The Apparent Velocity and Acceleration of Relativistically Moving Objects


Adrian Sfarti
387 Soda Hall, U.C. Berkeley, CA

ABSTRACT

The authors derive a formula for the perceived position $x_p$ of a relativistically moving object as a function of their retarded position $x'_p$ at the time of emitting a quantum of light. We show that the results are incorrect.

Keywords: apparent relativistic speed, apparent relativistic acceleration, relativistically moving objects

1. COMMENT

There are several errors in the paper:

a. The authors derive formulas (4) and (5):

$$x_p = \frac{vt}{1 \pm \frac{v}{c}}$$

For example, in the case of the particle approaching the observer, the authors conclude that:

$$x_p = \frac{vt}{c}$$

Based on (4) and (5) the authors proceed to the incorrect conclusion that "As $v \to c$, the apparent velocity $v_p = \frac{dx_p}{dt}$ approaches $c/2$ for the receding particle and approaches infinity for the approaching particle."

As we can see from fig.1, the correct derivation produces entirely different formulas and different conclusions. The particle is located in P when a quanta of light is emitted reaching the observer in O after an elapsed time $t$. When the particle has reached $P'$ traveling the same elapsed time $t$ but with the speed $v < c$, a second quanta of light is emitted, reaching the observer in O after an elapsed time $\tau$. The distance between O and P is what the authors labeled as $x_p$. The equations of motion are very simple:

$$x_p = vt + ct$$

$$x_p = ct$$

Eliminating $t$, we obtain:

$$x_p = \frac{ct}{1 - \frac{v}{c}}$$

Now, when $v \to c$ we can see that $\tau \to 0$ so $x_p = ct$, a finite value, and not $x_p \to \infty$. 

![Fig 1](image-url)
b. The authors derive the formula (10):

\[ x_p = \gamma (x'_p + \gamma vt) - \left( \frac{\gamma v}{c} \right)^2 d \pm \]

\[ \pm \sqrt{\left( \frac{\gamma v}{c} \right)^2 ((x'_p + \gamma vt)^2 + d^2 + (y'_p - e)^2 + (z'_p - f)^2) + \left( \frac{\gamma^2 v}{c} \right)^2 \left( \frac{vd}{c} \right)^2 - 2vt - 2 \frac{x'_p d}{\gamma}} \]

where \((x'_p, y'_p, z'_p)\) are the coordinates of the moving particle in the observer frame, \((x'_p, y'_p, z'_p)\) are the coordinates for the emission of the light pulse in the rest frame of the emitter with this frame moving with a velocity \(v\) in the \(x\) and \((d, e, f)\) are the coordinates of observer in its own (proper) frame. A simple test of the formula (10) shows that it is incorrect. We have shown that a circle moving at relativistic speed transforms in a family of ellipses moving with speed \(v\) and contracted by the factor \(\gamma\) in the direction of motion. If we set \((d, e, f) = (0,0,0)\) for simplicity, (10) becomes:

\[ x_p = \frac{1 \pm v/c}{\sqrt{1-mv/c}} (x'_p + \gamma vt) \]

i.e. the circle is neither moving at \(v\) nor is it contracted by the factor of \(\gamma\) in the direction of motion.

c. Since (10) is incorrect, it is obvious that (12), as a consequence of (10) is incorrect as well. Again, the correct formulas can be found in prior literature.

2. CONCLUSION

We have shown that incorrect assumptions as to how objects moving at relativistic speeds are visualized led to incorrect formulas.

REFERENCES

