assumed that the energy density for the coupled system ρ and ρ_p is positive, while the tension density λ may be positive or negative. Here u^i is the cloud four-velocity vector and ξ^i represent the direction of anisotropy, i.e., the direction of strings. If the particle density of the configuration is denoted by ρ_p then we have

$$\rho = \rho_p + \lambda \qquad (4)$$

The expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i{}_{,i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} = 3H, \qquad (5)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{2\dot{A}\dot{B}}{AB}\right), \qquad (6)$$

where H is Hubble parameter.

The Einstein field equations with gravitational units 8□G = 1 and variable cosmological term $\Lambda(t)$ in suitable units are

$$R_{ij} - \frac{1}{2}R g_{ij} = T_{ij} - \Lambda(t) g_{ij}, \qquad (7)$$

For the metric (1), Einstein's field equations can be written as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \xi\theta - \Lambda, \qquad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \xi\theta - \Lambda, \qquad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \lambda + \xi\theta - \Lambda, \qquad (10)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \rho + \Lambda, \qquad (11)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0, \qquad (12)$$

where dots on A and B represent the ordinary differentiation with respect to t.

From equation (12), we have

$$A = mB, \qquad (13)$$

where m is an integrating constant.

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From equation (13), without loss of generality we can take $m = 1$,

$$A = B, \quad (14)$$

To obtain the determinate model of the universe, we assume that the coefficient of bulk viscosity ξ is inversely proportional to the expansion scalar θ. This condition leads to

$$\xi\theta = K \quad (15)$$

where K is a proportionality constant. Therefore

$$\theta = \frac{3\dot{B}}{B} = 3H, \quad (16)$$

Using Eq. (14), Eqs. (8)-(11) become

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \xi\theta - \Lambda, \quad (17)$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = \lambda + \xi\theta - \Lambda, \quad (18)$$

$$\frac{3\dot{B}^2}{B^2} - \frac{1}{B^2} = \rho + \Lambda, \quad (19)$$

Now there are four independent equations (17)–(19) in five unknown parameters B, ξ, Λ, ρ and λ. Therefore, to solve the system of equations completely we use one more condition

$$\Lambda = aH, \quad (20)$$

Substituting Eqs (15) and (20) in equation (18) we get,

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = K - aH,$$

$$\Rightarrow \frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = K - a\frac{\dot{B}}{B},$$

$$\Rightarrow \frac{\ddot{B}}{\dot{B}} = \frac{1}{2}\left(K\frac{B}{\dot{B}} - a - \frac{\dot{B}}{B}\right), \quad (21)$$

Now integrating equation (21) we get,

$$\dot{B} = m_1 e^{\frac{-at}{2}} B^{-\left(\frac{K+1}{2}\right)}, \quad (22)$$

Again integrating, we obtain

$$B = \left(\frac{K+3}{2}\right)^{\frac{2}{K+3}} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{\frac{2}{K+3}}, \quad (23)$$

where m_1 and m_2 are constants of integration.

Thus

$$A = \left(\frac{K+3}{2}\right)^{\frac{2}{K+3}} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{\frac{2}{K+3}}, \quad (24)$$

Therefore the line element (1) reduce to

$$ds^2 = -\mathrm{dt}^2 + \left[\left(\frac{K+3}{2}\right)\left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)\right]^{\frac{4}{K+3}} \left(dx^2 + e^{2x} dy^2 + dz^2\right), \quad (25)$$

$$\Rightarrow \frac{\dot{B}}{B} = e^{\frac{-at}{2}} m_1 \left[\left(\frac{K+3}{2}\right)^{-1} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{-1}\right], \quad (26)$$

For the model of equation (23), the other physical and geometrical parameters can be easily obtained. The expressions for the energy density ρ, the string tension density λ, the scalar of expansion θ, Hubble parameter H, cosmological term Λ and the shear scalar σ^2 are respectively given by

$$\rho = 3{m_1}^2 e^{-at} \left(\frac{K+3}{2}\right)^{-2} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{-2}, \quad (27)$$

$$\lambda = -\left[\left(\frac{K+3}{2}\right)^{\frac{-4}{K+3}} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{\frac{-4}{K+3}}\right], \quad (28)$$

$$\theta = 3\, e^{\frac{-at}{2}} m_1 \left[\left(\frac{K+3}{2}\right)^{-1} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2\right)^{-1}\right], \quad (29)$$

$$H = 9\, e^{\frac{-at}{2}} m_1 \left[\left(\frac{K+3}{2} \right)^{-1} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2 \right)^{-1} \right], \quad (30)$$

$$\Lambda = 9a\, e^{\frac{-at}{2}} m_1 \left[\left(\frac{K+3}{2} \right)^{-1} \left(-\frac{2}{a} e^{\frac{-at}{2}} m_1 + m_2 \right)^{-1} \right],$$

$$\sigma^2 = 0$$

$$\Rightarrow \sigma = 0 \quad (31)$$

3. DISCUSSION

We have studied a new exact solution of Einstein's field equations for Bianchi type-III space-time with cloud of string in presence of bulk viscosity. We adopt an assumption that the coefficient of bulk viscosity ξ is inversely proportional to the expansion scalar θ. The model describes expanding, non-shearing and non rotating universe with a big bang start. Furthermore, the physical and geometrical aspects of the model are also discussed. As the time t increase, the rate of expansion θ decreases and $\sigma = 0$ implies that shear scalar does not exist. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$, therefore the model isotropic for large value of t.

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