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Bianchi Type-IX Universe with Magnetized Anisotropic Dark Energy

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ABSTRACT

A Bianchi type-IX model is studied in the presence of magnetized anisotropic dark energy. The energy momentum tensor consists of anisotropic fluid with anisotropic equation of state $p = \omega\rho$ and a uniform magnetic field of energy density ρ_b .The solutions to the Einstein field equations are obtained using the condition that expansion is proportional to the shear scalar. Also the physical properties of Bianchi type-IX model are discussed in the presence and absence of magnetic field.

Keywords: *Dark energy, Bianchi type-IX ; Anisotropic Fluid, Magnetic field.*

1. INTRODUCTION

The universe is highly homogeneous and isotropic on large scale structure [1] and with cosmic background radiation [2-6]. From the recent cosmological observations [7-9], cosmologists concluded that our universe is in an accelerated expansion. This acceleration is due to presence of more than 70% of dark energy (DE) in the universe. The paramount characteristic of the DE is constant or slightly changing as the universe expands, but we do not know the nature of DE very well [10-18]. The nature of DE and dark matter (DM) remains unknown and in the near future we can hope that the observations from Large Hadron Collider (LHC) will be able to provide hints on the nature of DM and DE. Consequently, the expansion of our present universe is accelerating rather than slowing down [19-23].

The Bianchi type homogeneous models are the simplest among anisotropic models whose spatial sections are flat but the contraction or expansion rate are direction dependent. In the last few years, most of the relativists are studying DE cosmological models. It is assumed that, these models which are spatially homogeneous and anisotropic generates an isotropic pressure and follows a simple equation of state in the form $p = \omega\rho$, where ρ is the energy density, p is the isotropic pressure and ω is the equation of state (EoS) parameter which is not necessarily constant. The models of universe with anisotropic background are under consideration of the theoretical arguments and recent experimental data which supports the existence of anisotropy phase. The studies of all the possible effects of anisotropy in the early universe on present day observations are mostly due to the simplicity and evolution of modern day universe into FRW universe. For studying the possible effects of anisotropy in the early universe of present day observations, many researchers [24-29] have investigated Bianchi models. From different point of view, the binary mixture of perfect fluid and DE has been studied [30]. Some authors [31-37] have studied anisotropic DE models with constant deceleration parameter.

The existence of magnetic fields on galactic scale is well-known today. The importance of a magnetic field for a variety of astrophysical phenomena is found out [38]. Also, the magnetic field could have a cosmological origin is suggested by Horrison[39]. Naturally it is possible to include magnetic fields in the energy momentum tensor of the early universe. The anisotropic cosmological models are more general in Einstein system of field equations than Robertson-Walker model [40]. Several authors [41-50] have discussed the presence of primordial magnetic fields in the early stages of evolution of the universe. The adiabatic compressions in clusters of galaxies are strongly responsible for the magnetic field. Large scale magnetic field gives rise to anisotropies in the universe. The evolution of the shear anisotropy is dominated by the anisotropic pressure created by the magnetic fields and it decays slower if the pressure was isotropic [51-52]. Such fields can be generated at the ends of an inflationary epoch [53-57]. The most significant contribution of the anisotropic magnetic field model is in the evolution of galaxies and stellar objects. Several authors [58-64] have investigated the Bianchi type cosmological models with a magnetic field in different context.

In this paper, we investigate the effects of magnetic field on the dynamics of Bianchi type-IX model in the presence of anisotropic DE. The solutions to the Einstein field equations are obtained using the condition that expansion is proportional to the shear scalar. The physical properties of Bianchi type-IX model are discussed in the presence and absence of magnetic field.

2. FIELD EQUATIONS

Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y\,dxdz \tag{1}$$

where a , b are scale factors and are functions of cosmic time t .

The model has one transverse direction x, two equivalent longitudinal directions y and z.

King and Coles [70] and Jacobs [62] used the magnetized perfect fluid energy momentum tensor to discuss the effects of magnetic field on the evolution of the universe filled with perfect fluid.

The energy-momentum tensor for the magnetized anisotropic DE fluid is in the form

$$T_j^i = diag\left[-\rho-\rho_b,\, p_x-\rho_b,\, p_y+\rho_b,\, p_z+\rho_b\right] \qquad (2)$$

where ρ_b is the energy density of magnetic fluid, ρ is the energy density of the fluid and p_x, p_y, p_z are pressures on x, y, z axes respectively.

The equation of state for an anisotropic fluid is taken of the form $p=\omega\rho$, where ω is not necessarily constant [17] :

$$T_j^i = diag\begin{bmatrix}-\rho-\rho_b, (\omega+\delta)\rho-\rho_b, (\omega+\gamma)\rho+\rho_b, \\ (\omega+\gamma)\rho+\rho_b\end{bmatrix}. \qquad (3)$$

Here ω is deviation free parameter, $\omega_x=\omega+\delta, \omega_y=\omega+\gamma, \omega_z=\omega+\gamma$ are the directional EoS parameters on x, y and z axes respectively. δ, γ and γ are the deviations on x, y and z axes respectively.

If deviation parameters are zero then equation (2) represents the energy momentum tensor for the isotropic fluid and magnetic field [70].

The Einstein field equations are given by

$$R_{ij}-\frac{1}{2}Rg_{ij}=-T_{ij}, \qquad (4)$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy-momentum tensor for a magnetized anisotropic fluid.

For Bianchi type-IX metric, using equation (3), the field equations (4) take the form

$$2\frac{\dot{a}\dot{b}}{ab}+\frac{1}{b^2}+\frac{\dot{b}^2}{b^2}-\frac{a^2}{4b^4}=\rho+\rho_b \qquad (5)$$

$$2\frac{\ddot{b}}{b}+\frac{1}{b^2}+\frac{\dot{b}^2}{b^2}-\frac{3a^2}{4b^4}=-(\omega+\delta)\rho+\rho_b \qquad (6)$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}+\frac{a^2}{4b^4}=-(\omega+\gamma)\rho-\rho_b, \qquad (7)$$

Where over dot $(\cdot)$ denotes the differentiation with respect to t.

The energy conservation equation $T^i_{j;i}=0$ gives two equations; one for anisotropic fluid and second for magnetic fluid [70]:

$$\dot{\rho}+(1+\omega)\rho\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)+\rho\left(\delta\frac{\dot{a}}{a}+2\gamma\frac{\dot{b}}{b}\right)=0 \qquad (8)$$

$$\rho_b=\frac{\beta}{b^4}, \qquad (9)$$

where β is a constant of integration.

The energy conservation equation $T^i_{j;i}=0$ for the anisotropic fluid can be decomposed into two parts:

$$T^i_{j;i}=T'^i_{j;i}+\tau^i_{j;i},$$

where $\tau^i_{j;i}$ is the last term in equation (8) which arises due to the anisotropy in the fluid and $T'^i_{j;i}$ represents the deviation free part of the $T^i_{j;i}$.

The average scale factor and the volume V is defined as:

$$A=(ab^2)^{1/3}, \qquad V=A^3=ab^2. \qquad (10)$$

The anisotropy parameter of the expansion Δ is characterized by the mean and directional Hubble parameters given by

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \qquad (11)$$

where the mean Hubble parameter H is given by

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right),$$

And H_i $(i=1,2,3)$ represent the directional Hubble parameters are defined as

$$H_x=\frac{\dot{a}}{a},\; H_y=H_z=\frac{\dot{b}}{b}.$$

For $\Delta\to 0$, the anisotropy of the expansion results in isotropic expansion of the universe.

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The scalar expansion Θ is given by

$$\Theta = u^a_{;a} = \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right). \quad (12)$$

Also, the shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ab}\sigma^{ab} = \frac{1}{3}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)^2. \quad (13)$$

It is mentioned here that any universe model become isotropic for the diagonal energy momentum tensor when $t \to \infty$ $\Delta \to 0$, $V \to \infty$ and ($T^{00} > 0$, for $\rho > 0$) [5, 33].

3. SOLUTION OF THE FIELD EQUATIONS

The field equations (5-7) are three linearly independent equations with six unknowns a, b, ρ, ω, δ, γ. Hence we need three extra conditions to solve the field equations completely.

Let us take as in Akarsu and Kilinc [33]

$$\tau^i_{j;i} = \rho\left(\delta\frac{\dot{a}}{a} + 2\gamma\frac{\dot{b}}{b}\right) = 0. \quad (14)$$

Using equation (14), equation (8) reduces to

$$T'^i_{j;i} = \dot{\rho} + (1+\omega)\rho\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right) = 0. \quad (15)$$

We conclude that the trivial solution of equation (14) is $\delta(t) = \gamma(t) = 0$ or equation (14) is satisfied when the ratio of expansion rate on x-axis to y-axis is equal to $-2\frac{\gamma}{\delta}$.

To obtain more general solution, the deviation parameter δ and γ are assumed to be [33] :

$$\delta(t) = n\frac{2}{3}\frac{\dot{b}}{b}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\frac{1}{\rho} \quad (16)$$

and

$$\gamma(t) = -n\frac{1}{3}\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\frac{1}{\rho}, \quad (17)$$

where δ & γ are dimensionless parameters and n is the real dimensionless constants that parameterizes the deviation from EoS parameter. The anisotropy of the DE is measured using the relation $\frac{\delta(t) - \gamma(t)}{\omega(t)}$ and for $n = 0$, DE is found to be isotropic.

Secondly, we use the condition that Θ in the model is proportional to the shear σ. Cosmic shear σ represents an effect of distortion of the image of distant galaxies due to deflection of light by matter, as predicted by general relativity. Metric expansion Θ is a key feature of Big Bang cosmology and is modeled mathematically with the Friedmann-Lemaitre-Walker (FLRW) metric. The metric expansion space is the averaged increase of metric (i.e measured) distance between distant objects in the universe with time. According to Throne [60], observations of velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately [63, 65] and red shift studies place the limit $\frac{\sigma}{H} \le .30$ where σ is the shear and H is the Hubble constant. Collins [5] discussed the physical significance of this condition for perfect fluid and baro tropic EoS in a more general case. In many papers [63, 67-69] the condition for the solutions of cosmological models is given

$$a = b^m, \quad (18)$$

where m is a positive constant and $m \ne 1$.

Subtracting equation (6) from equation (7) and using equations (16) and (17) there in, we get

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{b^2} - \frac{1}{b^2} + \frac{a^2}{b^4} = \frac{n}{3}\left(\frac{\dot{a}^2}{a^2} + 4\frac{\dot{a}\dot{b}}{ab} + 4\frac{\dot{b}^2}{b^2}\right) - \frac{2\beta}{b^4}. \quad (19)$$

Using (18), equation (19) reduces to

$$2\ddot{b} + \frac{2\lfloor 3(m^2-1) - nl \rfloor}{3(m-1)}\frac{\dot{b}^2}{b} = \frac{2}{b(m-1)} - \frac{4\beta}{b^3(m-1)} - \frac{2b^{2m}}{b^3(m-1)}, \quad (20)$$

where $l = (m+2)^2$.

Replacing $\dot{b} = f(b)$ in equation (20), it simplifies to

$$\frac{df^2}{db} + \frac{2\lfloor 3(m^2-1) - nl \rfloor}{3(m-1)}\frac{f^2}{b} = \frac{2}{b(m-1)} - \frac{4\beta}{b^3(m-1)}, - \frac{2b^{2m}}{b^3(m-1)} \quad (21)$$

whose solution is

$$f^2 = \left(\frac{db}{dt}\right)^2 = \frac{3}{3(m^2-1)-nl} - \frac{6\beta b^{-2}}{3m^2-3m-nl} + cb^{\frac{-2[3(m^2-1)-nl]}{3(m-1)}} - \left(\frac{3}{6m^2-6m-nl}\right)\frac{b^{2m}}{b^2}, \tag{22}$$

where c is a constant of integration.

Thus the space-time given in equation (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{3}{3(m^2-1)-nl} - \frac{6\beta T^{-2}}{3m^2-3m-nl} + cT^{\frac{-2[3(m^2-1)-nl]}{3(m-1)}} - \left(\frac{3}{6m^2-6m-nl}\right)\frac{T^{2m}}{T^2}\right]} + T^{2m}dx^2 + T^2dy^2 + (T^2\sin^2 y + T^{2m}\cos^2 y)dz^2 - 2T^{2m}\cos y\,dxdz \tag{23}$$

where $b = T$.

4. SOME PHYSICAL FEATURES OF THE MODEL

The directional and the mean Hubble parameters will take the form

$$H_x = mH_y = m\left(\frac{3T^{-2}}{3(m^2-1)-nl} - \frac{6\beta T^{-4}}{3m^2-3m-nl} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2}$$

$$H = \frac{(m+2)}{3}\left(\frac{3T^{-2}}{3(m^2-1)-nl} - \frac{6\beta T^{-4}}{3m^2-3m-nl} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2} \tag{24}$$

The Hubble parameter H and directional Hubble parameters H_x, H_y decrease with the increase in time and approach to zero as $T \to \infty$ *for* $n < \frac{3m^2+3m-6}{l}$.

The volume of the universe is given by

$$V = T^{m+2}. \tag{25}$$

The anisotropy expansion parameter becomes

$$\Delta = 2\frac{(m-1)^2}{(m+2)^2}. \tag{26}$$

The expansion scalar takes the form

$$\Theta = 3H = (m+2)\left(\frac{3T^{-2}}{3(m^2-1)-nl} - \frac{6\beta T^{-4}}{3m^2-3m-nl} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2} \tag{27}$$

and the shear scalar becomes

$$\sigma^2 = \frac{(m-1)^2}{3}\left(\frac{3T^{-2}}{3(m^2-1)-nl} - \frac{6\beta T^{-4}}{3m^2-3m-nl} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right) \tag{28}$$

We note that, spatial volume is zero at initial epoch and increases as $T \to \infty$. The expansion and shear scalar decrease with the increase in T. Thus the universe starts evolving with the zero volume at the initial epoch infinite rate of expansion which slows down for the later times of the universe. The shear scalar, shear expansion and Hubble parameter reduce due to the component of magnetic field.

Using equations (5) and (11), one can obtain most general form of the energy density

$$\rho = 3H^2\left(1-\frac{\Delta}{2}\right) + \frac{4b^2-a^2}{4b^4} - \frac{\beta}{b^4}. \tag{29}$$

The energy density for the model using equation (24) and (26) turns out to be

$$\rho = \left(\frac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\frac{1}{T^2} - \left(\frac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\frac{\beta}{T^4} + c(2m+1)T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \left(\frac{6m^2+18m-nl+12}{4(6m^2-6m-nl)}\right)\frac{T^{2m}}{T^4} \tag{30}$$

The energy condition $\rho \geq 0$ leads that energy density of the anisotropic DE is reduced by the magnetic field. This turns out to be infinite at the initial epoch, its value decreases with the increase in time and converges to

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zero as $T \to \infty$ with condition $n < \dfrac{3m^2+3m-6}{l}$ as shown in Figure 1(a). However energy density decreases after a big bang but it starts increasing and becomes infinite as $T \to \infty$ with condition $n > \dfrac{3m^2+3m-6}{l}$ as shown in Figure 1(b).

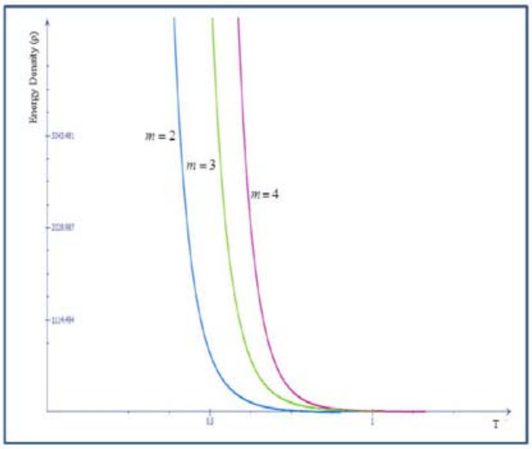

Figure 1 (a). Evolution of Energy Density (ρ), verses T with the condition $n < \frac{3m^2+3m-6}{l}$

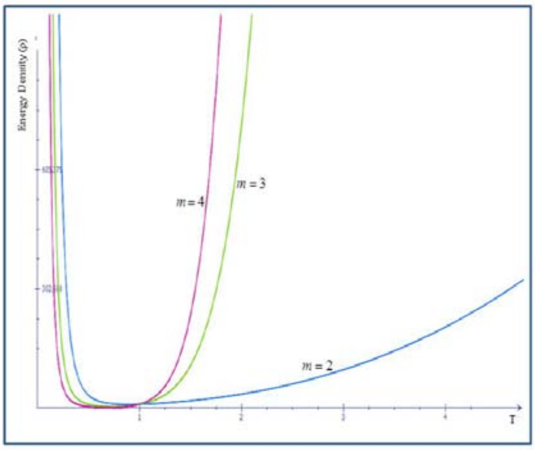

Figure 1 (b). Evolution of Energy Density (ρ) verses T with the condition $n > \frac{3m^2+3m-6}{l}$

Using equations (24) and (30), the deviation parameters $\delta(t)$ and $\gamma(t)$ become

$$\delta(T) = \frac{\dfrac{2n(m+2)}{3}\left\{\left(\dfrac{3}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} - \left(\dfrac{6}{3m^2-3m-nl}\right)\dfrac{\beta}{T^4} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \dfrac{3}{6m^2-6m-nl}\dfrac{T^{2m}}{T^4}\right\}}{\left\{\left(\dfrac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} - \left(\dfrac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\dfrac{\beta}{T^4} + c(2m+1)T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \left(\dfrac{6m^2+18m-nl+12}{4(6m^2-6m-nl)}\right)\dfrac{T^{2m}}{T^4}\right\}} \quad (31)$$

$$\gamma(T) = \frac{-\dfrac{mn(m+2)}{3}\left\{\left(\dfrac{3}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} - \left(\dfrac{6}{3m^2-3m-nl}\right)\dfrac{\beta}{T^4} + cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \dfrac{3}{6m^2-6m-nl}\dfrac{T^{2m}}{T^4}\right\}}{\left\{\left(\dfrac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} - \left(\dfrac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\dfrac{\beta}{T^4} + c(2m+1)T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \left(\dfrac{6m^2+18m-nl+12}{4(6m^2-6m-nl)}\right)\dfrac{T^{2m}}{T^4}\right\}} \quad (32)$$

The deviation free EoS parameter can be obtained by using the expression for directional Hubble parameter and energy density in equation (15) as

$$\omega(T) = -1 - \frac{\left\{-2\left(\dfrac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} + \left(\dfrac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\dfrac{4\beta}{T^4} - \dfrac{2c(2m+1)(3m^2+3m-nl-6)}{3(m-1)}T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \left(\dfrac{(2m-4)(6m^2+18m-nl+12)}{4(6m^2-6m-nl)}\right)\dfrac{T^{2m}}{T^4}\right\}}{(m+2)\left\{\left(\dfrac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\dfrac{1}{T^2} - \left(\dfrac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\dfrac{\beta}{T^4} + c(2m+1)T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}} - \left(\dfrac{6m^2+18m-nl+12}{4(6m^2-6m-nl)}\right)\dfrac{T^{2m}}{T^4}\right\}} \quad (33)$$

The anisotropy measure of anisotropic fluid is given by

$$\frac{\delta-\gamma}{\omega}=\frac{\frac{n(m+2)^3}{3}\left\{\left(\frac{3}{3(m^2-1)-nl}\right)\frac{1}{T^2}-\left(\frac{6}{3m^2-3m-nl}\right)\frac{\beta}{T^4}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right\}}{\left\{-m\left(\frac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\frac{1}{T^2}+(m-2)\beta\left(\frac{3m^2+9m-nl+6}{3m^2-3m-nl}\right)\frac{1}{T^4}+c(2m+1)\frac{(3m^2+3m-2nl-6)}{3(m-1)}T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}+\left(\frac{(3m-2)(6m^2+18m-nl+12)}{4(6m^2-6m-nl)}\right)\frac{T^{2m}}{T^4}\right\}} \tag{34}$$

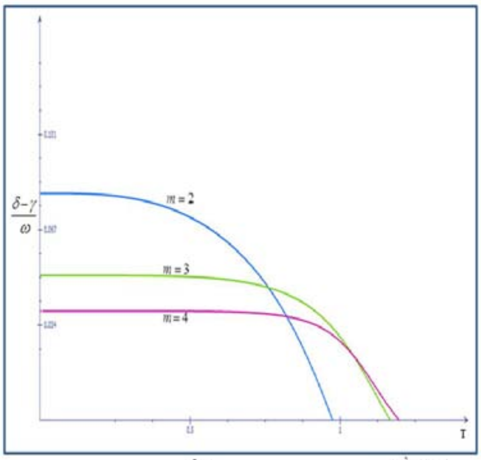

Figure 2 (a). Evolution of $\frac{\delta-\gamma}{\omega}$ verses T with the condition $n<\frac{3m^2+3m-6}{l}$

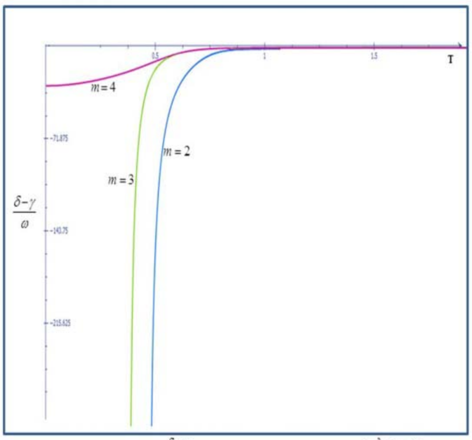

Figure 2 (b). Evolution of $\frac{\delta-\gamma}{\omega}$ verses T with the condition $n>\frac{3m^2+3m-6}{l}$

The deviation parameter $\delta(T)$ and $\gamma(T)$ are found to be finite at $T=0$ and converges to $\frac{2n(m+2)}{(3m^2+6m-nl)}$ and $\frac{-n\,m\,(m+2)}{(3m^2+6m-nl)}$ respectively as $T\to\infty$ land $n<\frac{3m^2+3m-6}{l}$. The anisotropy measure of DE $\frac{\delta-\gamma}{\omega}$ is constant at $T=0$ converges to $-\frac{(m+2)^3}{(3m^2+6m-nl)}$ for the later time of the universe with the condition that $n<\frac{3m^2+3m-6}{l}$. We note that the anisotropy of the dark energy does not vanish throughout the evolution of universe. Now we check the behavior of ω for $n<\frac{3m^2+3m-6}{l}$. For the earlier times of the universe, the deviation free EoS parameter of the DE is given by $\omega=-1+\frac{2(3m^2+3m-6-nl)}{3(m^2+m-2)}$ which shows that expansion in the universe may be in the quintessence region after big bang. Also $\omega\to-1+\frac{4}{(m-2)}$ as $T\to\infty$. Hence for later times, ω represent EoS of cosmological constant for $m=2$ and ω may result in quintessence region for $m>2$. Thus the model represents the accelerating expanding universe.

5. SPECIAL CASES

5.1 Model With $\beta=0$

In the absence of magnetic field (i.e. $\beta\to\infty$), the model (24) reduce to the form

$$ds^2=-\frac{dT^2}{\left[\frac{3}{3(m^2-1)-nl}+cT^{\frac{-2[3(m^2-1)-nl]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^2}\right]}+T^{2m}dx^2+T^2dy^2+(T^2\sin^2 y+T^{2m}\cos^2 y)dz^2-2T^{2m}\cos y\,dxdz \tag{35}$$

For the model (35), the energy density, the directional and the mean Hubble parameter is given by

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$$\rho=\left(\frac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\frac{1}{T^2}+c(2m+1)T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\left(\frac{6m^2+18m-nl+12}{4(6m^2-6m-nl)}\right)\frac{T^{2m}}{T^4} \quad (36)$$

$$H_x=mH_y=m\left(\frac{3T^{-2}}{3(m^2-1)-nl}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2}$$

$$H=\frac{m+2}{3}\left(\frac{3T^{-2}}{3(m^2-1)-nl}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2}. \quad (37)$$

We conclude that the dynamical H, H_x, H_y and ρ are infinite for earlier times and converge to zero as $T\to\infty$. The shear expansion and scalar are

$$\Theta=3H=(m+2)\left(\frac{3T^{-2}}{3(m^2-1)-nl}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right)^{1/2}. \quad (38)$$

From equation (29), we get

$$\sigma^2=\frac{(m-1)^2}{3}\left(\frac{3T^{-2}}{3(m^2-1)-nl}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right). \quad (39)$$

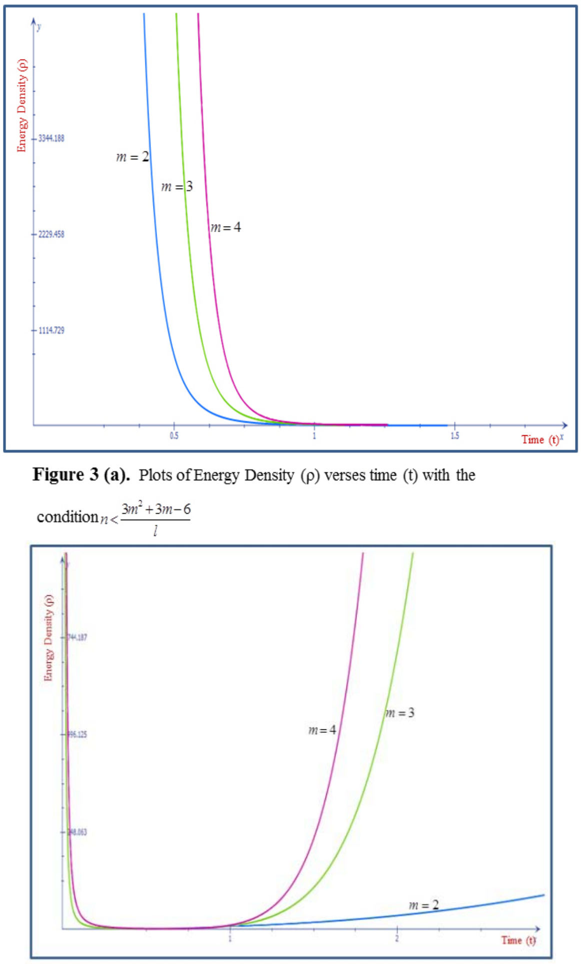

Figure 3 (a). Plots of Energy Density (ρ) verses time (t) with the condition $n<\frac{3m^2+3m-6}{l}$

Figure 3 (b). Plots of Energy Density (ρ) verses time (t) with the condition $n>\frac{3m^2+3m-6}{l}$

The expansion in the universe is infinite at the initial epoch and decreases with the increase in time for $n<\frac{3m^2+3m-6}{l}$ as shown in Figure 3(a), however if $n>\frac{3m^2+3m-6}{l}$ then the universe starts expanding at $T=0$ and expands indefinitely as $T\to\infty$ as shown in Figure 3(b).

The anisotropy expansion parameter of the expansion and anisotropy measure of fluid becomes

$$\Delta=2\frac{(m-1)^2}{(m+2)^2}. \quad (40)$$

From equation (34), we get

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$$\frac{\delta-\gamma}{\omega}=\frac{\frac{n(m+2)^3}{3}\left\{\left(\frac{3}{3(m^2-1)-nl}\right)\frac{1}{T^2}+cT^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}-\frac{3}{6m^2-6m-nl}\frac{T^{2m}}{T^4}\right\}}{\left\{-m\left(\frac{3m^2+6m-nl}{3(m^2-1)-nl}\right)\frac{1}{T^2}+c(2m+1)\frac{(3m^2+3m-2nl-6)}{3(m-1)}T^{\frac{-2[3m^2+3m-nl-6]}{3(m-1)}}+\left(\frac{(3m-2)(6m^2+18m-nl+12)}{4(6m^2-6m-nl)}\right)\frac{T^{2m}}{T^4}\right\}} \quad (41)$$

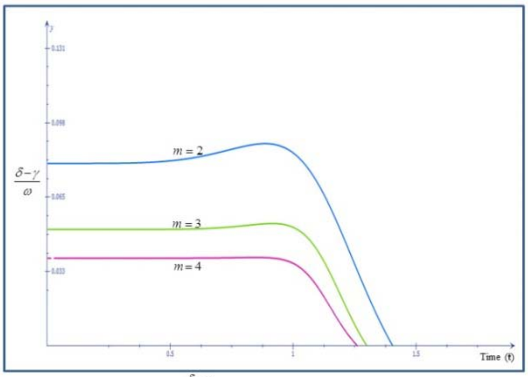

Figure 4 (a). Plots of $\frac{\delta-\gamma}{\omega}$ verses time (t) with the condition $n<\frac{3m^2+3m-6}{l}$

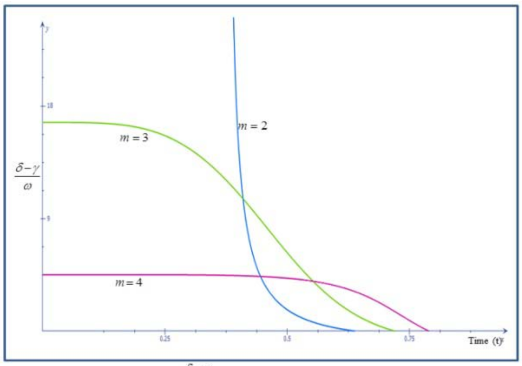

Figure 4 (b). Plots of $\frac{\delta-\gamma}{\omega}$ verses time (t) with the condition $n>\frac{3m^2+3m-6}{l}$

Here we note that, the value of $\frac{\delta-\gamma}{\omega}$ first remains and then decreases with increase in time as $T\to\infty$ with the condition that $n<\frac{3m^2+3m-6}{l}$ as shown in Figure 4(a), however the value of $\frac{\delta-\gamma}{\omega}$ decreases with the increase in time and becomes infinite as $T\to\infty$ with condition $n>\frac{3m^2+3m-6}{l}$ as shown in Figure 4(b).

5.2 Model With $m=1$

For $m=1$, from equation (18) we have

$$a=b\,, \quad (42)$$

also the metric (1) takes the form

$$ds^2=-dt^2+a^2(dx^2+dy^2+dz^2)-2a^2\cos y\,dxdz\,, \quad (43)$$

which represents homogeneous and isotropic universe.

Using equation (42) in equation (19) we get,

$$3n\frac{\dot{a}^2}{a^2}-\frac{2\beta}{a^4}=0\,, \quad (44)$$

whose solution is

$$a(t)=\sqrt{2}\left(\sqrt{\frac{2\beta}{3n}}\,t+c\right)^{1/2}. \quad (45)$$

The Hubble parameter is found to be

$$H=\frac{\dot{a}}{a}=\frac{\beta}{2\beta t+k} \quad (46)$$

where $k=c\sqrt{6n\beta}$

and the expansion scalar takes the form

$$\Theta=3H=\frac{3\beta}{2\beta t+k}. \quad (47)$$

While the energy density is

$$\rho=\frac{3\beta(2-n)}{8\beta t^2+12nc^2+8kt}\quad. \quad (48)$$

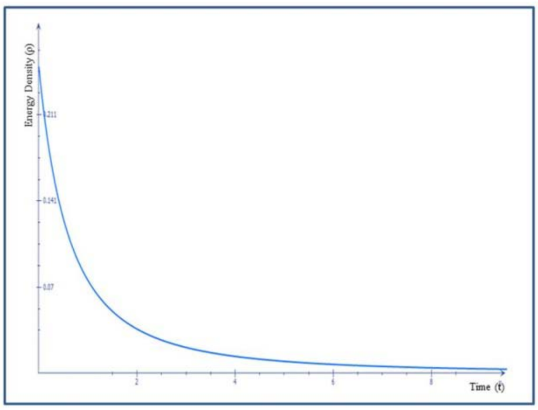

Figure 5. Evolution of (ρ) verses time (t) with n = 1.

We have found that, the Hubble parameter and expansion scalar are constant at initial epoch i.e. at $t = 0$ and decreases with increase in time. Hence, the universe is expanding for the earlier times of the universe. The energy density decreases with the increase in time as shown in Figure (3).

From equation (45), we conclude that for the radiation dominated universe $a(t)\,\alpha\, t^{1/2}$ [71-72] indicating the model represents expanding universe. The model is isotropic as the anisotropic parameter $\Delta = 0$. The deviation parameters $\delta(t)$, $\gamma(t)$ are constants and hence the anisotropy of the DE $\frac{\delta-\gamma}{\omega}$ is also constant for the model $m = 1$. The anisotropy of the DE vanishes if we choose the dimensionless constant n to be zero.

6. SUMMARY AND CONCLUSION

In this paper we have constructed Bianchi type-IX cosmological model with magnetized anisotropic DE fluid having anisotropic EoS $p = \omega\rho$. The deviation parameters δ and γ are obtained by assuming that conservation equation of DE consists of two separate conserved parts. The solution of the field equations are obtained using the condition that expansion scalar θ is proportional to σ. The physical aspects of the model are discussed in the presence and absence of magnetic field.
The component of magnetic field reduces the energy density, expansion, shear scalar and Hubble parameters. The expansion in the universe is found to be infinite at the initial epoch which decreases with the increase in time. In the absence of magnetic field, a similar behavior of expansion is observed for $n < \frac{3m^2 + 3m - 6}{l}$. However when $n > \frac{3m^2 + 3m - 6}{l}$, the universe starts expanding at $T = 0$ and expands indefinitely as $T \to \infty$. The anisotropy measure of the DE is dynamical and found to be finite for both earlier and later times of the universe. The isotropic DE can be recovered by choosing n to be null, where n parameterizes the deviation parameters. For $m \neq 1$, the universe model does not approach to isotropy since anisotropy parameter of expansion Δ is found to be constant in the presence of magnetic field. For $m = 1$, the Bianchi type-IX model is reduced to spatially flat FRW metric which represents homogeneous and isotropic universe for earlier times.

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