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Anisotropic Bianchi Type-V Cosmological Model with Perfect Fluid and Heat Flow in Sáez-Ballester Theory of Gravitation with Variable Deceleration Parameter

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***Abstract*—In this paper, we obtain a spatially homogeneous and anisotropic Bianchi type-V cosmological model of the universe for perfect fluid distribution with heat flow within the framework of scalar-tensor theory of gravitation proposed by Sáez and Ballester (Phys. Lett. 113:467, 1986). To prevail the deterministic solutions we consider time-dependent deceleration parameter (DP) which provides the value of scale factor as $a = [\sinh(\alpha T)]$, where α is arbitrary positive constant. This acclimates time-dependent deceleration parameter (DP) which affords a late time acceleration in the universe. The modified Einstein's field equations are solved exactly and the derived model is found to be in good concordance with recent observations. The physical significance of the cosmological model has also been discussed.**

***Index Terms*—Bianchi type-V universe, Exact solution, Alternative gravitation theory, Accelerating universe**
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I. INTRODUCTION

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar-tensor theories proposed by Brans and Dicke [1], Nordvedt [2], Wagoner [3], Rose [4], Dun [5], Sáez and Ballester [6], Barber [7], Lau and Prokhovnik [8] are most important among them. The scalar-tensor theories are the generalizations of Einstein's of gravitation in which the metric is generated by a scalar gravitational field together with non-gravitational field (matter). The scalar gravitational field itself is generated by the non-gravitational fields via a wave equation in curved space-time. The strength of the coupling between gravity and scalar field is determined by an arbitrary coupling function ω. Sáez- Ballester [6] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. The Scalar-Tensor theories of gravitation play a crucial role to remove elegant exit problem in the inflation era [9]. In earlier literature, cosmological models within the framework of Sáez-Ballester scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [10], [11], Reddy and Naidu [12], Rao et al. [13], [14], Adhav et al. [15], Singh [16], Pradhan and Singh [17]. Recently, Socorro and Sabido [18] and Naidu et al. [19], [20] (see the references therein) have studied cosmological models in Sáez and Ballester scalar tensor theory of gravitation in different context.

Ram et al. [21] obtained Bianchi type-V cosmological models with perfect fluid and heat flow in Sáez and Ballester theory by considering a variation law for Hubble's parameter with average scale factor which yields constant value of the deceleration parameter. In literature it is common to use a constant deceleration parameter as it duly gives a power law for metric function or corresponding quantity. But it is worth mentioned here that the universe is accelerated expansion at present as observed in recent observations of Type Ia supernova [22]− [26] and CMB anisotropies [27]− [29] and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [30]− [32]. So, in general, the DP is not a constant but time variable.

Recently, Pradhan et al. [33], [34] investigated some new exact Bianchi type-I cosmological models in scalar-tensor theory of gravitation with time dependent deceleration parameter. Motivated by these discussions and current observational facts, in this paper, we propose to study Bianchi type-V universe with perfect fluid and heat flow in Sáez-Ballester scalar-tensor theory of gravitation by considering a law of variation of scale factor as increasing function of time which yields a time dependent DP.

II. THE METRIC AND BASIC EQUATIONS

We consider anisotropic Bianchi type-V line element, given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right], \quad (1)$$

where A, B and C are metric functions and m is a constant.

We define the following parameters to be used in solving Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-V model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \tag{2}$$

A volume scale factor V is given by

$$V = a^3 = ABC. \tag{3}$$

In analogy with FRW universe, we also define the generalized Hubble parameter H as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{4}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively. Here, and also in what follows, a dot indicates ordinary differentiation with respect to t.

Further, the deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{5}$$

We introduce the kinematical quantities such as expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m), defined as follows:

$$\theta = u^i_{;i}, \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{7}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{8}$$

where $u^i = (0,0,0,1)$ is the matter 4-velocity vector and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^{\alpha}_j + u_{j;\alpha}P^{\alpha}_i\right) - \frac{1}{3}\theta P_{ij}. \tag{9}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j. \tag{10}$$

These dynamical scalars, in Bianchi type-V, have the forms

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{11}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}. \tag{12}$$

III. FIELD EQUATIONS

The field equations in the scalar-tensor theory, proposed by Seáz and Ballester [1], are given by

$$G_{ij} - \omega\phi^r\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij}, \tag{13}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ and $8\pi G = c = 1$. The scalar field ϕ satisfies the equation

$$2\phi^r\phi^{;i}_{;i} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0. \tag{14}$$

Here r is an arbitrary constant and ω is a dimensionless coupling constant. Comma and semi-colon respectively denote ordinary and covariant derivative with respect to cosmic time t. T_{ij} is the energy-momentum tensor of the matter. The energy-momentum tensor is the source of gravitational field through which the effect of the perfect fluid with heat flow in the evolution of the universe is performed. The energy-momentum tensor of a perfect fluid with heat flow has the form given by Singh [35]

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} + h_i u_j + h_j u_i, \tag{15}$$

where ρ is the energy density, p is the thermodynamic pressure, u_i is the four-velocity of the fluid and h_i is the heat flow vector satisfying

$$h^i u_i = 0. \tag{16}$$

We assume that the heat flow is in x direction only so that $h_i = (h_1, 0, 0, 0)$, h_1 being a function of time. Considering the form of the energy-momentum tensor (15), the Einstein's field equations (13), for the Bianchi type-V space-time (1) in Sáez-Ballester theory, are given explicitly as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{19}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{20}$$

$$m\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = h_1, \tag{21}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0. \tag{22}$$

From the energy conservation equation $T^j_{i;j} = 0$, we obtain

$$\dot{\rho} + (p + \rho)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2m}{A^2}q_1. \tag{23}$$

Equations (17)−(20) can be written in terms of H, q, σ^2 and ϕ as

$$p = H^2(2q - 1) - \sigma^2 + \frac{m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2. \tag{24}$$

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{25}$$

IV. SOLUTIONS OF FIELD EQUATIONS

Now, we follow the approaches of Saha [36], Saha and Rikhvitsky [37] and Singh and Chaubey [38] to solve the field equations (17)−(20). Subtracting Eq. (17) from (18), Eq. (17) from (19) and Eq. (18) from (19), we get the following relations respectively:

$$\frac{A}{B} = d_1 \exp\left(k_1\int\frac{dt}{a^3}\right), \tag{26}$$

$$\frac{A}{C} = d_2 \exp\left(k_2\int\frac{dt}{a^3}\right), \tag{27}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{28}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From Eqs.(26)-(28), the metric functions can be obtained explicitly as

$$A(t) = l_1 a \exp\left(\frac{X_1}{3} \int \frac{dt}{a^3}\right), \tag{29}$$

$$B(t) = l_2 a \exp\left(\frac{X_2}{3} \int \frac{dt}{a^3}\right), \tag{30}$$

$$C(t) = l_3 a \exp\left(\frac{X_3}{3} \int \frac{dt}{a^3}\right), \tag{31}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{d_1^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$X_1 = k_1 + k_2, \quad X_2 = k_3 - k_1, \quad X_3 = -(k_2 + k_3),$$

where the constants X_1, X_2, X_3 and l_1, l_2,l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0, \quad l_1 l_2 l_3 = 1. \tag{32}$$

The quadrature expression for the dimensionless scalar field function ϕ, from eq. (22), is found as

$$\phi = \left[\frac{\phi_0 (r+2)}{2} \int \frac{dt}{a^3}\right]^{2/(r+2)}, \tag{33}$$

where ϕ_0 is a constant.

It is clear from Eqs. (29)−(33) that once we get the value of the average scale factor a, we can easily calculate the metric functions A, B, C and the scalar function ϕ.

We define the deceleration parameter q as

$$q \equiv -\frac{\ddot{a}}{a}\left(\frac{\dot{a}}{c}\right)^{-2} = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(t) \quad \text{say}, \tag{34}$$

where a is the average scale factor of the universe defined by Eq. (2) and the dots indicate derivatives by proper time. The motivation to choose such time dependent DP is already described in the previous section of introduction. The expansion of the universe is said to be "accelerating" if $\ddot{a}$ is positive (recent measurements suggest it is), and in this case the DP will be negative. The minus sign and the name "deceleration parameter" are historical; at the time of definition q was thought to be positive, now it is believed to be negative. Recent observations [22]− [26] have suggested that the rate of expansion of the universe is currently accelerating, perhaps due to dark energy. This yields negative values of the DP.

Equation (34) may be rewritten as

$$\frac{\ddot{a}}{a} + b\frac{\dot{a}^2}{a^2} = 0. \tag{35}$$

In order to solve the Eq. (35), we assume $b = b(a)$. It is important to note here that one can assume $b = b(t) = b(a(t))$, as a is also a time dependent function. It can be done only if there is a one to one correspondences between t and a. But this is only possible when one avoid singularity like big bang or big rip because both t and a are increasing functions.

The general solution of Eq. (35) with the assumption $b = b(a)$, is obtained as

$$\int e^{\int \frac{b}{a} da} da = t + k, \tag{36}$$

where k is an integrating constant.

One cannot solve (36) in general as b is variable. So, in order to solve the problem completely, we have to choose $\int \frac{b}{a} da$ in such a manner that (36) be integrable without any loss of generality. Hence we consider

$$\int \frac{b}{a} da = \ln f(a), \tag{37}$$

which does not affect the nature of generality of solution. Hence from (36) and (37), we obtain

$$\int f(a) da = t + k. \tag{38}$$

Of course the choice of $f(a)$, in (38), is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider

$$f(a) = \frac{1}{\alpha\sqrt{1 + a^2}}, \tag{39}$$

where α is an arbitrary constant. In this case, on integrating Eq. (38) and neglecting the integration constant k, we obtain the exact solution as

$$a = (\sinh(\alpha T)), \tag{40}$$

where α is an arbitrary constant and $T = t + k$. We also note that $T = 0$ and $T = \infty$ respectively corresponding to the proper time $t = -k$ and $t = \infty$. The cosmic scale factor is a function of time which represents the relative expansion of the universe. For a greatest problem of observational cosmology during the coming decades − the determination of DP, q_0, − SNe la will play an important and perhaps decisive role. The relation (40) is recently by Pradhan et al. [39] in studying the dark energy models with anisotropic fluid in Bianchi type-VI_0 space-time. Relation (40) is also used by Amirhashchi et al. [40] to study the evolution of dark energy models in a spatially homogeneous and isotropic FRW space-time. Recently, relation (40) is also used by Pradhan et al. [41], [42] to study Bianchi type-I models in general relativity and FRW universe in scale-covariant theory respectively. Recently, Pradhan et al. [43−45] studied various cosmological models in general relativity by considering time dependent deceleration parameter.

From (5) and (40), we obtain the time varying deceleration parameter as

$$q = -(\tanh(\alpha T))^2. \tag{41}$$

Using (40) in Eqs. (29)−(31), we obtain the following expressions for scale factors:

$$A(t) = l_1(\sinh(\alpha T)) \exp\left(\frac{X_1}{3} F(T)\right), \tag{42}$$

$$B(t) = l_2(\sinh(\alpha T))^{1/n} \exp\left(\frac{X_2}{3} F(T)\right), \tag{43}$$

$$C(t) = l_3(\sinh(\alpha T))^{1/n} \exp\left(\frac{X_3}{3}F(T)\right), \quad (44)$$

where

$$F(T) = \coth(\alpha T)\text{cosech}(\alpha \text{T}) + \log\left(\tanh\left(\frac{\alpha T}{2}\right)\right). \quad (45)$$

Hence the geometry of the universe (1) is reduced to

$$ds^2 = dT^2 - l_1^2(\sinh(\alpha T))^2 \exp\left(\frac{2X_1}{3}F(T)\right)\, dx^2 - e^{2mx}\left[l_2^2(\sinh(\alpha T))^2 \exp\left(\frac{2X_2}{3}F(T)\right)\, dy^2 + l_3^2(\sinh(\alpha T))^2 \exp\left(\frac{2X_3}{3}F(T)\right)\, dz^2\right]. \quad (46)$$

V. SOME PHYSICAL AND GEOMETRIC PROPERTIES

The solution for scalar function ϕ, from (33), is obtained as

$$\phi = \left[\frac{\phi_0(r+2)}{2}F(T)\right]^{2/(r+2)}. \quad (47)$$

By using the values of the metric functions from Eqs. (42)−(44) into Eq. (21), the expression for the heat flow function h_1 is given by

$$h_1 = \frac{m\beta_1}{3(\sinh(\alpha T))^3}, \quad (48)$$

where $\beta_1 = 2X_1 - X_2 - X_3$.

From Eqs. (24) and (25), the pressure and energy density for the model (46) are given by

$$p = -\alpha^2(\coth(\alpha T))^2 + \left[\left(\frac{1}{2}\omega\phi_0^2 - \frac{\beta_2}{18}\right)\frac{1}{(\sinh(\alpha T))^6}\right] - 2\alpha^2 + \left[\frac{m^2}{l_1^2(\sinh(\alpha T))^2}\exp\left(\frac{-2X_1}{3}F(T)\right)\right], \quad (49)$$

$$\rho = 3\alpha^2(\coth(\alpha T))^2 + \left[\left(\frac{1}{2}\omega\phi_0^2 - \frac{\beta_2}{18}\right)\frac{1}{(\sinh(\alpha T))^6}\right] - \left[\frac{3m^2}{l_1^2(\sinh(\alpha T))^2}\exp\left(\frac{-2X_1}{3}F(T)\right)\right]. \quad (50)$$

In view of (32), it is observed that the above set of solutions satisfy the energy conservation equation (23) identically and hence represent exact solutions of the Einstein's modified field equations (17)−(22). From Eqs. (49) and (50), we observe that isotropic pressure p and the energy density ρ are always positive and decreasing function of time and both approach to zero as $t \to \infty$.

The physical parameters such as spatial volume (V), directional Hubble factors (H_i), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) for the model (46) are given by

$$V = (\sinh(\alpha T))^3, \quad (51)$$

$$H_i = \alpha\coth(\alpha T) + \frac{X_i}{3(\sinh(\alpha T))^3}, \quad (52)$$

$$\theta = 3H = 3\alpha\coth(\alpha T), \quad (53)$$

$$\sigma^2 = \frac{\beta_2}{18(\sinh(\alpha T))^6}, \quad (54)$$

$$A_m = \frac{\beta_2}{27\alpha^2}\frac{(\tanh(\alpha T))^2}{(\sinh(\alpha T))^6}, \quad (55)$$

where $\beta_2 = X_1^2 + X_2^2 + X_3^2$.

From Eqs. (51) and (53), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $T = 0$ which is big bang scenario. From Eqs. (42)−(44), we observe that the spatial scale factors are zero at the initial epoch $T = 0$ and hence the model has a point type singularity [46]). We observe that proper volume increases with time.

The dynamics of the mean anisotropic parameter depends on the constant $\beta_2 = X_1^2 + X_2^2 + X_3^2$. From Eq. (55), we observe that at late time when $t \to \infty$, $A_m \to 0$. Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations.

It is important to note here that $\lim_{T\to 0}\left(\frac{\rho}{\theta^2}\right)$ spread out to be constant. Therefore the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [47].

The flow of heat along the x-direction was maximum in early universe, and it diminishes as $T \to \infty$. From Eqs. (53) and (54), we also observe that $\frac{\sigma^2}{h_1^2}$ = constant which shows that shear scalar is proportional to heat conduction.

VI. CONCLUSIONS

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-V space-time within the framework of the scalar-tensor theory of gravitation proposed by Sáez and Ballester [6]. The field equations have been solved exactly with suitable physical assumptions. The solutions satisfy the energy conservation Eq. (23) identically. Therefore, new, exact and physically viable Bianchi type-V model has been obtained. To find the deterministic solution, we have considered a time dependent deceleration parameter which yields a scale factor as $a(t) = \sinh(\alpha T)$. In our derived model, it is observed that as $T \to \infty$, $q = -1$. This is the case of de Sitter universe. For $T \to 0$, $q = 0$. This shows that in the early stage the universe was decelerating where as the universe is accelerating at present epoch which is in good agreement with the recent supernovae Ia observation [22−26]. The parameter H_i, H, θ, and σ diverge at the initial singularity. There is a Point Type singularity [46] at $T = 0$ in the model. The rate of expansion slows down and finally tends to zero as $T \to 0$. The pressure, energy density and scalar field become negligible where as the scale factors and spatial volume become infinitely large as $T \to \infty$, which would give essentially an empty universe.

The main features of the models are as follows:

- The model is based on exact and new solutions of Einstein's modified field equations for the anisotropic Bianchi type-V space time filled with perfect fluid and heat flow.

• Our special choice of scale factor yields a time dependent deceleration parameter which represents a model of the Universe which is accelerating at present epoch. This scenario is consistent with recent observations [22−26].

• It has been observed that $\lim_{T\to 0}\left(\frac{\rho}{\theta^2}\right)$ turn out to be constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin.

• We also observe that $\frac{\sigma^2}{h_1^2}$ = constant which shows that shear scalar is proportional to heat conduction (i.e. $\sigma \propto h_1$).

Finally, the solutions presented here can be one of the potential candidates to describe the observed universe.

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