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# Bianchi Type-I Cosmological Models with Viscosity and Cosmological Term in General Relativity

Padminin Yadav[1], S. A. Faruqui[2], Anirudh Pradhan[3]
[1,2] Department of Mathematics, P. G. College, Ghazipur-233 001, U. P., India
[1] p_yadav91@yahoo.in
[3]Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India
[3]pradhan.anirudh@gmail.com

***Abstract*—Exact solutions of Einstein's field equations are obtained in a spatially homogeneous and anisotropic Bianchi type-I space-time in presence of a dissipative fluid with constant and time dependent cosmological term $\Lambda$. To prevail the deterministic solution we choose the scale factor $a(t) = \sqrt{te^t}$, which yields a time dependent deceleration parameter (DP), representing a model which generates a transition of the universe from the early decelerating phase to the recent accelerating phase. A barotropic equation of state together with the shear viscosity is proportional to expansion scalar, is also assumed. It is observed that the initial nature of singularity is not changed due to the presence of viscous fluid. The cosmological constant $\Lambda(t)$ is found to be a decreasing function of time and it approaches a small positive value at the present epoch which is corroborated by consequences from recent supernovae Ia observations. The cosmic jerk parameter in our descended models is also found to be in good concordance with the recent data of astrophysical observations under appropriate condition. The basic equation of thermodynamics have been deduced and the thermodynamic aspects of the models have been discussed. The physical and geometric properties of spatially homogeneous and anisotropic cosmological models are discussed.**

***Index Terms*—Bianchi type-I universe; Viscous fluid; Variable deceleration parameter; Variable cosmological constant**
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## I. INTRODUCTION

The cosmological constant $\Lambda$ is considered as one of the most important unsolved problems in cosmology. In general relativity, the cosmological constant was clearly established as a universal constant. The cosmological constant($\Lambda$) was introduced by Einstein in 1917 as the universal repulsion to make the Universe static in accordance with generally accepted picture of that time but a general expansion of the universe was observed by Hubble in 1927 subsequently. Recent observations [1]− [7] strongly favour a small and positive value of effective cosmological constant at the present epoch. Recent observations [1]− [7] strongly favour a significant and a positive value of effective cosmological constant $\Lambda$ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. Riess et al. [8], [9] have recently presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. They conclude to the evidence for present acceleration $q_0 < 0$ ($q_0 \approx -0.7$). Observations [8]− [10] of Type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating.

In general relativity, the Bianchi identities for the Einstein's tensor $G_{ij}$ and the vanishing covariant divergence of the energy momentum tensor $T_{ij}$ together with imply that the cosmological term $\Lambda$ is constant. In theories with a variable $\Lambda$-term, one either introduces new terms (involving scalar fields, for instance) in to the left hand side of the Einstein's field equations to cancel the non-zero divergence of $\Lambda g_{ij}$ [11], [12] or interprets $\Lambda$ as a matter source and moves it to the right hand side of the field equations [13], in which case energy momentum conservation is understood to mean $T^{*ij}_{;j} = 0$, where $T^*_{ij} = T_{ij} - (\Lambda/8\pi G)g_{ij}$. It is here that the first assumption that leads to the cosmological constant problem is made. It is that the vacuum has a non-zero energy density. If such a vacuum energy density exists, Lorentz invariance requires that it has the form $\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$. This allows to define an effective cosmological constant and a total effective vacuum energy density $\Lambda_{eff} = \Lambda + 8\pi G\langle\rho\rangle$ or $\rho_{vac} = \langle\rho\rangle + \Lambda/8\pi G$. Note at this point that only the effective cosmological constant, $\Lambda_{eff}$, is observable, not $\Lambda$, so the latter quantity may be referred to as a 'bare'. The two approaches are of course equivalent for a given theory [14]. A dynamic cosmological term $\Lambda(t)$ remains a focal point of interest in modern cosmological theories as it solves the cosmological constant problem in a natural way. For detail discussions, the readers are advised to see the references [15]− [20]. In an attempt to solve this problem, variable $\Lambda$ was introduced such that $\Lambda$ was large in the early universe and then decayed with evolution [21]. Cosmological scenarios with a time-varying $\Lambda$ were proposed by several researchers [22]− [42].

In recent years the introduction of viscosity in cosmic fluid content has been found useful in explaining many important physical aspects of the dynamics of homogeneous cosmological models. The dissipative mechanisms not only modify the nature of singularity, usually occurring for a perfect fluid, but also can successfully account for the large entropy per baryon in the present universe. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. The physical process such as decoupling of neutrinos during the radiation era and the recombination era [43], decay of massive super string modes into massless modes [44] and gravitational string production [45], [46]. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models. See Grøn [47] for a review on cosmological models with bulk viscosity. The first effort to create a theory of relativistic dissipative fluids, were made by Esckart [48] and Landau & Lifshitz [49]. Weinberg [50] established general expression for bulk and shear viscosity, and used them to evaluate the cosmological entropy production rate. A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [51]. Recently Pradhan et al. [52], [53] and Singh et al. [54] have studied viscous fluid cosmological models in Bianchi type-I, $VI_0$ and III space-times in different context.

The present day Universe is satisfactorily delineated by homogeneous and isotropic models given by the FRW space-time. But at smaller scales, the Universe is neither homogeneous and isotropic nor do we expect the Universe in its early stages to have these properties. Homogeneous and anisotropic cosmological models have been widely studied in the framework of general relativity in the search of a realistic picture of the Universe in its early stages. A spatially homogeneous Bianchi model necessarily has a three-dimensional group, which acts simply transitively on space-like three-dimensional orbits. Observations by the Differential Radiometers on NASA's Cosmic Background Explorer registered anisotropy in various angle scales. The simplest of anisotropic models, which, completely describe the anisotropic effects, are Bianchi type-I (BI) homogeneous models whose spatial sections are flat but the expansion or contraction rate is directional dependent. The advantages of these anisotropic models are that they have a substantial role in the description of the evolution of the early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models. The isotropy of the present-day universe makes the BI model a flower candidate for studying the possible effects of an anisotropy in the early universe. Recently, Pradhan et al. [55] and Dubey et al. [56] have studied cosmological models in anisotropic Bianchi type-I and $V$ space-times in different context.

Motivated by the above discussions, in this paper, we have investigated a new class of spatially homogeneous and anisotropic Bianchi type-I cosmological models with time dependent deceleration parameter and cosmological constant in presence of a dissipative fluid. The Einstein's field equations are solved explicitly. The outline of the paper is as follows: In Sect. 2, the basic equations are described. Section 3 deals with the solutions of the field equations by considering time dependent deceleration parameter. Section 4 describes the physical and geometric properties of the models. In Subsect. 4.1, we obtain the solution with variable $\Lambda$-term and constant $\xi$ and also discuss the thermodynamic equations. Subsection 4.2 deals with the models with variable $\Lambda$-term and $\xi \propto \rho$ and also discuss entropy production rate of model. In Subsect. 4.3, we describe the solution with constant $\Lambda$-term and time dependent $\xi$ along with the thermodynamic equation and its aspects of the models. Section 5 deals with cosmic jerk parameter. Finally, conclusions are in the last Sect. 6.

## II. THE BASIC EQUATIONS

We consider a spatially homogeneous and anisotropic Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where the metric potentials $A, B$ and $C$ are functions of cosmic time $t$ alone. This ensures that the model is spatially homogeneous.

We define the following parameters to be used in solving Einstein's field equations for t he metric (1).

The average scale factor $a$ of Bianchi type-I model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \quad (2)$$

A volume scale factor $V$ is given by

$$V = a^3 = ABC. \quad (3)$$

In analogy with FRW universe, we also define the generalized Hubble parameter $H$ and deceleration parameter $q$ as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad (4)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}, \quad (5)$$

where an over dot denotes derivative with respect to the cosmic time $t$.

Also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (6)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of $x$-, $y$- and $z$-axes respectively.

The Einstein's field equations (in gravitational unit $8\pi G = c = 1$) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}, \quad (7)$$

where $T_{ij}$ is the stress energy tensor of matter which, in case of viscous fluid and cosmological constant, has the form [57]

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \eta\mu_{ij} - \Lambda(t)g_{ij}, \quad (8)$$

with

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;i} = p - (3\xi - 2\eta)H \quad (9)$$

and

$$\mu_{ij} = u_{i;j} + u_{j;i} + u_i u^\alpha u_{j;\alpha} + u_j u^\alpha u_{i;\alpha}. \quad (10)$$

In the above equations, $\xi$ and $\eta$ stand for the bulk and shear viscosity coefficients respectively; $\rho$ is the matter density; $p$ is the isotropic pressure and $u^i$ is the four-velocity vector satisfying $u^i u_i = -1$.

In a co-moving coordinate system, where $u^i = \delta^i_0$, the field equations (7), for the anisotropic Bianchi type-I space-time (1) and viscous fluid distribution (8), yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + 2\eta\frac{\dot{A}}{A} + \Lambda, \tag{11}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + 2\eta\frac{\dot{B}}{B} + \Lambda, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + 2\eta\frac{\dot{C}}{C} + \Lambda, \tag{13}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho + \Lambda. \tag{14}$$

Here, and also in what follows, a dot designates ordinary differentiation with respect to $t$.

Equations (11)-(14) can also be written as

$$p - \xi\theta - \Lambda = H^2(2q-1) - \sigma^2, \tag{15}$$

$$\rho + \Lambda = 3H^2 - \sigma^2, \tag{16}$$

where $\sigma$ is shear scalar given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \tag{17}$$

where

$$\sigma_{ij} = u_{i;j} + \frac{1}{2}(u_{i;k}u^k u_j + u_{j;k}u^k u_i) + \frac{1}{3}\theta(g_{ij} + u_i u_j). \tag{18}$$

The expansion scalar ($\theta$) and the anisotropy parameter ($A_m$) are defined as

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} \tag{19}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{20}$$

The energy conservation equation $T^{ij}_{\ \ ;j} = 0$, leads to the following expression:

$$\dot{\rho} = -(p+\rho)\theta + \xi\theta^2 + 4\eta\sigma^2 - \dot{\Lambda}. \tag{21}$$

It follows from (21) that for contraction, that is, $\theta < 0$, we have $\dot{\rho} > 0$ so that the matter density increases or decreases depending on whether the viscous heating is greater or less than the cooling due to expansion.

The Raychaudhuri equation is obtained as

$$\dot{\theta} = -\frac{1}{2}[\rho + 3(p - \xi\theta)] - \frac{1}{3}\theta^2 - 2\sigma^2 + \Lambda. \tag{22}$$

We have a system of four independent equations (11)−(14) and eight unknown variables, namely $A, B, C, p, \rho, \xi$, $\eta$ and $\Lambda$. So for complete determinacy of the system, we need four appropriate relations among these variables that we shall consider in the following section and solve the field equations.

## III. SOLUTIONS OF FIELD EQUATIONS

Subtracting (11) from (12), (11) from (13), (12) from (13) and taking second integral of each, we get the following three relations

$$\frac{A}{B} = d_1 \exp\left(x_1 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{23}$$

$$\frac{A}{C} = d_2 \exp\left(x_2 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{24}$$

$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{25}$$

where $d_1, x_1, d_2, x_2, d_3$ and $x_3$ are constants of integration.

From (23)−(25), the metric functions can be explicitly written as

$$A(t) = a_1\ a \exp\left(b_1 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{26}$$

$$B(t) = a_2\ a \exp\left(b_2 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{27}$$

$$C(t) = a_3\ a \exp\left(b_3 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{28}$$

where

$$a_1 = \sqrt[3]{d_1 d_2}, \quad a_2 = \sqrt[3]{d_1^{-1} d_3}, \quad a_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$b_1 = \tfrac{x_1 + x_2}{3}, \quad b_2 = \tfrac{x_3 - x_1}{3}, \quad b_3 = \tfrac{-(x_2 + x_3)}{3}.$$

These constants satisfy the following two relations

$$a_1 a_2 a_3 = 1, b_1 + b_2 + b_3 = 0. \tag{29}$$

Thus the metric functions are found explicitly in terms of the average scale factor $a$. It is clear from Eqs. (26)−(28) that once we get the value of the average scale factor $a$, we can easily calculate the metric functions $A$, $B$, $C$.

Following Amirhashchi et al. [58] and Pradhan et al. [59], we consider the DP as a variable for which we consider the variation of scale factor $a$ with cosmic time $t$ by the relation

$$a(t) = \sqrt{te^t}. \tag{30}$$

Recently, Pradhan et al. [59] examined the relation (30) when studying accelerating cosmological model in scalar tensor theory of gravitation. It is worth mentioning here that one can also select many other ansatzes than Eq. (30) which mimic an accelerating universe. However, one should also be careful to check the physical acceptability and stability of their corresponding solutions, otherwise they do not prove any relation of such solutions with the observable universe. Equation (30) yields physically plausible solutions.

From (5) and (30), we get the time varying DP as

$$q(t) = \frac{2}{(1+t)^2} - 1. \tag{31}$$

From Eq. (31), we observe that $q > 0$ for $t < \sqrt{2} - 1$ and $q < 0$ for $t > \sqrt{2} - 1$. It is observed that our model is evolving from deceleration phase to acceleration phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 1 graphs the deceleration parameter ($q$) versus time which gives the behaviour of $q$ from decelerating to accelerating phase. Thus our derived models have accelerated expansion at present epoch which is consistent with recent observations of Type Ia supernova (Riess et al. [3]; Perlmutter et al. [2]) and CMB anisotropies (Bennett et al. [60]; de Bernardis et al. [61]; Hanany et al. [62]).

Next, we assume that the coefficient of shear viscosity ($\eta$) is proportional to the expansion scalar ($\theta$) i.e. $\eta \propto \theta$, which leads to

$$\eta = \eta_0 \theta, \tag{32}$$

where $\eta_0$ is proportionality constant. Such relation has already been proposed in the physical literature as a physically plausible relation [34], [35].

Finally to conveniently specify the source, we assume the perfect gas equation of state, which may be written as

$$p = \gamma\rho, 0 \leq \gamma \leq 1. \tag{33}$$

Using Eqs. (30) and (32) into (26)−(28), we get the following expressions for the scale factors

$$A = a_1\sqrt{te^t}\exp\left[b_1\int\left[\left(te^t\right)^{-\frac{3}{2}(1+2\eta_0)}\right]dt\right], \tag{34}$$

$$B = a_2\sqrt{te^t}\exp\left[b_2\int\left[\left(te^t\right)^{-\frac{3}{2}(1+2\eta_0)}\right]dt\right], \tag{35}$$

$$C = a_3\sqrt{te^t}\exp\left[b_3\int\left[\left(te^t\right)^{-\frac{3}{2}(1+2\eta_0)}\right]dt\right]. \tag{36}$$

## IV. PHYSICAL AND GEOMETRIC PROPERTIES

The physical parameters such as directional Hubble factors ($H_i$), Hubble parameter ($H$), expansion scalar ($\theta$), spatial volume ($V$), deceleration parameter ($q$), anisotropy parameter ($A_m$) and shear scalar ($\sigma$) are given by

$$H_i = \frac{1}{2}\left(\frac{t+1}{t}\right) + b_i\left(te^t\right)^{-\frac{3}{2}(2\eta_0+1)}, \tag{37}$$

$$H = \frac{1}{2}\left(\frac{t+1}{t}\right), \tag{38}$$

$$\theta = \frac{3}{2}\left(\frac{t+1}{t}\right), \tag{39}$$

$$V = \left(te^t\right)^{\frac{3}{2}}, \tag{40}$$

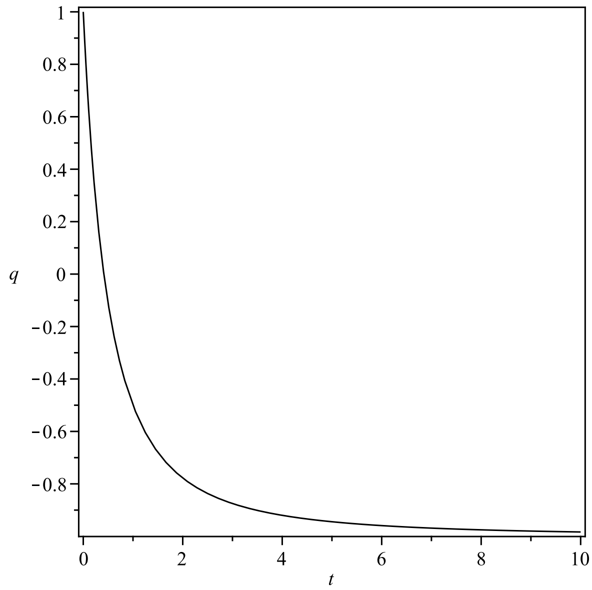


Figure 1. The plot of deceleration parameter $q$ versus $t$

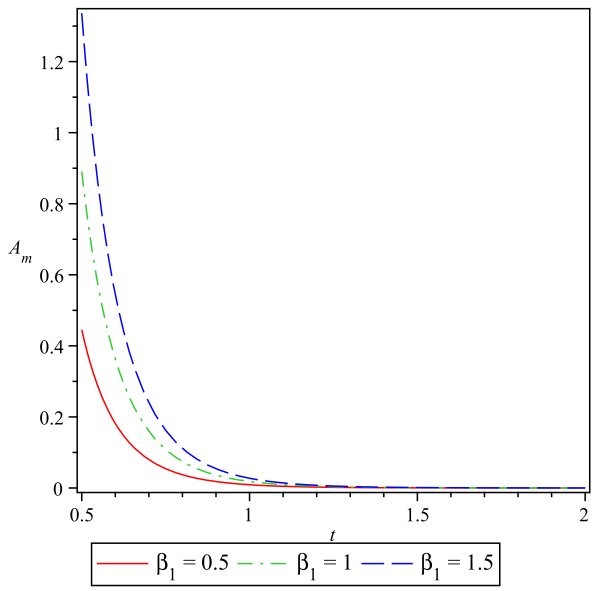


Figure 2. The plot of anisotropy parameter $A_m$ versus $t$. Here $\eta_0 = 0.1$

$$A_m = \frac{4}{3}\beta_1\left(\frac{t}{t+1}\right)^2\left(te^t\right)^{-3(2\eta_0+1)}, \tag{41}$$

$$\sigma^2 = \frac{1}{2}\beta_1\left(te^t\right)^{-3(2\eta_0+1)}, \tag{42}$$

where

$$\beta_1 = b_1^2 + b_2^2 + b_3^2,$$
$$\beta_2 = b_1b_2 + b_2b_3 + b_3b_1,$$
$$\beta_3 = b_2^2 + b_3^2 + b_2b_3. \tag{43}$$

The shear viscosity of the model reads as

$$\eta = \frac{3}{2}\eta_0\left(\frac{t+1}{t}\right). \tag{44}$$

Equations (15) and (16) lead to

$$p-\frac{3}{2}\xi\left(\frac{t+1}{t}\right)-\Lambda=-\frac{23}{36}\left(\frac{t+1}{t}\right)^2-\frac{1}{2}\beta_1\left(te^t\right)^{-3(2\eta_0+1)}, \tag{45}$$

$$\rho+\Lambda=\frac{3}{4}\left(\frac{t+1}{t}\right)^2-\frac{1}{2}\beta_1\left(te^t\right)^{-3(2\eta_0+1)}, \tag{46}$$

From Eqs. (40) and (39), we observe that the spatial volume is zero at $t=0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $t=0$ which is a big bang scenario. From Eqs. (34)–(36), we observe that the spatial scale factors are zero at the initial epoch $t=0$ and hence the model has a point type singularity [63]. We observe that proper volume increases exponentially as time increases. Thus, the models represent the inflationary scenario.

The dynamics of the mean anisotropic parameter depends on

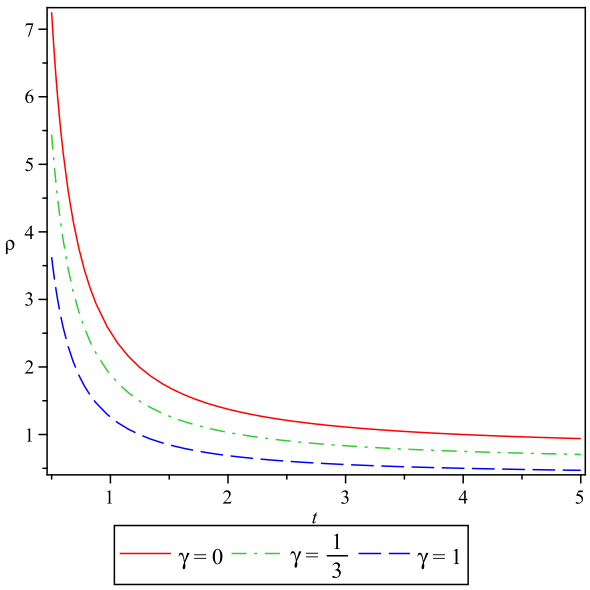


Figure 3. The plot of energy density $\rho$ versus $t$ for $\xi_0=\eta_0=0.1$, $\beta_2=1$, $\beta_3=0.5$

the constant $\beta_1=b_1^2+b_2^2+b_3^2$. From Eq. (41), we observe that at late time when $t\to\infty$, $A_m\to 0$. Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations. Figure 2 depicts the variation of anisotropy parameter ($A_m$) versus cosmic time $t$. From the figure, we observe that $A_m$ decreases with time and tends to zero as $t\to\infty$. Thus, the observed isotropy of the universe can be achieved in our model at present epoch.

Here, we solve the Eqs. (45) and (46) with (33) in the following cases:

### A. MODELS WITH VARIABLE Λ-TERM and CONSTANT ξ

Let us assume that the coefficient of bulk viscosity is constant, i.e. $\xi(t)=\xi_0=$ constant. Then the Eqs. (45) and

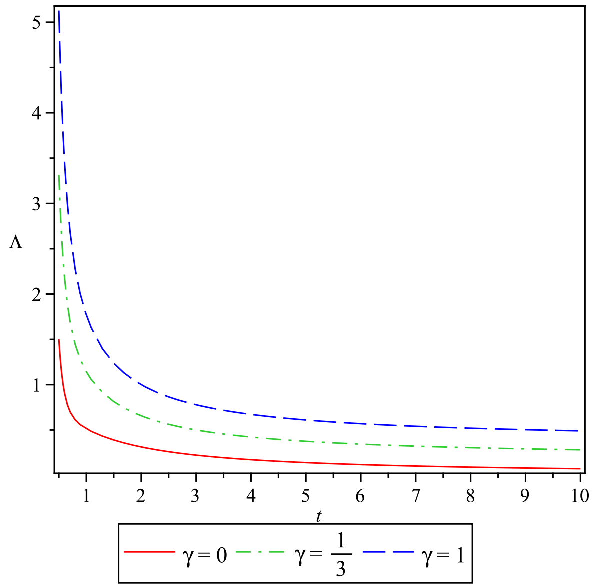


Figure 4. The plot of cosmological constant $\Lambda$ versus $t$ for $\xi_0=\eta_0=0.1$, $\beta_2=1$, $\beta_3=0.5$

(46) together with (33) leads the following expressions for energy density, pressure and cosmological constant:

$$\rho=\frac{1}{(1+\gamma)}\left[(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)}+\frac{1}{t^2}+\frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right], \tag{47}$$

$$p=\frac{\gamma}{(1+\gamma)}\left[(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)}+\frac{1}{t^2}+\frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right], \tag{48}$$

$$\Lambda=\frac{3}{4}\left(\frac{t+1}{t}\right)^2+\frac{1}{(1+\gamma)}\left[(\beta_2\gamma+\beta_3)\left(te^t\right)^{-3(2\eta_0+1)}-\frac{1}{t^2}-\frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right]. \tag{49}$$

From above relations (47)–(49), we can obtain four types of models:

- When $\gamma=0$, we obtain empty model.
- When $\gamma=\frac{1}{3}$, we obtain radiating dominated model.
- When $\gamma=-1$, we have the degenerate vacuum or false vacuum or $\rho$ vacuum model [64].
- When $\gamma=1$, the fluid distribution corresponds with the equation of state $\rho=p$ which is known as Zeldovich fluid or stiff fluid model [13], [65].

From Eq. (47), it is observed that the energy density $\rho$ is a decreasing function of time and $\rho>0$ always. The energy density has been graphed versus time in Figure 3 for $\gamma=0,\frac{1}{3},1$. It is apparent that the energy density remains positive in all three types of models. However, it decreases more sharply with the cosmic time in Zeldovich universe, compare to radiating dominated and empty fluid universes. Also it can be seen from the figure that $\rho$ decreases more sharply with time in radiating dominated universe, compare to empty universe.

Figure 4 describes the variation of cosmological term $\Lambda$ with time ($\rho$ and $\Lambda$ are in geometric units in entire paper) for $\gamma = 0, \frac{1}{3}, 1$. This is taken to be a representative case of physical viability of the models. In all three types of models, we observe that $\Lambda$ is decreasing function of time $t$ and it approaches a small positive value at late time (i.e. at present epoch). However, it decreases more sharply with the cosmic time in empty universe, compare to radiating dominated and stiff fluid universes. The $\Lambda$-term also decreases more sharply in radiating dominated universe, compare to stiff fluid universe. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure and drives the accelerated expansion of the universe. Recent cosmological observations [1]− [10] suggest the existence of a positive cosmological constant $\Lambda$ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological $\Lambda$-term. Thus, the nature of $\Lambda$ in our derived models are supported by recent observations.

*1) THERMODYNAMIC EQUATIONS:* The energy in a comoving volume is $U = \rho V$. The equation for production of entropy $S$ in a comoving volume due to the dissipative effects in a fluid with temperature $\mathbf{T}$ is given by

$$\mathbf{T}\dot{S} = \dot{U} + p\dot{V} = 3V(3\xi + 2\eta A_m)H^2. \tag{50}$$

In a cosmic fluid where the energy density and pressure of the cosmic fluid are functions of temperature only, $\rho = \rho(\mathbf{T})$, $p = p(\mathbf{T})$ and where the cosmic fluid has no net charge, we obtain easily [66]

$$S = \frac{V}{\mathbf{T}}(\rho + p). \tag{51}$$

From (50) and (51), we get the following expression for the entropy production rate in viscous Bianchi type-I universe

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta A_m)H^2}{\rho + p}. \tag{52}$$

For a fluid obeying the equation of state (33), Eqs. (51) and (52) reduce to

$$S = \frac{V}{\mathbf{T}}(1+\gamma)\rho, \tag{53}$$

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta A_m)H^2}{(1+\gamma)\rho}. \tag{54}$$

Equation (54) can be rewritten as

$$\frac{\dot{S}}{S} = \frac{\xi + 4\eta(\sigma^2/\theta^2)}{(1+\gamma)(\rho/\theta^2)}, \tag{55}$$

which gives the rate of change of entropy with time.

Let the entropy density be $s$ so that

$$s = \frac{S}{V} = \frac{(1+\gamma)\rho}{\mathbf{T}}. \tag{56}$$

It defines the entropy density in terms of the temperature.

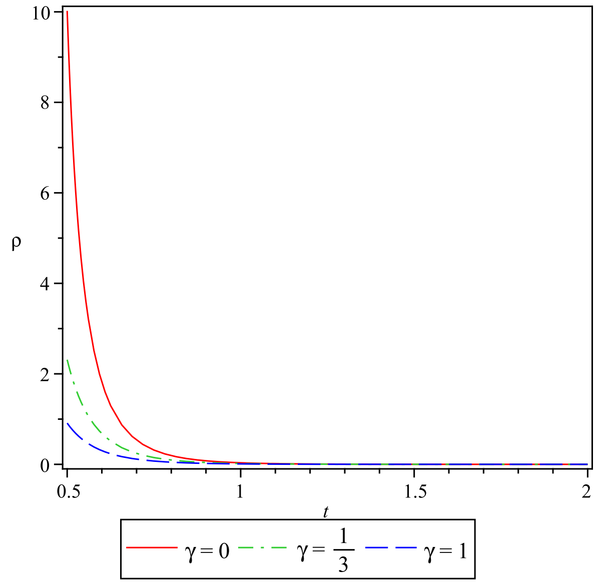


Figure 5. The plot of energy density $\rho$ versus $t$ for $\xi_0 = \eta_0 = 0.1$, $\beta_2 = 1$, $\beta_3 = 0.5$

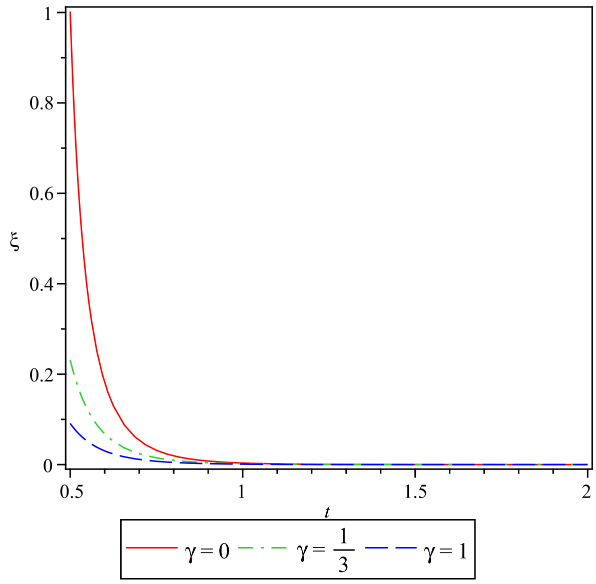


Figure 6. The plot of bulk viscosity coefficient $\xi$ versus $t$ for $\xi_0 = \eta_0 = 0.1$, $\beta_2 = 1$, $\beta_3 = 0.5$

The first law of thermodynamics may be written as

$$d(\rho V) + \gamma\rho dV = (1+\gamma)\mathbf{T}d\left(\frac{\rho V}{\mathbf{T}}\right), \tag{57}$$

which on integration, yields

$$\mathbf{T} \sim \rho^{\left(\frac{\gamma}{1+\gamma}\right)}. \tag{58}$$

From (56) and (57), one can get

$$s \sim \rho^{\left(\frac{1}{1+\gamma}\right)}. \tag{59}$$

The entropy in a comoving volume then varies according to

$$S \sim sV. \tag{60}$$

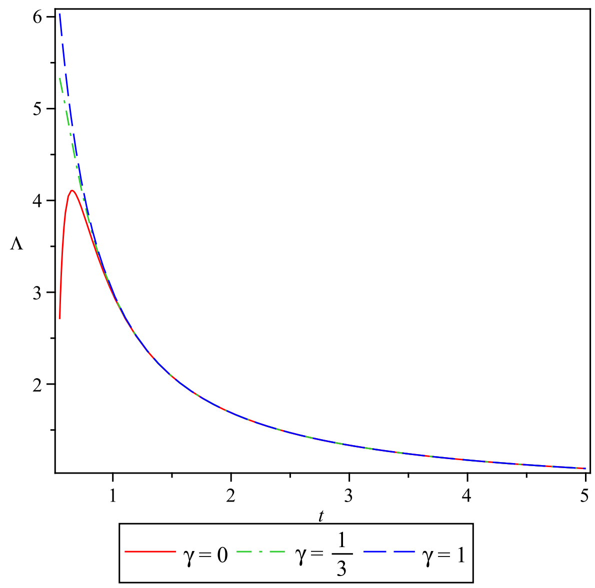


Figure 7. The plot of cosmological constant $\Lambda$ versus $t$ for $\xi_0 = \eta_0 = 0.1$, $\beta_2 = 1$, $\beta_3 = 0.5$

These equations are not valid for a vacuum fluid with $\gamma = -1$. For a Zel'dovich fluid ($\gamma = 1$), we get

$$\mathbf{T} \sim \rho^{\frac{1}{2}} \quad \text{and} \quad s \sim \rho^{\frac{1}{2}}, \tag{61}$$

so that the entropy density is proportional to the temperature.

Using (40) and (47), we find the respective temperature ($\mathbf{T}$), entropy density ($s$) and total entropy ($S$) from (58)−(60), as

$$\mathbf{T} = \mathbf{T}_0\left[\frac{1}{(1+\gamma)}\left\{(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)} + \frac{1}{t^2} + \frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right\}\right]^{\frac{\gamma}{1+\gamma}}, \tag{62}$$

$$s = s_0\left[\frac{1}{(1+\gamma)}\left\{(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)} + \frac{1}{t^2} + \frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right\}\right]^{\frac{1}{1+\gamma}}, \tag{63}$$

$$S = S_0\left(te^t\right)^{\frac{3}{2}}\left[\frac{1}{(1+\gamma)}\left\{(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)} + \frac{1}{t^2} + \frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)\right\}\right]^{\frac{1}{1+\gamma}}, \tag{64}$$

where $\mathbf{T}_0$, $S_0$ and $s_0$ are positive constants.

Now the entropy production rate of the model is given by

$$\frac{\dot{S}}{S} = \frac{3\left[\frac{3}{4}\xi_0\left(\frac{t+1}{t}\right)^2 + \eta_0\beta_1\left(\frac{t+1}{t}\right)\left(te^t\right)^{-3(2\eta_0+1)}\right]}{(\beta_2-\beta_3)\left(te^t\right)^{-3(2\eta_0+1)} + \frac{1}{t^2} + \frac{3}{2}\xi_0\left(\frac{t+1}{t}\right)}. \tag{65}$$

From above equation (65), we observe that the rate of change of entropy with time, i.e. the relation $\frac{\dot{S}}{S} > 0$. This also implies that the total entropy increases with time in Bianchi type-I model presented in this paper. This is in good agreement with second law of thermodynamics.

### B. MODELS WITH VARIABLE Λ-TERM AND $\xi \propto \rho$

Let us consider that $\xi = \xi_0\rho$. In this case we obtain the expressions for energy density, pressure, bulk viscosity and cosmological constant as follows:

$$\rho = \frac{1}{\left\{1+\gamma-3\left(\frac{t+1}{t}\right)\right\}} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right], \tag{66}$$

$$p = \frac{\gamma}{\left\{1+\gamma-3\left(\frac{t+1}{t}\right)\right\}} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right], \tag{67}$$

$$\xi = \frac{\xi_0}{\left\{1+\gamma-3\left(\frac{t+1}{t}\right)\right\}} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)} Biggr\right], \tag{68}$$

$$\Lambda = \frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \left[\beta_2 - \frac{\beta_4}{\left\{1+\gamma-3\xi_0\left(\frac{t+1}{t}\right)\right\}}\right]\left(te^t\right)^{-3(2\eta_0+1)}, \tag{69}$$

where

$$\beta_4 = b_1b_2 + b_3b_1 - b_2^2 - b_3^2.$$

In this case, we find the respective temperature ($\mathbf{T}$), entropy density ($s$) and total entropy ($S$) as given by

$$\mathbf{T} = \frac{\mathbf{T}_0}{\left\{1+\gamma-3\xi_0\left(\frac{t+1}{t}\right)\right\}} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right]^{\frac{\gamma}{1+\gamma}} \tag{70}$$

$$s = \frac{s_0}{\left\{1+\gamma-3\xi_0\left(\frac{t+1}{t}\right)\right\}} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right]^{\frac{1}{1+\gamma}} \tag{71}$$

$$S = \frac{S_0\left(te^t\right)^{\frac{3}{2}}}{\left(1+\gamma-3\xi_0\left(\frac{t+1}{t}\right)\right.} \times \left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right]^{\frac{1}{1+\gamma}} \tag{72}$$

The expression for entropy production rate of model is given by

$$\frac{\dot{S}}{S} = \frac{3\left[\frac{3}{4}\xi_0\left(\frac{t+1}{t}\right)^2 + \eta_0\beta_1\left(\frac{t+1}{t}\right)\left(te^t\right)^{-3(2\eta_0+1)}\right]}{\frac{(1+\gamma)}{\left\{1+\gamma-\frac{3}{2}\left(\frac{t+1}{t}\right)\right\}}\left[\beta_4\left(te^t\right)^{-3(2\eta_0+1)}\right]}. \tag{73}$$

Figures 5 and 7 depict the variation of energy density $\rho$ and cosmological term $\Lambda$ versus time for $\gamma = 0, \frac{1}{3}, 1$ respectively. The nature of $\rho$ and $\Lambda$ in this model are same as described in previous section 3.1.

Figure 6 plots the variation of bulk viscosity coefficient $\xi$ with time $t$. From this figure we observe that $\xi$ is a positive decreasing function of time and it approaches to a constant quantity which is near to zero for all three types of models $\gamma = 0, \frac{1}{3}, 1$. This is in good agreement with physical behaviour of $\xi$. However, it decreases more sharply with the cosmic time in empty universe, compare to radiating dominated and Zeldovich universes. It is also observed from the figure that $\xi$ decreases more sharply with time in radiating dominated universe, compare to empty universe.

From Eq. (73), we observe that $\frac{\dot{S}}{S} > 0$. This implies that the total entropy increases with time. This is reproducible with second law of thermodynamics.

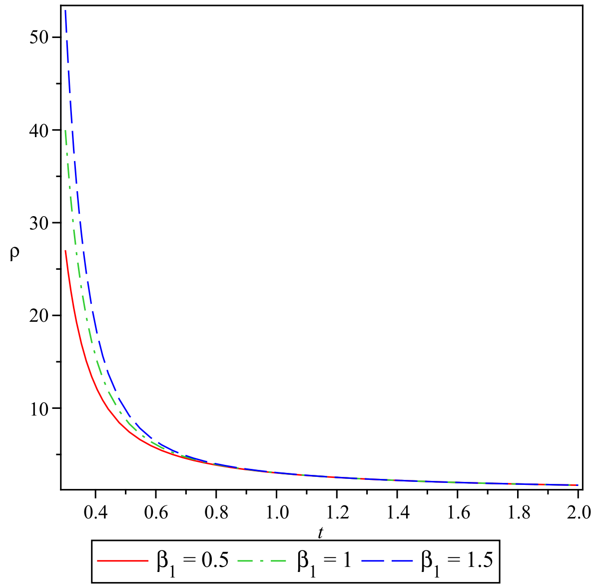


Figure 8. The plot of energy density $\rho$ versus $t$ for $\eta_0 = 0.1, \Lambda = 0$

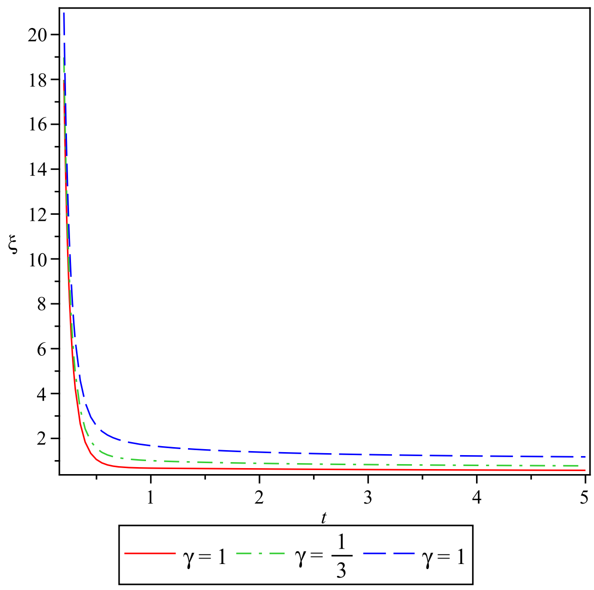


Figure 9. The plot of bulk viscosity coefficient $\xi$ versus $t$ for $\eta_0 = 0.1$, $\beta_4 = 1$, $\Lambda = 0$

*C. MODELS WITH CONSTANT* $\Lambda$*-TERM AND* $\xi(T)$

In this case, the expressions for energy density, isotropic pressure and bulk viscosity coefficient are respectively, given by

$$\rho = \frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda, \tag{74}$$

$$p = \gamma\left[\frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda\right], \tag{75}$$

$$\xi = \frac{(1+\gamma)(t+1)}{2t} + \frac{2\beta_4\, t}{3(t+1)}\left(te^t\right)^{-3(2\eta_0+1)} - \frac{2}{3t(t+1)} - \frac{2(1+\gamma)\Lambda\, t}{3(t+1)}. \tag{76}$$

In this case, we find the respective temperature ($\mathbf{T}$), entropy density ($s$), total entropy ($S$) and entropy production rate of model are given by

$$\mathbf{T} = \mathbf{T}_0\left[\frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda\right]^{\frac{\gamma}{(1+\gamma}}, \tag{77}$$

$$s = s_0\left[\frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda\right]^{\frac{1}{(1+\gamma}}, \tag{78}$$

$$S = S_0\left(te^t\right)^{\frac{3}{2}}\left[\frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda\right]^{\frac{1}{(1+\gamma}}, \tag{79}$$

$$\frac{\dot{S}}{S} = \frac{\frac{3}{4}\left(\frac{t+1}{t}\right)\left[\frac{3}{2}(1+\gamma)\left(\frac{t+1}{t}\right)^2 + 2(\beta_4 + 2\eta_0\beta_1)\left(te^t\right)^{-3(2\eta_0+1)} - \frac{2}{t^2} - 2(1+\gamma)\Lambda\right]}{(1+\gamma)\left[\frac{3}{4}\left(\frac{t+1}{t}\right)^2 + \beta_2\left(te^t\right)^{-3(2\eta_0+1)} - \Lambda\right]} \tag{80}$$

From Eq. (74), it is observed that the energy density $\rho$ is a decreasing function of time and $\rho > 0$ always. Figure 8 depicts the variation of energy density with cosmic time which shows this nature of energy density.

Figure 9 plots the variation of bulk viscosity coefficient $\xi$ with time $t$. From this figure we observe that $\xi$ is a positive decreasing function of time and it approaches to a constant quantity which is near to zero for all three types of models $\gamma = 0, \frac{1}{3}, 1$. This is in good agreement with physical behaviour of $\xi$. However, it decreases more sharply with the cosmic time in empty universe, compare to radiating dominated and Zeldovich universes. It is also observed from the figure that $\xi$ decreases more sharply with time in radiating dominated universe, compare to empty universe.

From Eq. (80), we observe that $\frac{\dot{S}}{S} > 0$. This implies that the total entropy increases with time. This is reproducible with second law of thermodynamics.

## V. COSMIC JERK PARAMETER

A convenient method to describe models close to $\Lambda$ CDM is based on the cosmic jerk parameter $j$, a dimensionless third derivative of the scale factor with respect to the cosmic time [67]− [71]. A deceleration-to-acceleration transition occurs for models with a positive value of $j_0$ and negative $q_0$. Flat $\Lambda$ CDM models have a constant jerk $j = 1$. The jerk parameter in cosmology is defined as the dimensionless third derivative of the scale factor with respect to cosmic time

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}. \tag{81}$$

and in terms of the scale factor to cosmic time

$$j(t) = \frac{(a^2H^2)''}{2H^2}. \tag{82}$$

where the 'dots' and 'primes' denote derivatives with respect to cosmic time and scale factor, respectively. One can rewrite Eq. (81) as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{83}$$

Eqs. (31) and (83) reduce to

$$j(t) = 1 - \frac{6}{(t+1)^2} + 64. \tag{84}$$

This value overlaps with the value $j \simeq 2.16$ obtained from the combination of three kinematical data sets: the gold sample of type Ia supernovae [8], the SNIa data from the SNLS project [72], and the X-ray galaxy cluster distance measurements [73] for

$$t = 3.45 \times 10^{-2}\kappa - \frac{50}{\kappa} - 1, \tag{85}$$

where $\kappa = 10^4[8.41 + 1.45\sqrt{(14.4 + 33.6)}]^{\frac{1}{3}}$.

## VI. CONCLUSIONS

In this paper, a class of cosmological models are presented with variable deceleration parameter $q$ and cosmological term $\Lambda$ in spatially homogeneous and anisotropic Bianchi type-I space-time in presence of bulk and shear viscosity. To find the explicit solution, we have considered a time dependent deceleration parameter which yields a scale factor as $a(t) = \sqrt{te^t}$. We obtain that in the early stage the universe was decelerating where as the universe is accelerating at present epoch which is corroborated from the recent supernovae Ia observation [1−10]. The parameter $H_i$, $H$, $\theta$, and $\sigma$ diverge at the initial singularity. There is a Point Type singularity [63] at $t = 0$ in the model. The rate of expansion slows down and finally tends to zero as $T \to \infty$. The pressure, energy density and scalar field become negligible whereas the scale factors and spatial volume become infinitely large as $T \to \infty$, which would give essentially an empty universe.

The main features of the models are as follows:

- The models are based on exact solutions of the Einstein's field equations for the anisotropic Bianchi-I space-time in presence of a dissipative fluid with variable $\Lambda$-term.
- The model represents expanding, accelerating, shearing and non-rotating universe.
- The cosmological constant has been assumed to represent the energy density of vacuum, which has a potentially important contribution in the dynamics of the evolution of universe. The cosmological constant is observed to have a small and positive value at late times. The nature of decaying vacuum energy density $\Lambda(t)$ in our derived models are supported by recent cosmological observations. These observations on magnitude and red-shift of type Ia supernova [1−10] suggest that our universe may be an accelerating one with induced cosmological density through the cosmological $\Lambda$-term.
- In literature it is a common practice to consider constant deceleration parameter to solve the field equations. Now for a Universe which was decelerating in past and accelerating at present epoch, the DP must show signature flipping. Thus, our consideration of DP to be variable is physically justified. Our derived model is decelerating in past and accelerating at present epoch.
- The present models have a transition of the universe from the early deceleration phase to the recent acceleration phase (see, Figure 1) which is in good agreement with recent observations [74].
- The cosmic jerk parameter in our descended models is also found to be in good agreement with the recent data of astrophysical observations namely the gold sample of type Ia supernovae [8], the SNIa data from the SNLS project [72], and the X-ray galaxy cluster distance measurements [73].
- The shear viscosity is observed to be responsible for the faster removal of initial anisotropies in the universe. This can be seen from the expression of anisotropy parameter (41). Hence, the isotropy observed in the present universe, is a possible consequence of viscous effects in the cosmic fluid.

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