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Characteristics of Quantum Noise in Semiconductor Lasers Operating in Single Mode

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ABSTRACT

The performance of a semiconductor laser is adversely affected by the presence of noise. A self-consistent numerical model is applied to analyze the intensity noise of 850 nm GaAs laser. A systematic technique has been demonstrated to introduce the Langevin noise sources on the photon number. The time varying profile of the fluctuating photon number and carrier number are analyzed. The frequency spectrum of the intensity noise is calculated with the help of Fast Fourier Transform (FFT). Transient behavior of semiconductor laser is also described that is significant in determining the noise characteristics of the laser output. In this paper, we aim to study the characteristics of quantum noise of semiconductor lasers operating in single mode since quantum noise is an intrinsic property of the semiconductor lasers and impossible to control in principle. The rate equations for photon numbers and carrier numbers for semiconductor laser operating in single mode are obtained by considering self-suppression coefficient. The parameters of the rate equations for GaAs are obtained and the noise effect is described through numerical simulation of the relative intensity noise (RIN). Photon number and carrier number variations with injection current are also demonstrated. Correspondence between this simulation results with practical data is also demonstrated.

Keywords: *Quantum noise, semiconductor laser, Langevin noise, relative intensity noise, self-suppression coefficient, laser rate equation.*

1. INTRODUCTION

Among optoelectronic devices semiconductor, lasers are the most important. In the four decades since the development of the first laser diode, remarkable achievements have been made so far. Today, various types of semiconductor lasers are mass-produced and widely used as coherent light sources for a variety of applications, including optical fiber communication systems and optical disk memory systems [1]. Quantum-mechanical effects often set the limits for optical systems. This is basically because of owing to the high optical frequencies the photon energy in the optical domain. Early calculations of noise were based on small-signal analysis. This concept was developed by McCumber [2] and was applied to semiconductor lasers by Haug [3]. Linearization of the rate equations following the small signal approximation brings about the analytical treatment which was applied in most of the previous calculations [4]-[11]. However, information concerning the instantaneous fluctuations of the photon and carrier numbers was missed in such small signal calculations. Direct numerical integration of the rate equations has been applied to overcome the limitations of the small signal analysis [12]-[22].

The theoretical model used in this paper for numerical simulation and analysis of quantum noise is the rate equation model. The model has been applied to a solitary 850 nm GaAs laser assuming that only the fundamental transverse mode exists. The rate equations of the modal photon number $S_p(t)$ and number of injected electrons N(t) are used to study the modal dynamics and quantum noise characteristics of semiconductor lasers. In order to include fluctuated spontaneous emission, a random term known as Langevin noise source has been added to the rate equation for photon number. Frequency spectra of the intensity noise are calculated with the help of the fast Fourier transform (FFT).

The next section lays down the theoretical model of our analysis. This section forms the basis of the work to be done in this paper. It talks about the rate equations for photon number and carrier number and also about the introduction of noise source for photon number. Then we move on to the development of the algorithm for numerical simulation in Section III. Next in section IV, we discuss the various results of the numerical simulation for the 850 nm GaAs laser and analyze them. Lastly, we conclude this paper with some concluding remarks on the results in the last section.

2. THE RATE EQUATION MODEL

2.1 Laser Rate Equations

The laser rate equations can be stated as follows. [23]

For photon number:

$$\frac{dS}{dt} = (G - G_{tho})S + \frac{a\xi N}{V} + F_S(t) \qquad (1)$$

For injected carrier (electron) number:

$$\frac{dN}{dt} = -AS - \frac{N}{\tau_S} + \frac{I}{e} \qquad (2)$$

where G=A-BS is the gain of single mode laser with wavelength λ,

G_{tho} is the threshold gain of solitary laser given by

$$G_{tho} = \frac{c}{n_r}\left[k + \frac{1}{2L}\ln\frac{1}{R_f R_b}\right] \tag{3}$$

The noise source in this model is $F_s(t)$, which is considered to be a Langevin noise source,

$$F_s(t) = \sqrt{\frac{V_{SS}}{\Delta t}}\, g_S \tag{4}$$

This Langevin noise source is considered to be a function in inducing instantaneous fluctuations in photon number due to spontaneous emission and recombination process. $F_s(t)$ can be approximated as a Gaussian distribution with a zero mean value. The photon number S(t) and electron number N(t) suffers sufficient fluctuation due to this noise source.

The other parameters are

A is the linear gain given by

$$A = \frac{a\xi}{V}(N - N_g) \tag{5}$$

B is the self-suppression coefficient written as

$$B = \frac{9}{4}\frac{\hbar\omega}{\varepsilon_0 n_r^2}\left(\frac{\xi\tau_{in}}{\hbar V}\right)^2 a R_{cv}^2 (N - N_S) \tag{6}$$

$$V_{SS} = \left[\frac{a\xi}{V}(N + N_g) + G_{tho}\right]S + \frac{a\xi N}{V} \tag{7}$$

In the above equations, it has been considered that a is the differential gain coefficient, ξ is the field confinement factor, V is the volume of the active region, τ_s is the injected carrier (electron) lifetime, I is the injection current, e is the electron charge and k is the internal loss in the laser cavity.

In (4), g_s is the Gaussian random variable in the range of

$$-1 \le g_s \le 1 \tag{8}$$

The Gaussian random variable is generated using the Box-Muller transformation [24] method in which two uniformly distributed random numbers u_1 and u_2 are taken in the range between -1 to +1. The following equation is then used to obtain g_s.

$$g_s = \frac{1}{5}\left(\sqrt{-2\log u_1}\cos(2\pi u_2)\right) \tag{9}$$

Other parameters are: N_g is the electron number at transparency, ħ is the reduced Planck constant, b is the width of the linear gain coefficient, τ_{in} is the intra-band relaxation time, R_{cv} is the dipole moment and N_s is the electron number characterizing the self-suppression coefficient.

2.2 Output Power of Semiconductor Laser

The output power P(t) from the front facet of the semiconductor lasers is given by

$$P(t) = \frac{h\nu c}{2n_r L}\frac{\ln(1/(R_f R_b))(1 - R_f)}{(1 - \sqrt{R_f R_b})(1 + \sqrt{R_f R_b})}S(t) \tag{10}$$

where c is the speed of light in vacuum and hν is the photon energy of emitted light.

The lasing power fluctuation may be defined as

$$\delta P(t) = P(t) - \overline{P} \tag{11}$$

where $\overline{P}$ is the time average laser power.

2.3 Intensity Noise Calculation

In this analysis, the time fluctuating components are transformed into Fourier frequency components. The noise is then calculated from the Fourier frequency components.

The numerical approach used here is somewhat different from that used by McCumber and Haug. In this analysis, the relative intensity noise (RIN) is calculated from the laser power fluctuations $\delta P(t) = P(t) - \overline{P}$ that can be obtained from the time integration of the rate equations for photon number and carrier number. The RIN spectrum can be originally defined as the Fourier transform of the autocorrelation function written as,

$$RIN = \frac{1}{\overline{P}^2}\int_0^\infty \delta P(t)\delta P(t+\tau)e^{j\omega\tau}d\tau \tag{12}$$

3. NUMERICAL SIMULATION

Our aim here is to obtain the instantaneous photon number S(t), carrier number N(t) and corresponding relative intensity noise through numerical simulation. The typical values for GaAs laser parameters considered for numerical simulation are listed in Table 1.

The fourth-order Runge-Kutta algorithm has been used to solve the rate equations to obtain the results. For the numerical integrations, a short time interval of Δt = 5 ps has been used. Such a small value of Δt produces noise sources that can approximately describe a white noise spectrum up to a frequency of 200GHz. This is so taken such that the behavior of the laser both before and after relaxation frequency can be studied. Extending the integration to a time period as long as 2.5μs allows us to demonstrate the intensity noise as low as 400kHz.

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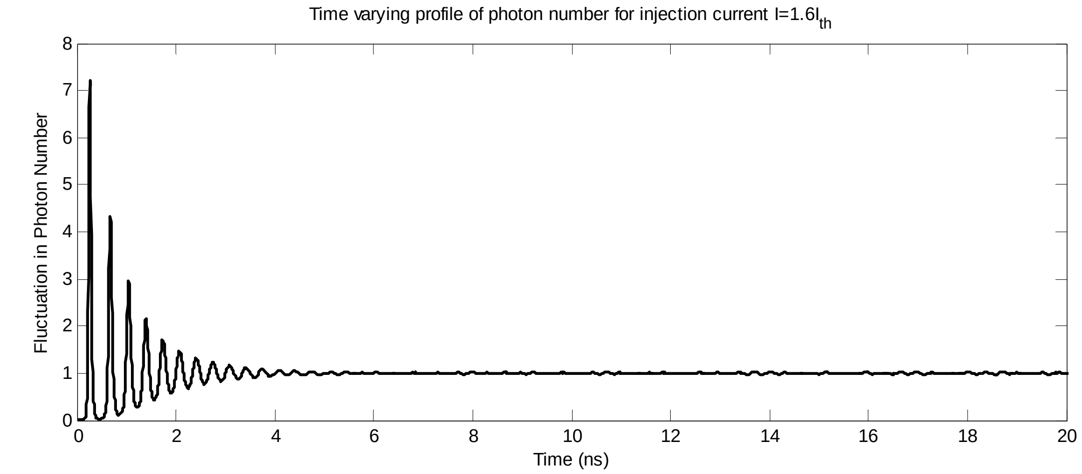
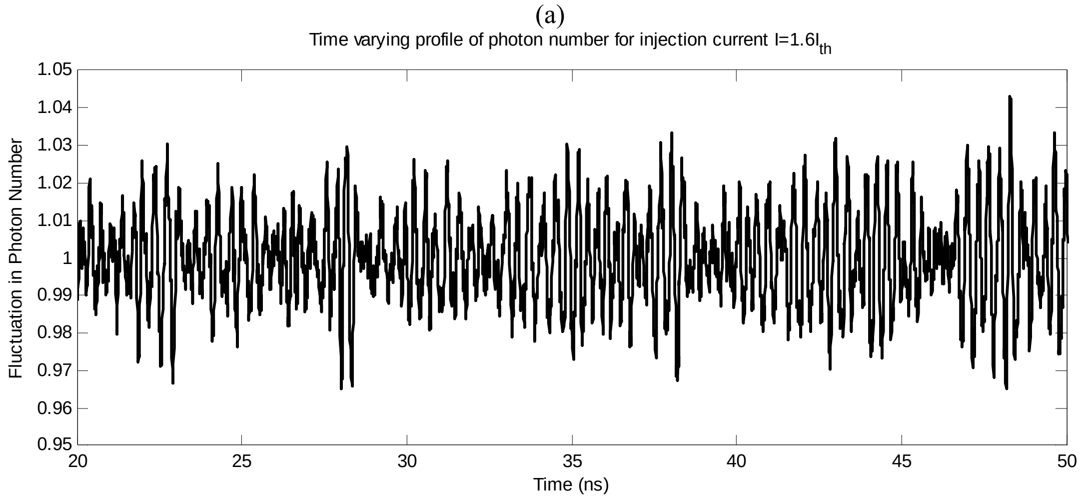
Table 1: Typical values of GaAs laser parameters [25]

Symbol	Definition	Value	Unit
a	tangential gain coefficient	2.75×10^{-12}	m^3s^{-1}
$\lvert R_{cv}\rvert^2$	Squared value of absolute value of dipole moment	2.8×10^{-57}	C^2m^2
ξ	confinement factor of field	0.2	-
τ_{in}	electron intraband relaxation time	0.1	ns
τ_S	average electron lifetime	2.79	ns
N_S	electron number characterizing non-linear gain	1.7×10^8	-
N_g	electron number at transparency	2.1×10^8	-
V	volume of the laser active region	100	μm^3
d	thickness of the laser active region	0.11	μm
L	length of the laser active region	300	μm
n_r	refractive index of laser active region	3.6	-
k	internal loss in the laser cavity	10	cm^{-1}
R_f	reflectivity of front facet	0.3	-
R_b	reflectivity of back facet	0.6	-



(a)



(b)

Fig 1: (a) Time varying profile of photon number with transient, (b) Time varying profile of photon number after termination of transient.

We have talked earlier about the Gaussian random variable g_s that is needed to obtain the appropriate Langevin noise source. We have used the Box-Muller transformation method [24] to generate the Gaussian variable g_s. In this method, two uniformly distributed random variables u_1 and u_2 between -1 and +1 are first taken. These random variables are obtained by two random number generators in Matlab. Then by using Box-Muller transformation method, the Gaussian random variable for photon number is obtained from u_1 and u_2 using equation (9).

4. RESULTS AND DISCUSSION

4.1 Fluctuations of the Photon and Carrier Numbers

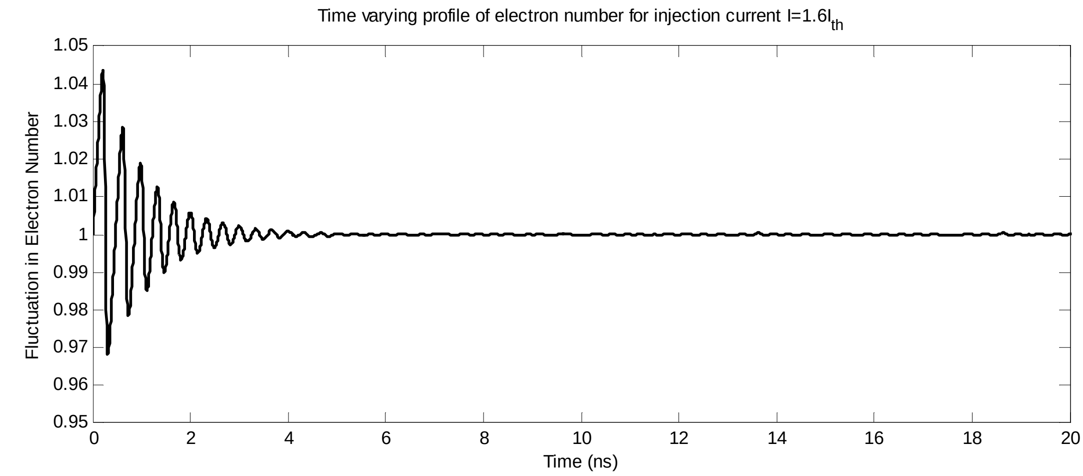
Through the numerical simulation, it was possible to obtain and plot the time varying profiles of the photon number S(t) and the carrier number N(t). Figs 1 and 2 show these profiles for an injection current I equal to 1.6 times the threshold value I_{th}. Figs 1(a) and 2(a) show the profiles before termination of transients for the photon number and carrier number variations, respectively. Figs 1(b) and 2(b) show the profiles after termination of transients for the photon number and carrier number fluctuations, respectively.

Fig. 1 shows that the fluctuation in photon number has a transient response initially; but after the termination of transients, it fluctuates close to its average value $\overline{S}$. This is the effect of driving the rate equations by the Langevin noise sources that cause these physical quantities to fluctuate around their dc values. These fluctuations do not die away, but continue with time even after the transient response has ended, as can be observed from Fig. 1(b).

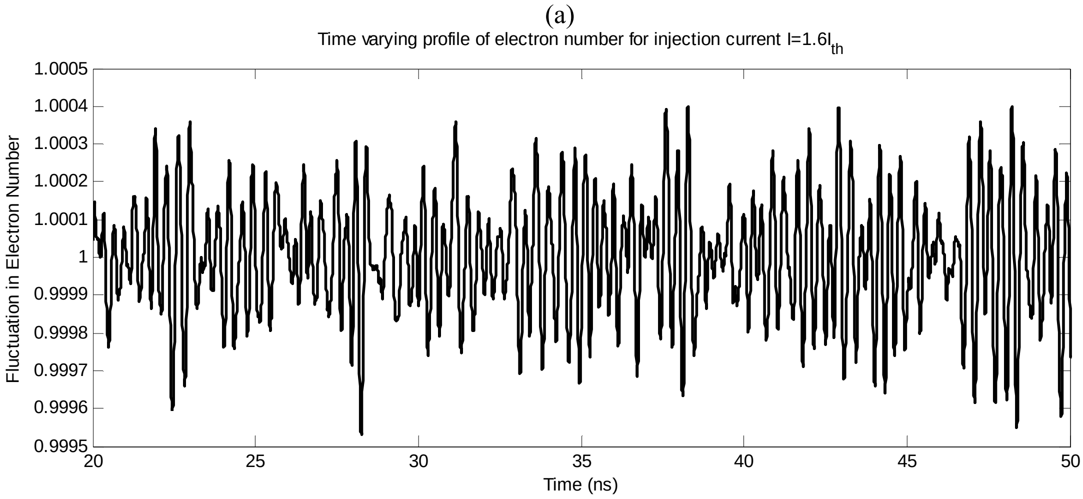
The same phenomena can be observed for the time varying profile of the carrier (electron) number as shown in Fig. 2. The fluctuations are obtained through (N/ $\overline{N}$).

To enable comparison and to observe the effect of change in injection current on the time varying profile of photon numbers and electron numbers, Fig. 3 was plotted for two different injection currents, one for $I=1.0I_{th}$ and another for $I=1.6I_{th}$. The behavior of these physical quantities before and after the transient response is shown separately.

In order to explain the transient response of the semiconductor laser, a physical model needs to be considered. It is important to understand that semiconductor lasers respond much more rapidly to changes in the pump power as compared to other types of lasers. Pulses at bit rates of up to 1-2Gbps can be produced by semiconductor lasers by direct and efficient modulation by the pump current. This makes semiconductor an excellent candidate for applications in communication systems. This behavior can be attributed to two factors namely, the short time constant related to carrier injection and to the high volume excitation rate due to which a small optical cavity can be used. [26]



(a)



(b)

Fig 2: (a) Time varying profile of electron number with transient, (b) Time varying profile of electron number after termination of transients

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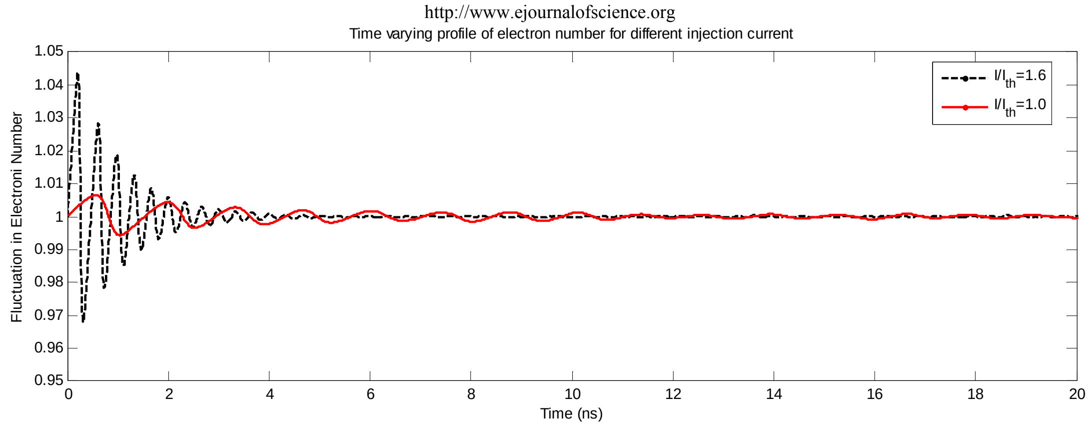

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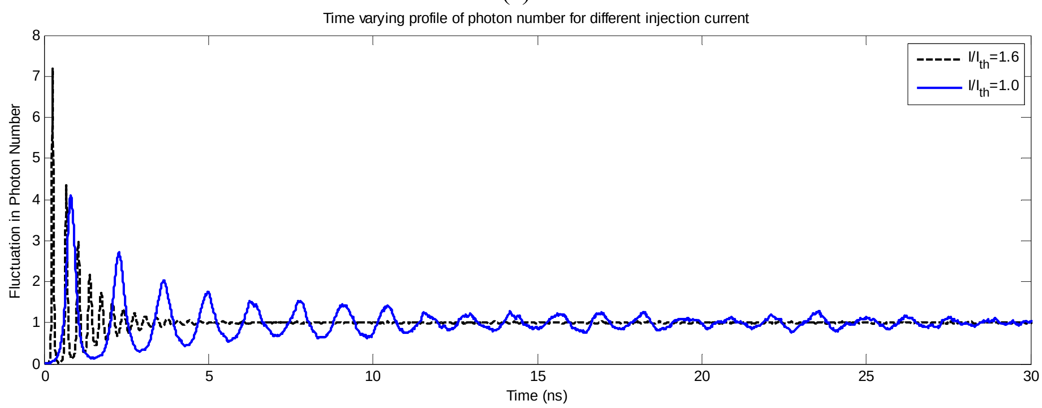

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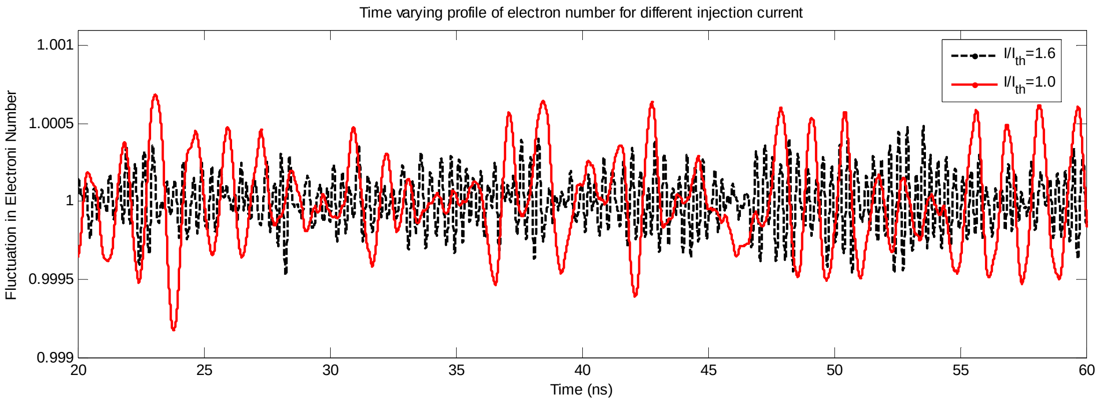

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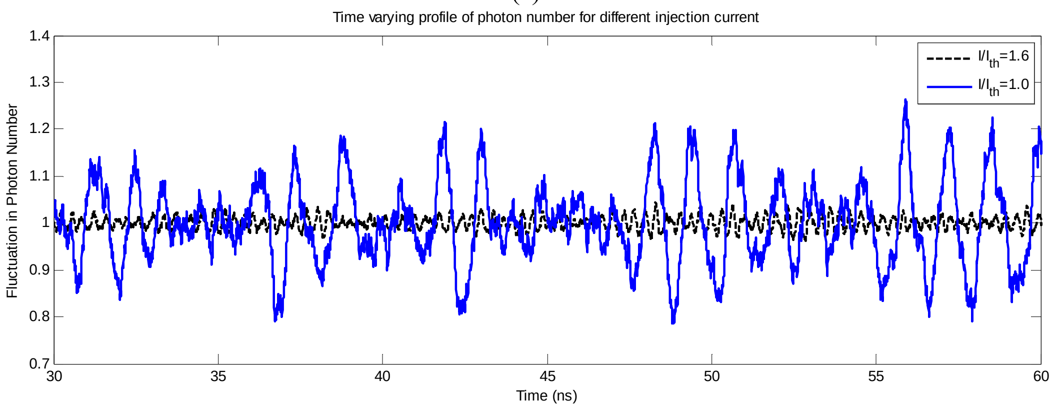

(d)

Fig 3: Time varying profiles of photon and carrier numbers before and after transient response for two values of injection current ($I=1.6I_{th}$ and $I=1.0I_{th}$); (a) Time varying profile of the carrier number with transient response. (b) Time varying profile for the photon number with transient response. (c) Time varying profile of the carrier number after transient response dies out. (d) Time varying profile for the photon number after transient response dies out.

In any kind of laser, semiconductor or otherwise, spontaneous emission contributes to a relatively insignificant amount of output light. It is the stimulated emission that produces the main output. The way that the output light intensity responds to the injection current depends on both the interaction between the photon population in the cavity and the fraction of injected carriers that are in excess of the equilibrium threshold value. Hence, we can say that this process is somewhat dependent on the time constants associated with both the carriers and the photons. It therefore becomes necessary to study the changes in the concentration of both injected carriers and the photons in the resonator, if we wish to analyze the effect of changes in the drive current. When the injected current is increased, the initial effect is that the injected carrier concentration, and hence, the local rate of light emission increases. A noteworthy point here is that in a laser, a distinction has to be made between the rate of light emission into the laser resonator and the rate of light output from the end faces of the resonator. The light output from the end faces of the resonator depends on the size of the photon population that has accumulated in the resonator, not on the rate of generation of photons. The rise in photon population lags behind the increase in the photon generation rate and it is this delay that is responsible for the response mechanism of the laser. This delay, although small, determines the way the photons supply feedback to the stimulated emission. The result is that the speed of response is very high in lasers. The fast response speed can be attributed to the fact the average photon lifetime in the resonator is short (about 5ps). [27] Thus the photon population can rise extremely fast in proportion to its initial value, which in turn immediately enhances the stimulated emission rate, and thus speeds up the response to the injected carrier concentration.

Figs 1(a) and 2(a) show the response of both the photon and injected carrier populations just after the laser is switched on. The dependence of the photon population on the carrier population can be explained as follows. During the switch on process of the laser, the injected carrier concentration rises significantly above its equilibrium level. This causes the photon population to exceed its equilibrium level at the end of the first phase of switch on, which in turn results on the transient oscillation.

4.2 Intensity Noise

As stated earlier the RIN spectrum can be originally defined as the Fourier transform of the autocorrelation function.

$$RIN = \frac{1}{\bar{S}^2}\frac{\Delta t^2}{T}\left|FFT[\delta S(t_i)]\right|^2 \qquad (13)$$

where $\bar{S}$ = time average photon number,

Δt = time-step of the calculation,

T = total time period of the calculation and

$\delta S(t_i) = \bar{S} - S(t_i)$ is the instantaneous photon number fluctuation.

Fig. 4 shows the stimulated spectrum of quantum RIN for injection current $I=1.6I_{th}$. From the figure we see that the RIN shows a pronounced peak around the relaxation oscillation frequency.

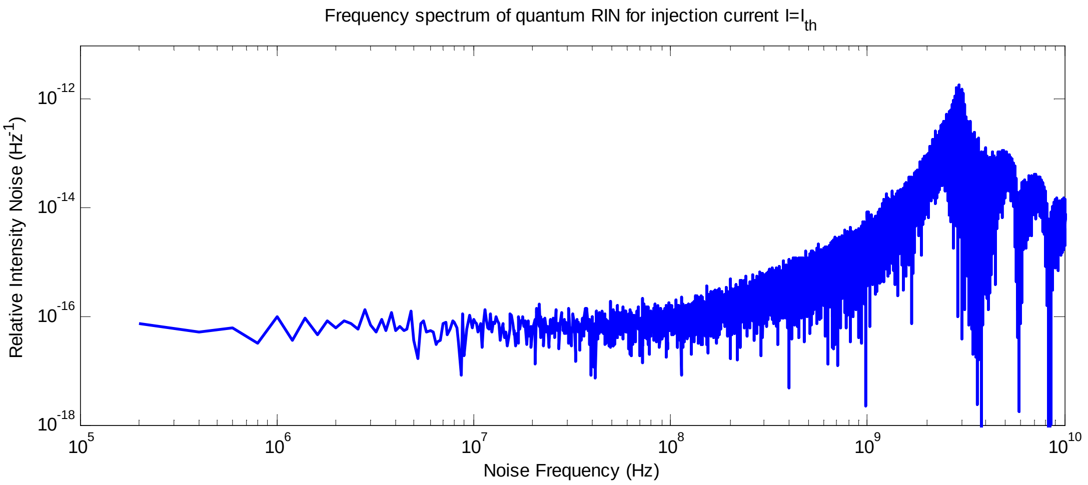

Fig 4: Frequency Spectrum of Quantum RIN for $I=1.6I_{th}$.

The noise characteristics of the laser are affected by the natural resonance of electron and photon populations. Over most of the frequency range up to resonance, the photon quantum noise and electron shot noise of the laser do not show a remarkable variation. At the resonant frequency, however, the noise is enhanced by a factor of up to 5 or 10 [26]. After enhancement in the resonant band it rapidly dies away.

In the output of all types of lasers, noise fluctuations are present that are greater than the natural quantum noise of the photon stream. This is because the quantum fluctuations in the electron and photon population are amplified in the optical resonator. This in turn may be attributed to the discrete and random nature of the emission and recombination processes. The effect of fluctuations in the photon and carrier populations on the output of the laser is similar to that in which would be produced by deliberate modulation of the two populations.

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Complementary processes of photon emission or absorption and electron recombination or generation are correlated in time. These complementary processes must be taken into consideration while analyzing the overall effect. The response of the laser to the fluctuation depends on the injection current. There is a resonant interaction that magnifies the noise over a certain band of frequencies around the relaxation frequency.

For a comparative study of the effect of injection current on the RIN spectrum, Fig. 7 is obtained for three different values of injection current, $I=0.9I_{th}$, $I=1.0I_{th}$ and $I=1.6I_{th}$. Observable peaks are obtained at injection currents I greater than threshold current. For injection currents below the threshold, the RIN Spectrum does not show any resonance frequency peak. This may be attributed to the fact that for injection current I greater than I_{th}, the noise is caused by photon fluctuation and for I less than I_{th}, the noise is caused by electron fluctuation.

4.3 Effects of Injection Current

The effect of injection current on the noise characteristics, the output power of the laser, the average electron number and the average photon number are analyzed in this section.

Let us first study the effect on the noise characteristics. Fig. 5 shows that as the laser current is increased from threshold, the relative output noise below 100 MHz or so show a maximum around threshold and then decreases with a further increase in current. Noise centered at frequencies higher than a few hundred megahertz is additionally affected by the resonant enhancement.

Output power P(t) varies with different injection currents I as shown in Fig. 6. The plotted fluctuations are far from the relaxation regime. The plot features that the repetition of the fluctuation becomes faster with increasing current I, which indicates that the relaxation frequency has increased.

The corresponding spectrum of relative intensity noise, RIN is plotted in Fig. 7. We have seen and discussed about the characteristics of noise shown in Fig. 5, which included injection currents both above and below the threshold level. Now we talk only about injection currents greater than the threshold level. The variation of the noise characteristics shown is in correspondence with that of the fluctuation of P(t). Increasing I causes shift of the relaxation oscillation peak towards higher frequency and decreases of the noise level. The shift of the peak frequency of RIN is followed by the increase of the repetition of the fluctuations with increasing I. On the other hand, the suppression of the fluctuations leads to a decrease in the level of RIN as shown in Fig. 7.

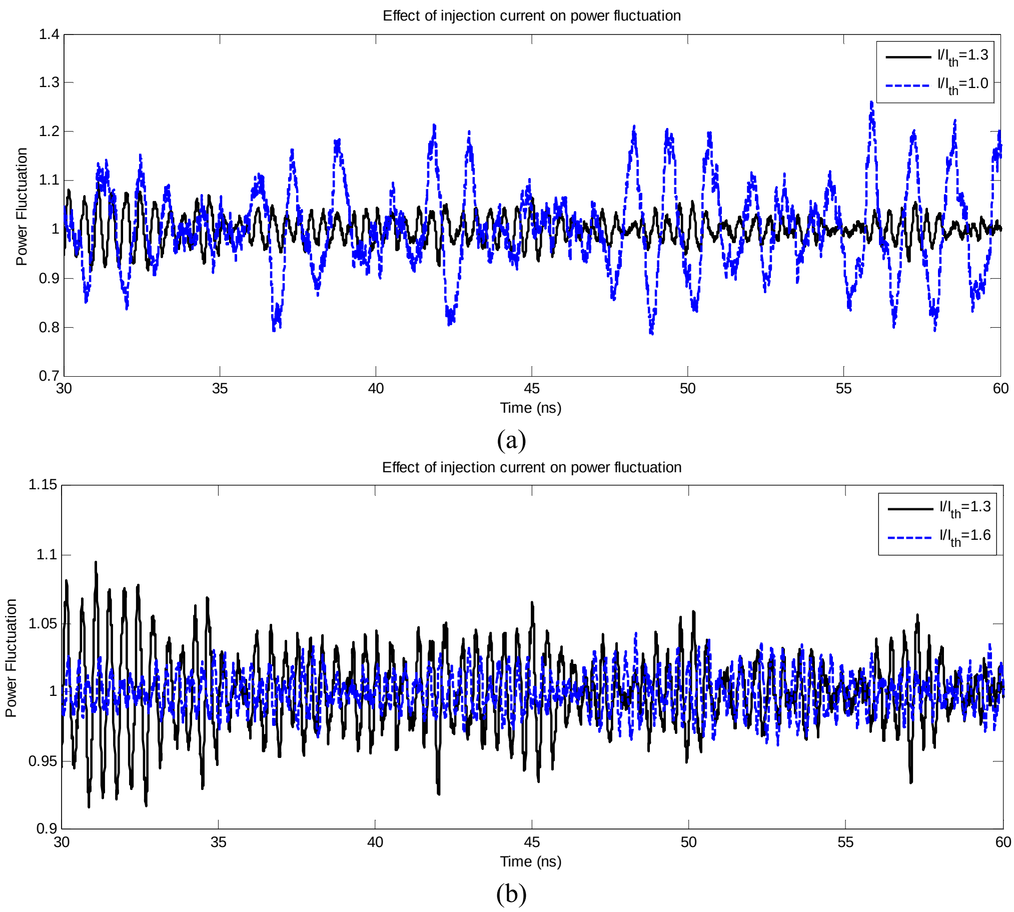


Fig 5 & 6: Effect of injection current on power fluctuation P(t). (a) For $I=1.0I_{th}$ and for $I=1.3I_{th}$. (b) For $I=1.3I_{th}$ and for $I=1.6I_{th}$.

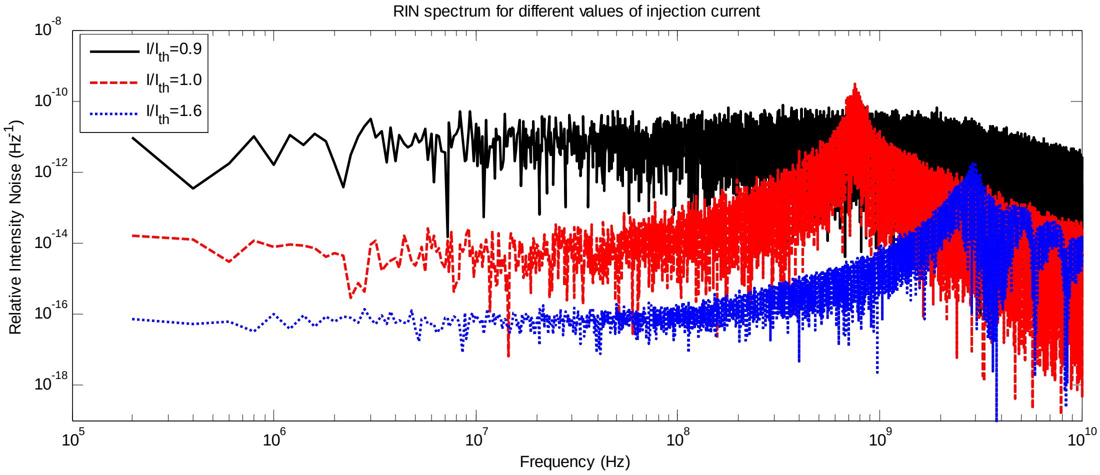
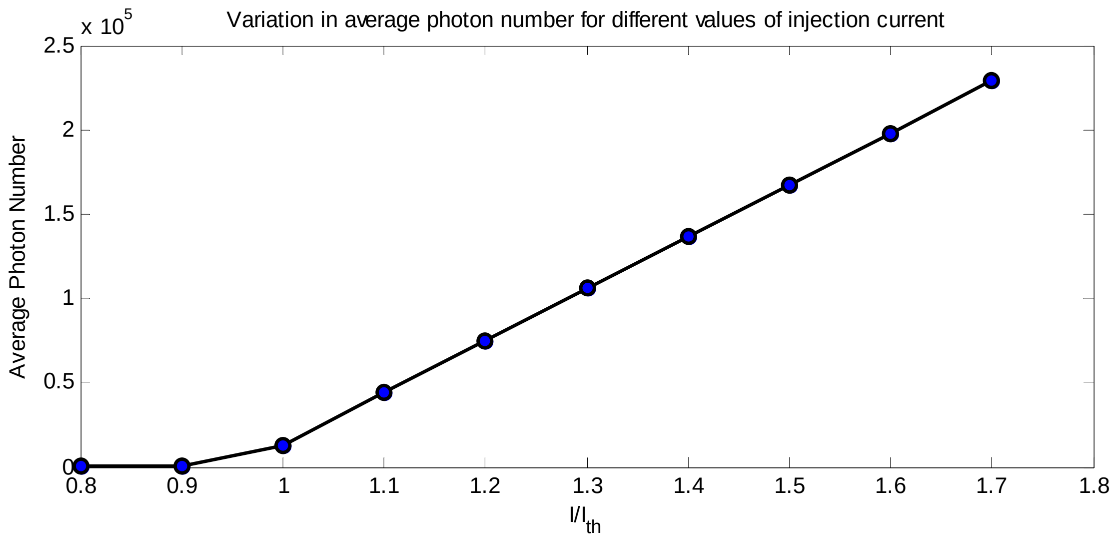
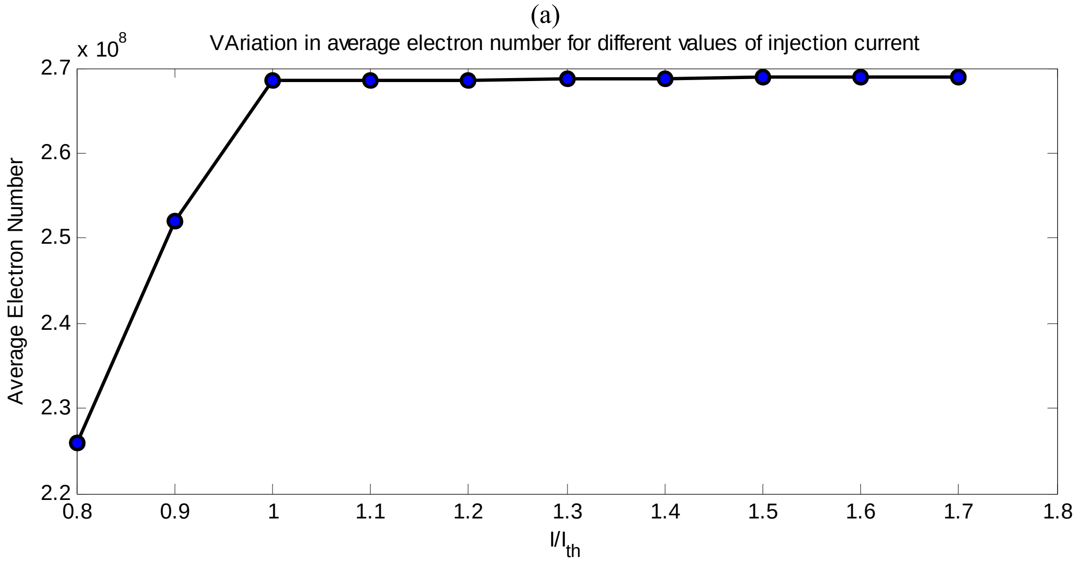


Fig 7: RIN Spectrum for different values of injection current

The variation of the average photon number and carrier number with injection current is shown in Figs 8(a) and 8(b), respectively. It shows that after threshold as the lasing starts time average photon numbers increases with carrier injection, though the average electron number remains constant.

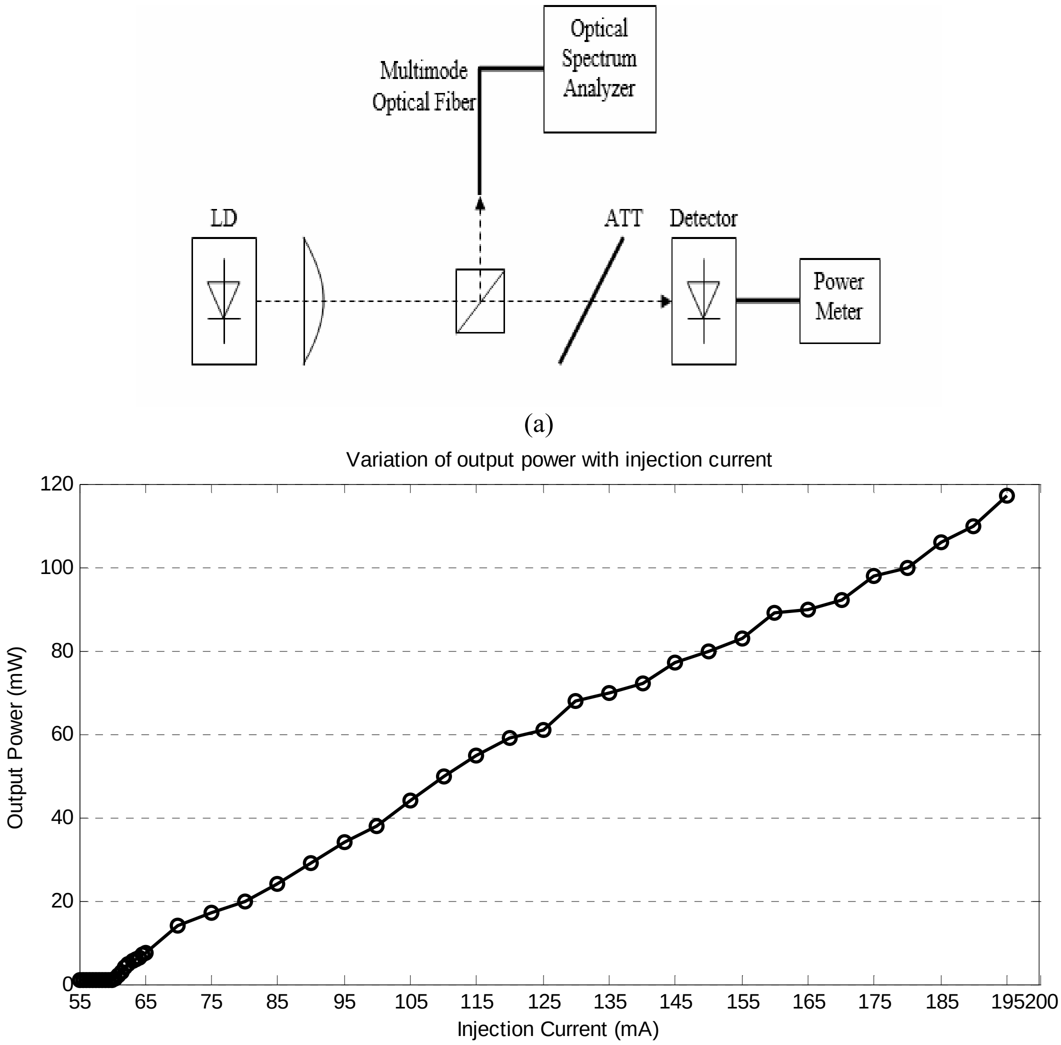


(a)



(b)

Fig 8: (a) Variation of average photon number with injection current, (b) Variation of average electron number with injection current.



(b)

Fig 9: (a) Practical setup to measure optical power of laser diodes, (b) Variation of output power with input current injection.

Fig. 9 shows practical setup to measure optical output of a semiconductor laser diode. The data were taken at constant temperature T=25^{0}C. We get different output power for different injection current that is plotted in Fig. 9(b). It shows that the output power is changing linearly with injection current, same as photon number changes shown in Fig. 8(a).

5. CONCLUSION

In this paper, the rate equations for photon numbers and carrier numbers have been obtained for semiconductor laser operating in single mode by considering the self-suppression coefficient. Then the numerical simulation of the rate equations have been done for GaAs lasers by choosing the optimum values of the parameters for typical 850 nm GaAs lasers. The quantum noise effect of the solitary laser has been described through numerical simulation of the relative intensity noise (RIN). The photon number and carrier number variations with injection current and the variation of output power characteristics have been obtained and explained in detail with the simulation results. A systematic technique has been demonstrated to introduce the Langevin noise sources on the photon number and carrier number. The time varying profile of the fluctuating photon number and carrier number are analyzed. The frequency spectrum of the intensity noise is calculated with the help of FFT.

The intensity noise present in the output of laser diodes limit their reliability when applied as light sources in optical communication systems, optical discs, etc. Analysis of the laser noise thus becomes crucial in improving the efficiency of semiconductor lasers. We have theoretically analyzed the quantum noise of semiconductor laser through numerical simulation of the laser rate equations including Langevin noise source for the photon number that account for the generation of the fluctuations.

It has been found that both photon number and carrier number show transient response at the start of laser and after few ns that transients die out. After transients die out the photon number and carrier number fluctuate around their average values. With the increase of current injection, the quantum noise level of the laser decreases and the relaxation oscillation peak shifted towards higher frequency. All these quantum noise characteristics are numerically presented using the laser rate equations and all the large signal simulation results are also explained satisfactorily.

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