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Cosmological Consequences with Time Dependent Λ-Term in Bianchi Type-I Space-Time—Revisited

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***Abstract*—Einstein's field equations with variable cosmological constant Λ in presence of perfect fluid for a homogeneous and anisotropic Bianchi type-I space-time has been studied by generalizing recent results (Pradhan et al., in JBAP 2: 50, 2013). Einstein's field equations are exactly solved by considering a scale factor $a(t) = \sqrt{t^n e^t}$, where n is a positive constant which yields a time-dependent deceleration parameter (DP) $q = -1 + \frac{2n}{(n+t)^2}$, representing a model which generates a transition of the universe from the early decelerating phase to the recent accelerating phase. The cosmological constant Λ is found to be a decreasing function of time and it approaches a small positive value at the present epoch which is corroborated by consequences from recent supernovae Ia observations. From recently developed Statefinder diagnostic, the behaviour of different stages of the evolution of the universe has been studied. The physical and geometric properties of the cosmological models have also been described.**

***Index Terms*—Cosmology, Variable cosmological term, Perfect fluid models, Statefinder parameters**
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I. INTRODUCTION

The discovery that the accelerated expansion of the Universe is driven by the dark energy from the type Ia supernovae (SN Ia) observations [1−5] greatly astonished the world. To explain this phenomena the notion known as dark energy (DE) with large negative pressure is proposed. At present there are a lot of theoretical models of DE. But the most suitable models of DE is the cosmological constant. According of the modern observational cosmology, the present value of cosmological constant is $10^{-55}cm^{-2}$. At the same time, the particle physics tells us that its value must be 10^{120} times greater than this factor. It is one main problem modern cosmology and known as the cosmological constant problem. From recent cosmological observations we obtained $\Omega_M \approx 0.3$, $\Omega_\Lambda \approx 0.7$, which substantially rejected the traditional $(\Omega_M, \Omega_\Lambda)$ = $(1, 0)$ universe. This value of the density parameter Ω_Λ corresponds to a cosmological constant that is small, nevertheless, nonzero and positive, that is, $\Lambda \approx 10^{-52}m^{-2} \approx 10^{-35}s^{-2}$. An intense search is going on, in both theory and observations, to bring out the true nature of this acceleration. Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe, i.e., $\rho_v = \frac{\Lambda}{8\pi G}$, where Λ is the cosmological constant. However, a constant Λ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. In an attempt to solve this problem, Dolgov [6] introduced variable λ such that Λ was large in the early universe and then decayed with evolution. Cosmological scenarios with a time-varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during last two decades [7]− [15].

In general relativity, the Bianchi identities for the Einstein's tensor G_{ij} and the vanishing covariant divergence of the energy momentum tensor T_{ij} together with imply that the cosmological term Λ is constant. In theories with a variable Λ-term, one either introduces new terms (involving scalar fields, for instance) in to the left hand side of the Einstein's field equations to cancel the non-zero divergence of Λg_{ij} (Bergmann [16]; Wagoner [17]) or interprets Λ as a matter source and moves it to the right hand side of the field equations (Zeldovich [18]), in which case energy momentum conservation is understood to mean $T^{*ij}_{;j} = 0$, where $T^*_{ij} = T_{ij} - (\Lambda/8\pi G)g_{ij}$. It is here that the first assumption that leads to the cosmological constant problem is made. It is that the vacuum has a non-zero energy density. If such a vacuum energy density exists, Lorentz invariance requires that it has the form $\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$. This allows to define an effective cosmological constant and a total effective vacuum energy density $\Lambda_{eff} = \Lambda + 8\pi G\langle\rho\rangle$ or $\rho_{vac} = \langle\rho\rangle + \Lambda/8\pi G$. Note at this point that only the effective cosmological constant, Λ_{eff}, is observable, not Λ, so the latter quantity may be referred to as a 'bare'. The two approaches are of course equivalent for a given theory as examined by Vishwakarma [19]. A dynamic cosmological term $\Lambda(t)$ remains a focal point of interest in modern cosmological theories as it solves the cosmological constant problem in a natural way. For detail discussions, the readers are advised to see the references (Carroll et al. [20]; Abdussattar and Vishwakarma [21]; Peebles and Ratra [22]; Lima [23]; Sahni and Starobinsky [24]; Padmanabhan [25], [26]; Singh et al. [27], Li et al. [28]).

For studying the possible effects of anisotropy in the early universe on present day observations many researchers [29−36] have investigated Bianchi type-I models from different point of view. Recently, Pradhan et al. [37] studied cosmological consequences with time dependent Λ-term in Bianchi type-I space-time by considering scale factor $a(t) = te^t$. In this paper, we have revisited this solution and obtained Bianchi type-I cosmological models with time dependent deceleration parameter and cosmological Λ-term in presence of perfect fluid which generalizes [37]. The out line of the paper is as follows: In Sect. II, the metric and basic equations are described. Section III deals with the solutions of the field equations. In Sect. IV, the physical and geometric aspects of the models has been discussed. In Sect. V, Statefinder diagnostic pair is briefly discussed. Finally, conclusions are summarized in the last Sect. VI.

II. THE METRIC AND BASIC EQUATIONS

We consider the space-time admitting Bianchi type-I group of motion in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \quad (1)$$

As already discussed in Introduction, in theories with a variable Λ-term, the energy momentum conservation is understood to mean

$$T_{ij} - \left(\frac{\Lambda}{8\pi G}\right) g_{ij} = 0. \quad (2)$$

Overduin [38] pointed out that the two approaches were equivalent for a given theory. Here we follow the later approach and assume that the cosmic matter is represented by the energy momentum tensor of perfect fluid augmented with the Λ-term as

$$T_{ij} = (\rho + p)u_i u_j + \left(p - \frac{\Lambda}{8\pi G}\right) g_{ij}, \quad (3)$$

together with a perfect gas equation of state

$$p = \gamma\rho, \;\; 0 \leq \gamma \leq 1, \quad (4)$$

where ρ, p are the energy density, thermodynamical pressure and u_i is the four-velocity vector of the fluid comporting the relation

$$u_i u^i = -1. \quad (5)$$

The Einstein's field equations with time-dependent G and Λ are

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi G T_{ij}. \quad (6)$$

For the metric (1) and energy-momentum tensor (3) in comoving coordinate system, the field equation (6) proceeds a set of four independent equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda, \quad (7)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -8\pi G p + \Lambda, \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda, \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = 8\pi G\rho + \Lambda. \quad (10)$$

Here, and also in what follows, a dot designates ordinary differentiation with respect to t.

The energy conservation equation $T^{ij}{}_{;j} = 0$, leads to the following expression:

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0, \quad (11)$$

which is a consequence of the field equations (7)-(10).

We define the following parameters to be used in solving Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-I model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \quad (12)$$

A volume scale factor V is given by

$$V = a^3 = ABC. \quad (13)$$

In analogy with FRW universe, we also define the generalized Hubble parameter H as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), \quad (14)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively. Here, and also in what follows, a dot indicates ordinary differentiation with respect to t.

Further, the deceleration parameter q is defined by

$$q \equiv -\frac{\ddot{a}}{a}\left(\frac{\dot{a}}{a}\right)^{-2} = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (15)$$

The expansion of the universe is said to be "accelerating" if $\ddot{a}$ is positive (recent measurements suggest it is), and in this case the DP will be negative. The minus sign and the name "deceleration parameter" are historical; the time of definition q was thought to be positive, now it is believed to be negative. Recent observations [1]− [4] have suggested that the rate of expansion of the universe is currently accelerating, perhaps due to dark energy. This yields negative values of the DP.

We introduce the kinematical quantities such as expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m), defined as follows:

$$\theta = u^i_{;i}, \quad (16)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \quad (17)$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{18}$$

where $u^i = (0,0,0,1)$ is the matter 4-velocity vector and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P_j^\alpha + u_{j;\alpha}P_i^\alpha\right) - \frac{1}{3}\theta P_{ij}. \tag{19}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j. \tag{20}$$

These dynamical scalars, in Bianchi type-I, have the forms

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{21}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}. \tag{22}$$

III. SOLUTIONS OF FIELD EQUATIONS

Subtracting (7) from (8) and integrating, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC} = k_1 a^{-3}, \tag{23}$$

where k_1 is a constant of integration. Again integrating (23), we get

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right), \tag{24}$$

where d_1 is an integrating constant.

Similarly subtracting (7) and (8) from (9), and continuing as above we get two more relations:

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{25}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{26}$$

where d_2, d_3, k_2, and k_3 are constants of integration.

From Eqs. (24)-(26), the metric functions can be obtained explicitly as

$$A(t) = l_1 a \exp\left(m_1 \int \frac{dt}{a^3}\right), \tag{27}$$

$$B(t) = l_2 a \exp\left(m_2 \int \frac{dt}{a^3}\right), \tag{28}$$

$$C(t) = l_3 a \exp\left(m_3 \int \frac{dt}{a^3}\right), \tag{29}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{d_1^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$3m_1 = k_1 + k_2, \quad 3m_2 = k_3 - k_1, \quad 3m_3 = -(k_2 + k_3),$$

where the constants m_1, m_2, m_3 and l_1, l_2,l_3 satisfy the relations

$$m_1 + m_2 + m_3 = 0, \quad l_1 l_2 l_3 = 1. \tag{30}$$

It is clear from Eqs. (27)-(29) that once we get the value of the average scale factor a, we can easily calculate the metric functions A, B, C.

Following Saha et al. [39], Pradhan and Amirhashchi [40], we take following *ansatz* for the scale factor, where increase in term of time evolution is

$$a(t) = \sqrt{t^n e^t}, \tag{31}$$

where n is a positive constant. If we put $n = 0$, Eq. (31) reduces to $a(t) = \sqrt{e^t}$ i.e. exponential law of variation. It is worth mentioned here that $a(t)$ is a unit less function. In (31), a is a function of $\tau = \frac{t}{t_1}$, where t_1 is a constant of unit [Time]. As a result, being a function of time, a still remains unit less. For simplicity here and further we write a as a function of t with t now being unit less. Saha et al. [39], and Pradhan & Amirhashchi [40] examined the relation (31) in studying two-fluid scenario for dark energy in an FRW universe and accelerating dark energy models in Bianchi type-V space-times respectively. This *ansatz* generalized the one proposed by Amirhashchi et al. [41]. The choice of scale factor (31) yields a time-dependent deceleration parameter (see Eq. (42)).

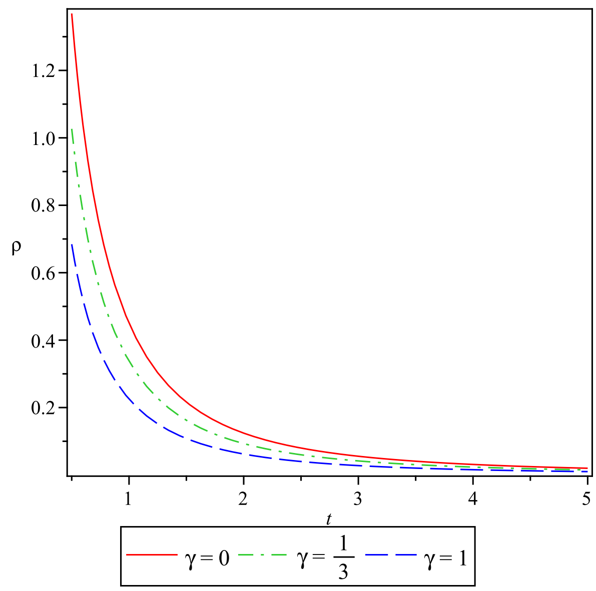

Fig. 1. The plot of energy density ρ versus t for $8\pi G = 1$, $n = 0.5$, $\beta_1 = 1$

Using Eq. (31) into (27)–(29), we get the fallowing expression for scale factors:

$$A(t) = l_1\sqrt{t^n e^t}\exp\left[m_1 \int \left(t^n e^t\right)^{-\frac{3}{2}} \mathrm{d}t\right] = l_1\sqrt{t^n e^t}\exp\left(m_1 K\right), \tag{32}$$

$$B(t) = l_2\sqrt{t^n e^t}\exp\left[m_2 \int \left(t^n e^t\right)^{-\frac{3}{2}} \mathrm{d}t\right] = l_2\sqrt{t^n e^t}\exp\left(m_2 K\right), \tag{33}$$

$$C(t) = l_3\sqrt{t^n e^t}\exp\left[m_3 \int \left(t^n e^t\right)^{-\frac{3}{2}} \mathrm{d}t\right] = l_3\sqrt{t^n e^t}\exp\left(m_3 K\right), \tag{34}$$

where $K = \left[\left(\frac{2}{3}\right)^{-\frac{3n}{2}+1}\Gamma\left(-\frac{3n}{2}+1\right)\right]$. Here, we have taken the limits of integration for cosmic time t from $t = 0$ to

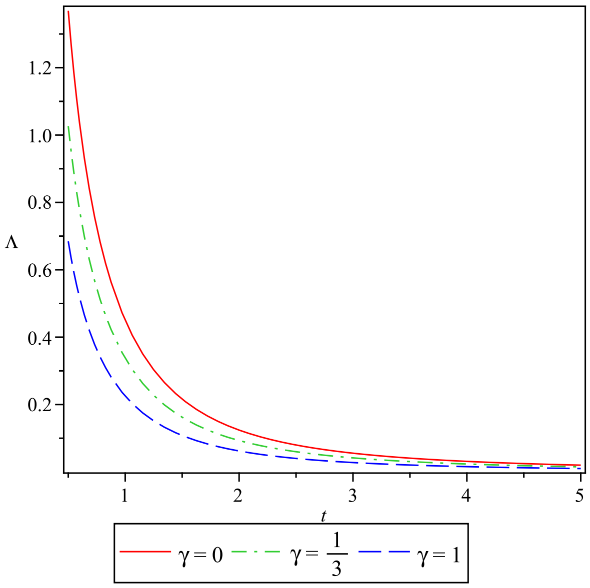

Fig. 2. The plot of cosmological constant Λ versus t for $n = 0.5$, $\beta_2 = 1$

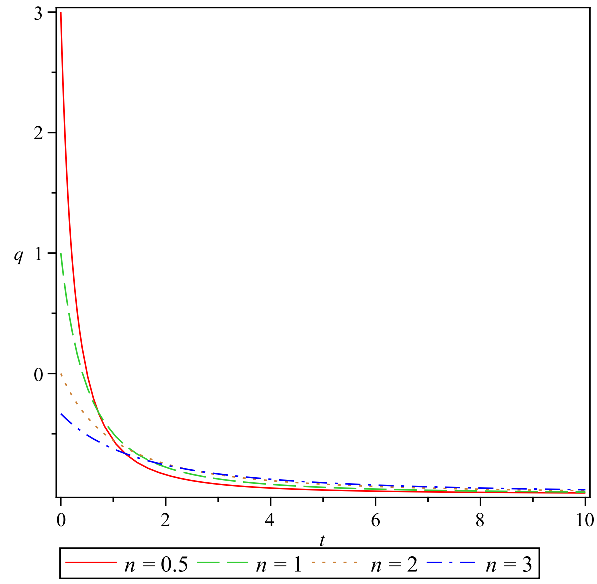

Fig. 4. The variation of DP q vs t

$t = \infty$ which help us to get the above exact solutions.

Hence the geometry of the universe (1) is reduced to

$$ds^2 = -dt^2 + l_1^2(t^n e^t)\exp(2m_1 K)\,\mathrm{d}x^2 + l_2^2(t^n e^t)\exp(2m_2 K)\,\mathrm{d}y^2 + l_3^2(t^n e^t)\exp(2m_3 K)\,\mathrm{d}z^2. \quad (35)$$

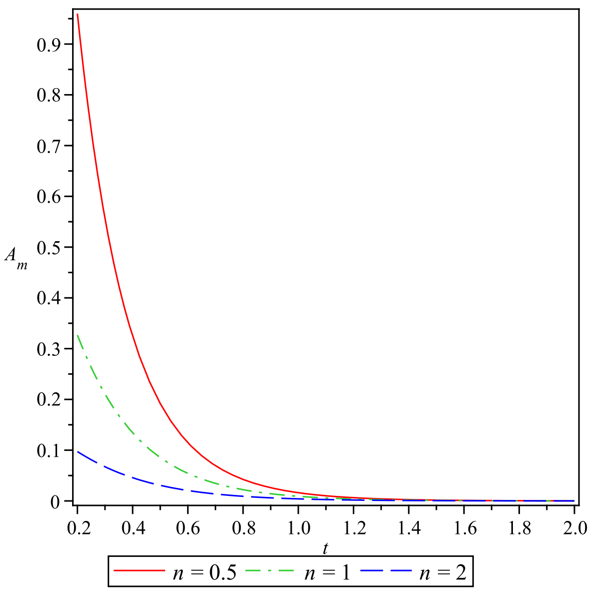

Fig. 3. The plot of anisotropic parameter A_m versus t for $\beta_1 = 1$

IV. SOME PHYSICAL AND GEOMETRIC PROPERTIES

Substituting (32)−(34) into Eqs. (9) and (10), and solving with (4), we get the expressions for pressure (p), energy density (ρ) and cosmological term Λ for the model (35) as

$$p = \frac{\gamma}{8\pi G(1+\gamma)}\left[\frac{n}{t^2} - \beta_1(t^n e^t)^{-3}\right], \quad (36)$$

$$\rho = \frac{1}{8\pi G(1+\gamma)}\left[\frac{n}{t^2} - \beta_1(t^n e^t)^{-3}\right], \quad (37)$$

$$\Lambda = \frac{3}{4}\left(\frac{n}{t}+1\right)^2 + \frac{1}{(1+\gamma)}\left[\beta_2(t^n e^t)^{-3}\right], \quad (38)$$

where

$$\beta_1 = m_1^2 + m_2^2 + m_3^2,$$

$$\beta_2 = m_1^2 + m_2^2 + m_1 m_2 + \gamma(m_1 m_2 + m_2 m_3 + m_3 m_1). \quad (39)$$

From above relations (36)−(38), we can obtain four types of models:

- When $\gamma = 0$, we obtain empty model.
- When $\gamma = \frac{1}{3}$, we obtain radiating dominated model.
- When $\gamma = -1$, we have the degenerate vacuum or false vacuum or ρ vacuum model (Cho [42]).
- When $\gamma = 1$, the fluid distribution corresponds with the equation of state $\rho = p$ which is known as Zeldovich fluid or stiff fluid model (Zeldovich [43]; Barrow [44]).

From Eq. (37), it is observed that the energy density ρ is a decreasing function of time and $\rho > 0$ always. The energy density has been graphed versus time in Fig. 1 for $\gamma = 0, \frac{1}{3}$ and 1. It is evident that the energy density remains positive in all three types of model. However, it decreases more sharply with the cosmic time in Zeldovich universe, compare to radiating dominated and empty fluid universes.

Figure 2 is the plots of cosmological term Λ versus time for $\gamma = 0, \frac{1}{3}$ and 1. In all three types of models, we observe that Λ is decreasing function of time t and it approaches a small positive value at late time (i.e. at present epoch). However, it decreases more sharply with the cosmic time in empty universe, compare to radiating dominated and stiff fluid universes. Recent cosmological observations [1]– [5] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, the nature of Λ in our derived model is supported by recent observations.

The physical parameters such as spatial volume (V), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) for the model (35) are given by

$$V = (t^n e^t)^{\frac{3}{2}}, \tag{40}$$

$$\theta = 3H = \frac{3}{2}\left(\frac{n}{t} + 1\right), \tag{41}$$

$$q = -1 + \frac{2n}{(n+t)^2}, \tag{42}$$

$$\sigma^2 = \frac{1}{2}\beta_1 (t^n e^t)^{-3}, \tag{43}$$

$$A_m = \frac{4}{3}\beta_1 (t^n e^t)^{-3} \frac{t^2}{(n+t)^2} \tag{44}$$

From Eqs. (40) and (41), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. From Eqs. (32)−(34), we observe that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity (see, MacCallum [45]). We observe that proper volume increases with time.

The dynamics of the mean anisotropic parameter depends on the constant $\beta_1 = m_1^2 + m_2^2 + m_3^2$. From Eq. (44), we observe that at late time when $t \to \infty$, $A_m \to 0$. Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations. Figure 3 depicts the variation of anisotropic parameter (A_m) versus cosmic time t. From the figure, we observe that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in our model at present epoch.

From Eq. (42), we observe that $q > 0$ for $t < \sqrt{2n} - n$ and $q < 0$ for $t > \sqrt{2n} - n$. It is observed that for $0 < n < 2$, our model is evolving from deceleration phase to acceleration phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 4 graphs the deceleration parameter (q) versus time which gives the behaviour of q from decelerating to accelerating phase for different values of n.

It is important to note here that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ spread out to be constant. Therefore, the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [46].

V. STATEFINDER DIAGNOSTIC

Over the past few decades, a simple cosmological model called the lambda cold dark-matter (ΛCDM) model has emerged as the best fit to the current observational data. ΛCDM stands for cosmological constant which is currently associated with a vacuum energy or dark energy inherent in empty space that explains the current expansion of space against the attractive (collapsing) effects of gravity. In order to explain the cosmic acceleration a form of negative-pressure matter called dark energy was suggested. The simplest and most popular candidate is Einstein's cosmological constant. Many other candidates for dark energy have been proposed, including scalar fields with a time dependent equation of state, quintessence, modified gravity, branes, etc. Confrontation between these models and currently observational data does not say much (Evans et al. [47]), mainly because most of them have ΛCDM as a limiting case in the redshift range already observed. The SNAP (Super Novae Acceleration Probe) satellite is expected to observe $\sim$ 2000 supernovae per year with redshift up to $z = 1.7$. Nowadays several cosmological models of dark energy are available which cannot be excluded by current observational data. Recently, Sahni et al. [48] and Alam et al. [49] have introduced a pair of new cosmological parameters (so-called "statefinder parameters") that seem to be promising candidates for this purpose. This is based on the dimensionless parameters r,s, and are given entirely in terms of the scale factor and its derivatives with respect to the cosmic time, up to the third order. Nowadays the most accepted model in cosmology which explains the evolution of the Universe is known as ΛCDM. In this model 4 per cent of the total content of the Universe is baryonic matter, 22 per cent is nonbaryonic dark matter (DM) and the rest is in some form of cosmological constant. ΛCDM has achieved several observations with outstanding success. For a current review, see Maga$\tilde{n}$a et al. [50].

In fact, trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. The statefinder parameters can effectively differentiate between different form of dark energy and provide simple diagnosis regarding whether a particular model fits into the basic observational data. The above statefinder diagnostic pair has the following form:

$$r = \frac{\dddot{a}}{aH^2} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})}\,. \tag{45}$$

For our model, the parameters $\{r, s\}$ can be explicitly written in terms of T as

$$r = 1 - \frac{2n[3(n+t)-4]}{(n+t)^3}\,, \quad s = -\frac{4n[3(n+t)-4]}{3(n+t)[4n-3(n+2)^2]}\,. \tag{46}$$

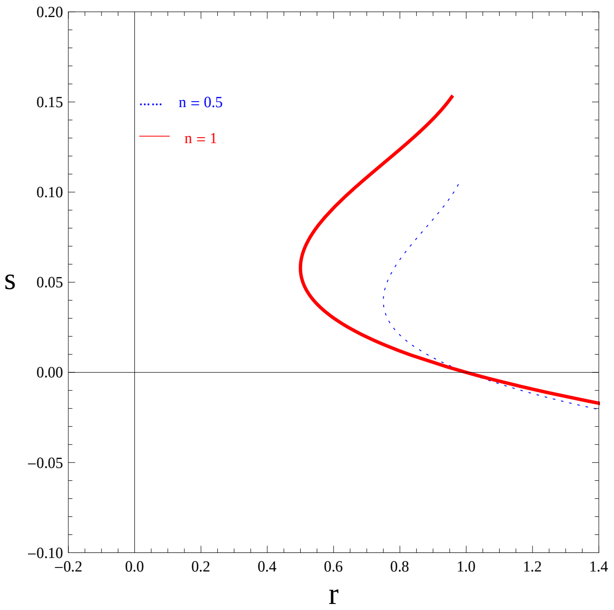

Fig. 5. The variation of s against r.

Figure 5 depicts the variation of s against r. From this figure, we observe that s is negative when $r > 1$. We also observe that the universe starts from an asymptotic Einstein static era ($r \to \infty, s \to -\infty$) and for the sake of comparison, we note that the ΛCDM model corresponds to the point (shown in the Fig. 5) $r = 1, s = 0$.

VI. Conclusions

In this paper, a class of cosmological models is presented with variable cosmological term Λ in spatially homogeneous and anisotropic Bianchi type-I space-time in the presence of a perfect fluid. To find the deterministic solution, we have considered a scale factor $a(t) = \sqrt{t^n e^t}$ which yields a time dependent deceleration parameter so that in the early stage the universe was decelerating where as the universe is accelerating at present epoch which is corroborated from the recent supernovae Ia observation [1−5]. The parameters H, θ, and σ diverge at the initial singularity. There is a Point Type singularity [45] at $t = 0$ in the model. The rate of expansion slows down and finally tends to zero as $t \to 0$. The pressure, energy density and cosmological term Λ become negligible where as the scale factors and spatial volume become infinitely large as $t \to \infty$, which would give essentially an empty universe.

The main features of the models are as follows:

- The models are based on exact solutions of the Einstein's field equations for the anisotropic Bianchi-I space-time filled with perfect fluid with variable Λ-term and generalize the recent results obtained by Pradhan et al. [37].
- The model represents expanding, shearing and non-rotating universe.
- The nature of decaying vacuum energy density $\Lambda(t)$ in our derived models is supported by recent cosmological observations. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term.
- In literature it is a plebeian practice to consider constant deceleration parameter. Now for a Universe which was decelerating in past and accelerating at present epoch, the DP must show signature flipping as already discussed in Section 2. Therefore, our consideration of scale factor which provides a time-dependent DP to be variable is physically justified. Our derived model is accelerating at present epoch (Fig. 4)
- For different choice of n, we can generate a class of cosmological models in Bianchi type-I space-time. It is observed that such particular cosmological models are also in good harmony with current observations. Thus, the solutions demonstrated in this paper may be useful for better understanding of the characteristic of anisotropic cosmological models in the evolution of the universe within the framework of Bianchi type-I space-time.
- $\{r, s\}$ diagram (Fig. 5) shows that the evolution of the universe starts from asymptotic Einstein static era ($r \to \infty, s \to -\infty$) and approaches to ΛCDM model ($r = 1, s = 0$). So, from the Statefinder parameter $\{r, s\}$, the behaviour of different stages of the evolution of the universe have been generated.

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