

Kaluza Klein Cosmological Model with String, SQM and Time Varying Λ

¹Namrata Jain, ²S.S. Bhoga, ³G.S. Khadekar

¹Department of Physics, M. D. College, Parel, Mumbai-400 012

²Department of Physics, RTM Nagpur University, Nagpur-4400033

³ Department of Mathematics, RTM Nagpur University, Nagpur-4400033

¹nam_jain@rediffmail.com, ²msrl.physics1@gmail.com, ³gkhadekar@yahoo.com

ABSTRACT

In this paper, exact solutions of Einstein field equations of five dimensional Kaluza-Klein cosmological model with strange quark matter and string cloud have been obtained by using $\Lambda = \alpha H^2 + \beta R^{-2}$, where $R(t)$ is scale factor and H is Hubble parameter. To get solutions, Quark matter is assumed to behave like perfect fluid and its equation of state is given by $p = 1/3(\rho - 4B_C)$. In order to obtain exact solutions, extra dimension is assumed to be equal to n th power of $R(t)$ where $n < 1$. This assumption is followed because the Universe is anisotropic at early stages, although as per cosmic principle, it is believed to be homogeneous and isotropic. The exact solutions thus obtained led to anisotropic and nonsingular model.

Keywords: String cosmology. cosmological model. cosmological constant. strange quark matter. Kaluza Klein metric.

1. INTRODUCTION

Recently, lot of attention has been gained by string cosmology, which is the key factor for getting information about early Universe. Since Big-Bang, early universe phenomenology is major curiosity among many cosmologists. The idea of string theory came up to describe events of early stage of evolution of the Universe.

At the very early stages of evolution of the Universe, it is assumed that when the Universe passed through phase transition at critical temperature, Symmetry of the Universe is broken spontaneously. This gave rise to topological defects, such as Magnetic monopoles, Strings and Domain walls. The strings are actually the consequences of Quantum field theory, which were used to explain hadron scattering through Feynman diagrams. The concept of strings in Feynman diagrams was first explained by Gabriele Veneziano [1] in 1968. Cosmic strings are topologically stable defects which might be found during a phase transition of the early Universe (Kibble [2]). It is also believed that strings are density perturbations which lead to the formation of galaxies (Zel'dovich [3]). These strings are also useful in the study of gravitational effects as they have stress energy and can be coupled to gravitational effects (Letelier [4]). The gravitational effects of the strings can be studied, if strings are supposed to be consisted of string cloud with particles. Stachel [5] had also explained string dust model where null strings were taken to be massive. Vilenkin [6] had studied the gravitational effects of domain walls with strings.

Strings are also considered important for discussing quantum gravity and useful in obtaining information about graviton. Gasperini [7] reviewed the string cosmology for pre Big-Bang scenario and concluded that the strings can alone be useful to explain early universe phenomena.

In this paper, we investigate higher dimensional string cosmological model in presence of quark matter and time varying cosmological constant in Kaluza-Klein metric.

After the Big-Bang, Universe had undergone another phase transition i.e. Quark phase to hadron phase when $T_C \sim 200\text{MeV}$. This state is also called as Quark-Gluon-Plasma (QGP) state. This phase transition which plays an important role at early Universe had been discussed by Witten [8], Fahri & Jaffe [9]. They also pointed out the role of Quark matter at early Universe. Gerlach [10], Ivanecka et al [11], Bodmer [12], Itoh [13] had discussed the importance of Quarks during phase transition. The concept of Quark matter had emerged from the Quantum Chromo-dynamics (QCD). It is also thought that there is possibility of so called Quark star or compact star which is smaller than neutron star and which is supported by degenerated quark pressure.

It is plausible to attach Quark matter to strings as strings are free to vibrate in different modes and different modes represent different particles, and also different modes of vibrations are observed as different masses or spins. Charged strange quark matter, attached to string cloud in cylindrical space time admitting conformal motion, had been studied by Mak & Harko [14], Sanjay Oli [15], Pradhan A. et al [16], Bali & Pradhan [17], Mahanta et al [18], Singh & N. K. Sharma [19], Banerjee et al [20], Yavuz I. and Tarhan I. [21] had discussed the string cosmological model attached to quark matter in Bianchi space time. The papers by S. K. Tripathi [22], D.R. K. Reddy [23], M. Glovanini [24], Kanti Jotania [25], Bijan Saha et al [26], G. P. Singh & T. Singh [27] had studied Bianchi type string cosmological model in presence of either electro-magnetic or magnetic field in different context. The papers by D.D. Pawar [28], Bali Raj [29] had

<http://www.ejournalofscience.org>

obtained the solutions of Einstein field equations for the string cosmological model with viscous fluid and bulk viscous fluid respectively. While Xing-Xang Wang [30] had discussed BI string with Bulk Viscosity & magnetic field.

Since concept of higher dimension has originated from string cosmology so it plays an important role in the study of early Universe. Higher dimensions also gained attention for unifying gravitation and particle interaction, electromagnetism, gauge theories etc. which were first put forth by Kaluza [31] and Klein [32] independently.

There are many reporting which studied higher dimension for exploring the idea of Brane Cosmology, scalar-tensor field theory, parameterization of mass etc. Paul Wesson [33] has enlightened Kaluza-Klein theory so as to develop the new idea of Space-Time - Matter theory and hence applied to many phenomenon i.e. particle interactions, Gravitation electromagnetism etc.

Study of Higher dimension cosmological model with the string cloud and strange quark matter will be fruitful for bringing information at early universe stage.

Cosmological constant ' Λ ' has played the key role in the study of Universe as it represents vacuum energy. Observational data of SN Ia by Perlmutter [34] and Reiss [35] in 1998 when combined with CMB measurements implied that Universe is accelerating, which indicate towards finite and small positive value of Cosmological Constant. However, this value differs from the value which was predicted by Standard model of Particle Physics. This problem is now known as Cosmological Constant Problem (CCP). Chen and Wu [36] suggested that time varying cosmological constant can solve this problem by assuming ansatz $\Lambda \propto R^{-2}$ where $R(t)$ is scale factor. There are number of papers which studied cosmological models with time varying cosmological constant in different context. A fine review had been done by Overduin and Cooperstock [37], Varun Sahani [38] and T. Padmanabhan [39], Arbab I. Arbab [40].

Recently it has been suggested in many papers that the value of Cosmological Constant Λ may be large at the time of evolution of Universe, when strings are also assumed to be dominating. There are papers by Pradhan et. al [41], R.K. Tiwari [42], Abbasi & Rajmi [43], Anil Yadav [44] which had discussed string cosmological model with time varying cosmological constant. Ray, Rahman & Mukhopadhyay [45] had explained scenarios of Cosmic strings with variable Cosmological constant by assuming $\Lambda = 3r^{-2}$, $\Lambda = 8\pi\rho$, $\rho_s = 3r^{-2}$ here r has its usual meaning. Later on, to reconcile age parameter and to deal with low density problem with observational data, lambda had been further generalized by Carvalho & Lima [46] as

$$\Lambda = \alpha H^2 + \beta R^{-2}.$$

Yilmiz [47], Katore [48] had explained string cosmological model in presence of quark matter in Kaluza-Klein metric without cosmological constant.

Motivated by above discussion, in this paper, exact solutions of Einstein field equations of Kaluza Klein Cosmological model with string in presence of quark matter and time varying Cosmological Constant have been obtained.

The paper is organized as follows. In section II, Einstein field equations of string cosmological model in Kaluza Klein metric are obtained. In Section III we derive solutions for string cosmological model in presence of quark matter and time varying cosmological constant. Expressions for densities, pressure and other physical parameters are also obtained with the help of solutions. With the help of expressions of physical parameters, it is possible to draw some conclusions on the cosmological model and its behavior which seems to describe physical situations at early stage.

2. EINSTEIN FIELD EQUATIONS

The line element for five dimensional Kaluza-Klein model is given by

$$ds^2 = -dt^2 + R^2(t) \left[\frac{dr^2}{(1-kr^2)} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] + A^2(t) d\Psi^2 \quad (1)$$

Where k is curvature parameter, which is equal to 0, 1, -1 for flat, closed and open universe respectively. $R(t)$ and $A(t)$ are scale factors. Ψ is fifth dimension. As per Cosmic principle, $\hbar = c = 8\pi G = 1$ is assumed.

$$T_j^i = (\rho + p) u^i u_j + p g_j^i - \lambda x^i x_j \quad (2)$$

Where p is isotropic pressure; ρ is the proper energy density for a cloud of strings with particles attached to them; λ is the string tension density; $U^i = (0, 0, 0, 0, 1)$ is five velocity and time like vector; x^i is a unit space like vector such that $x^i x_i = 1$ in the direction of $x^i = \delta_4^i$ which represents the directions of cloud i.e. directions of anisotropy. We also consider $U^i U_i = -1$ and $U^i U_j = -1$, $U^i x_j = 0$.

Proper energy density for strings attached to particles can also be written as $\rho = \rho_p + \lambda$. As per Bag model, quark density $\rho_p = \rho_q + B_c$ and quark pressure $p_q = \rho_q/3$, $p_p = p_q - B_c$, where B_c is the Bag constant. As per Bag model, it is the difference between the energy density of the

<http://www.ejournalofscience.org>

perturbative and non- perturbative QCD vacuum. Thus, proper density is given by $\rho = \rho_q + B_c + \lambda$.

To obtain Einstein field equations, consider the following equation

$$G_j^i = R_j^i - \frac{1}{2} R g_j^i + \Lambda g_j^i \quad (3)$$

Where R_j^i is Ricci tensor, R is Ricci scalar, g_j^i is metric element. We know that $G_j^i = -T_j^i + \Lambda g_j^i$, where $T_j^i = \text{diag}(-\rho, p, p, p, p-\lambda)$, T_j^i is the stress -energy tensor. From equation (1) Einstein field equations are obtained as follows

$$G_1^1 = 2 \frac{\ddot{R}}{R} + 2 \frac{\dot{R} \dot{A}}{R A} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\ddot{A}}{A} = -p + \Lambda \quad (4)$$

$$G_4^4 = 3 \frac{\ddot{R}}{R} + 3 \frac{\dot{R}^2}{R^2} + 3 \frac{k}{R^2} = -p + \lambda + \Lambda \quad (5)$$

$$G_5^5 = 3 \frac{\dot{R}^2}{R^2} + 3 \frac{\dot{R} \dot{A}}{R A} + 3 \frac{k}{R^2} = \rho + \Lambda \quad (6)$$

Conservation of energy momentum tensor is given by

$$\left(T_j^i + \Lambda g_j^i \right)_{;j} = \left(T_j^i \right)_{\text{eff};j} = 0$$

From here, we obtain,

$$\dot{\rho} + (p + \rho) \left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A} \right) + \dot{\lambda} - \lambda \frac{\dot{A}}{A} = 0 \quad (7)$$

Above equation can be obtained from field equation also. In above equation, we have three equations and $R, A, p, \rho, \lambda, \Lambda$ are unknowns. To solve above equations explicitly, we need three more equations. We assume generalized Λ as a function of time, i.e.

$$\Lambda = \alpha \frac{\dot{R}^2}{R^2} + \beta \frac{1}{R^2}$$

Since Quarks are considered to be mass less particle and quark fluid is supposed to be perfect fluid as per Bag model, we consider Equation Of State (EOS) for quark matter, which is given by,

$$p = \frac{1}{3} (\rho - 4B_c) \quad (8)$$

To obtain the scale factors $R(t)$ and $A(t)$, we solve field equations as follows. From equations (4) and (5), we have following relations

$$2 \frac{\ddot{R}}{R} + \frac{\ddot{A}}{A} - \frac{\dot{R} \dot{A}}{R A} - 2 \frac{\dot{R}^2}{R^2} - 2 \frac{k}{R^2} = -(p + \rho) \quad (9)$$

$$\lambda = \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2k}{R^2} - \frac{2\dot{R} \dot{A}}{R A} - \frac{\ddot{A}}{A} \quad (10)$$

We will also derive an expression for physical variables, expansion factor and shear scalar in five dimensional metric, which are defined as

$$\theta = 3 \frac{\dot{R}}{R} + \frac{\dot{A}}{A} \quad (11)$$

$$\sigma^2 = \frac{3}{8} \left(\frac{\dot{R}}{R} - \frac{\dot{A}}{A} \right)^2 \quad (12)$$

Using equation of state for quark matter, we get,

$$2 \frac{\ddot{R}}{R} + \frac{\ddot{A}}{A} - \frac{\dot{R} \dot{A}}{R A} - 2 \frac{\dot{R}^2}{R^2} - 2 \frac{k}{R^2} = -\frac{4\rho}{3} + \frac{4B_c}{3} \quad (13)$$

Substituting equation (6) in above equation and simplifying it, we obtain following equation as,

$$2 \frac{\ddot{R}}{R} + \frac{\ddot{A}}{A} + 3 \frac{\dot{R} \dot{A}}{R A} + 2 \frac{\dot{R}^2}{R^2} + 2 \frac{k}{R^2} - \frac{4\Lambda}{3} = \frac{4B_c}{3} \quad (14)$$

In order to solve above equation, we assume ansatz $A = R^n$. This power law equation is considered due to the fact that there is still anisotropy for the flat and homogeneous Universe and $\theta \propto \sigma_{ij}$ (shear tensor), so we use polynomial relation between metric coefficients. Thus Equation (14) is simplified as follows,

$$(n+2) \frac{\ddot{R}}{R} + (n^2 + 2n + 2) \frac{\dot{R}^2}{R^2} + 2 \frac{k}{R^2} - \frac{4\Lambda}{3} = \frac{4B_c}{3} \quad (15)$$

$$\frac{\ddot{R}}{R} + \frac{(n^2 + 2n + 2) \dot{R}^2}{(n+2) R^2} + \frac{2k}{(n+2) R^2} - \frac{4\Lambda}{3(n+2)} = \frac{4B_c}{3(n+2)} \quad (16)$$

<http://www.ejournalofscience.org>

In next section we will take time variable cosmological constant which was generalized, first put forth by Carvalho and Lima [46]. We will derive the solution of above field equation and determine physical parameters i.e. density, pressure, string tension density, quark density, quark pressure etc.

3. SOLUTIONS OF EINSTEIN FIELD

EQUATIONS FOR $\Lambda = \alpha \frac{\dot{R}^2}{R^2} + \beta \frac{1}{R^2}$

To find the exact solution of field equation for flat Universe, let $k=0$. Substituting $\Lambda = \alpha \frac{\dot{R}^2}{R^2} + \beta \frac{1}{R^2}$ & $k=0$ in the equation (16), We get,

$$\frac{\ddot{R}}{R} + \frac{(3n^2+6n+6-4\alpha)\dot{R}^2}{3(n+2)R^2} + \frac{(-4\beta)}{3(n+2)} \frac{1}{R^2} = \frac{4B_C}{3(n+2)} \quad (17)$$

In above equation we assume that,

$$m = \frac{3n^2+6n+6-4\alpha}{3(n+2)}, \quad k_1 = \frac{4\beta}{3(n+2)},$$

$$\text{and } k_2 = \frac{4B_C}{3(n+2)},$$

Equation (17) now will be simplified as

$$\frac{\ddot{R}}{R} + m \frac{\dot{R}^2}{R^2} - k_1 \frac{1}{R^2} - k_2 = 0 \quad (18)$$

First order integral solution of above equation is given by

$$\dot{R}^2 = \frac{k_1}{m} + \frac{k_2}{(m+1)} R^2 + \frac{C_0}{R^{2m}} \quad (19)$$

For simplicity let us assume $m=1$, the solution of above equation has been obtained as

$$\dot{R}^2 = k_1 + \frac{k_2}{2} R^2 + \frac{C_0}{R^2} \quad (20)$$

Where C_0 is constant of integration. Hence scale factor $R(t)$ for Kaluza – Klein metric is obtained as follows.

$$R^2 = \sqrt{\frac{2C_0}{k_2} - \left(\frac{k_1}{k_2}\right)^2} \sinh \sqrt{2k_2} (t+c) - \frac{k_1}{k_2} \quad (21)$$

$$\text{Let } a = \sqrt{\frac{2C_0}{k_2} - \left(\frac{k_1}{k_2}\right)^2} = \sqrt{\frac{3(n+2)C_0}{2\beta} - \left(\frac{B_C}{\beta}\right)^2},$$

$$\varphi = \sqrt{2k_2} (t+c) \quad \text{and} \quad k_3 = \frac{k_1}{k_2}, \quad \text{Now equation (21) is}$$

rewritten as follows

$$R^2 = a \sinh \varphi - k_3 \quad (22)$$

$$R(t) = (a \sinh \varphi - k_3)^{\frac{1}{2}} \quad (23)$$

Knowing $R(t)$ we calculate other physical parameters as follows

$$A(t) = (a \sinh \varphi - k_3)^{\frac{n}{2}} \quad (24)$$

$$\rho(t) = \frac{[3(n+1)-\alpha]k_2 a^2 \cosh^2 \varphi + (-2\beta)(a \sinh \varphi - k_3)}{2(a \sinh \varphi - k_3)^2} \quad (25)$$

$$p(t) = \frac{1}{3}(\rho - 4B_C) \quad (26)$$

$$H(t) = \sqrt{\frac{k_2}{2}} a \cosh \varphi (a \sinh \varphi - k_3)^{-1}$$

$$\Lambda(t) = \frac{[\alpha]k_2 a^2 \cosh^2 \varphi + 2\beta(a \sinh \varphi - k_3)}{2(a \sinh \varphi - k_3)^2} \quad (27)$$

$$q(t) = - \left| \frac{R \ddot{R}}{\dot{R}^2} \right| = - \left| \frac{a + k_3 \sinh \varphi}{a \cosh^2 \varphi} \right| \quad (28)$$

$$(29)$$

To find string tension, density consider equation (9), since $k=0$ equation (8) is rewritten as

$$\lambda = \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} - \frac{2\dot{R}\dot{A}}{RA} - \frac{\ddot{A}}{A}$$

on substitution of $A(t) = R^n(t)$, we get

$$\lambda = (1-n) \left(\frac{\ddot{R}}{R} + (2+n) \frac{\dot{R}^2}{R^2} \right) \quad (30)$$

<http://www.ejournalofscience.org>

$$\lambda = (1-n) \left(\frac{k_2(n+1)a^2 \text{Cosh}^2\varphi + 2ak_2 \text{Sinh}\varphi (a \text{Sinh}\varphi - k_3)}{2(a \text{Sinh}\varphi - k_3)^2} \right) \quad (31)$$

From above equation $n \neq 1$. To find quark density we consider proper density as $\rho = \rho_q + B_c + \lambda$.
 $\therefore \rho - \lambda = \rho_q + B_c$.

$$\rho_q = \left(\frac{k_2(n^2+3n+2-\alpha)a^2 \text{Cosh}^2\varphi - [2\beta+(1-n)2ak_2 \text{Sinh}\varphi](a \text{Sinh}\varphi - k_3)}{2(a \text{Sinh}\varphi - k_3)^2} \right) - B_c \quad (32)$$

Quark pressure can be obtained as follows.

$$p_q = \frac{\rho_q}{3} = \left(\frac{k_2(n^2+3n+2-\alpha)a^2 \text{Cosh}^2\varphi - [2\beta+(1-n)2ak_2 \text{Sinh}\varphi](a \text{Sinh}\varphi - k_3)}{6(a \text{Sinh}\varphi - k_3)^2} \right) - \frac{B_c}{3} \quad (33)$$

Expression for expansion factor is found out with the help of equation (11) as follows,

$$\theta = (3+n) \frac{\dot{R}}{R} = (3+n) a \sqrt{2k_2} \text{Cosh}\varphi (a \text{Sinh}\varphi - k_3)^{-1} \quad (34)$$

Expression for shear scalar can be found using equation (12),

$$\sigma^2 = \frac{3}{8} (1-n)^2 \left(\frac{\dot{R}}{R} \right)^2$$

$$\therefore \sigma = \frac{3}{16} (1-n)^2 a^2 k_2 \text{Cosh}^2\varphi (a \text{Sinh}\varphi - k_3)^{-2} \quad (35)$$

4. DISCUSSION

From equation (25), we observe that the total energy density decreases with time and it is always positive. From equation (26), we observe that in absence of B_c , our model reaches to the radiation dominated phase.

From Eq. (28), we observe that the cosmological constant decreases with time and approaches to a small positive value at late time, which is in good agreement with the recent astrophysical observations (Perlmutter [34], Reiss [35]).

From Eq. (29), at $t \rightarrow 0$, $q(t) = -1$. This implies that the model is de-Sitter in the early stage of the evolution of the Universe. We also observe that when $t \rightarrow \infty$, $q(t) = 0$.

This implies that the model reaches to steady state at late time, so Universe expansion rate falls down if time increases.

It is also observed from Eq. (31) that string density ' λ ' decreases with time and it is always positive. From equations (34) & (35), we find that

$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \rightarrow \text{constant}$ and, hence it leads to anisotropic model.

5. CONCLUSION

In this paper, we derived exact solutions for the higher dimensional Cosmological model with strange quark matter and a variable cosmological term Λ . The model is also nonsingular and represents inflationary phase, if k_1 is assumed to be zero. It is the generalized model in which dimensionless parameter α & β defines the behavior of the model. For real solution we must have,

$$\frac{3C_0(n+2)}{B_c} > \left(\frac{\beta}{B_c} \right)^2 \quad \text{i.e. } C_0 > \frac{\beta^2}{3(n+2)B_c} \quad \text{and } n \neq -2.$$

With $\beta=0$, our work generalizes the recent work done by Ozel et.al [49] in absence of string cloud. The model is expanding and it disappears at late time. Our derived model also generalizes the work of Yilmiz [47] in absence of any pressure and density.

ACKNOWLEDGEMENTS

We highly appreciate the facilities given by IUCAA library. Also, thanks to TIFR library and Haffkin's Institute for the facilities they provide. My sincere gratitude to Dr. Anirudh Pradhan for providing me his valuable suggestions during this work.

REFERENCES

- [1] Gabriele Veneziano, "Scale factor for classical and quantum strings" *Phy.Lett. B*, 265, 287, (1991)
- [2] T.W.B. Kibble, "Topology of Cosmic domain strings" *J. Phy. A* 9,1387 (1976)
- [3] Zel'dovich, "Cosmological consequence of spontaneous break down of direct symmetry" *Sov. Phys. JETP* 40, 1(1975).
- [4] P.S. Letelier, "String Cosmologies" *Phy. Rev. D* 28, 2414 (1983)

<http://www.ejournalofscience.org>

- [5] Stachel John, “Thickening the strings . II. Null string dust”, *Phy. Rev. D* 21, 2171 (1980)
- [6] A. Vilenkin, “Cosmic strings and domain walls” *Phy.Rep.*121,263(1985)
- [7] M. Gasperini, “A thermal interpretation of the Cosmological Constant” *Class. & Quant. Gravity* 5,521 (1988)
- [8] E. Witten “Cosmic separation of phases” *Phys. Rev. D* 30,272 (1984)
- [9] E. Fahri, R.L Jaffe, “Strange matter” *Phys. Rev.D* 30, 2379(1984)
- [10] Gerlach Ulrich H., “Equation of state , a super nuclear existence of a third family of super dense stars” *Phy. Rev.* 172, 1325 (1968)
- [11] D. Ivanenko ,D.F. Kurdgelze, “Remarks on quark stars” *letters Nuovo Cimento*, 2, 13, (1969)
- [12] A. R. Bodmer, “Collapsed Nuclei”, *Phys. Rev. D* 4, 1601 (1971).
- [13] N. Itoh , Hydrostatic equilibrium of hypothetical quark stars” *Prog. Theor. Phys.* 44, 291(1970)
- [14] M. K. Mak & T. Harko, “Quark stars admitting a one parameter group of conformal Motions” *Int. J. Mod. Phys. D* 13,149 (2004)
- [15] Sanjay Oli, “String Cosmological Models in five dimensional spacetimes”, *Chin. Phy. Lett.* 20, 010404, (2009)
- [16] A. Pradhan et al, “Massive String Cosmology in Bianchi Type III Space-Time with Electromagnetic Field” *Commun.Theor.Phys.*51, 367(2009)
- [17] Raj Bali, A. Pradhan, “Bianchi type –III Cosmological Models with Time dependent Bulk Viscosity” *Chin. Phys. Lett.* , vol. 24(2), 585, (2007)
- [18] Mahanta et.al , “string cloud and Domain walls with Quark matter in Lyra Geometry” *Journal of Modern Physics*, 3, 1479 (2012)
- [19] J. K.Singh , N. K. Sharma, “some Exact Solutions of String Cosmological Models in Bianchi type II space-time” *Int. J. Theo. Phys.* 49, 2902 (2010)
- [20] A. Banerjee, Abhik kumar Sanyal and Subenoy Chakraborty “String Cosmology in Bianchi I space time” *Pramana*, 34, 1 (1990)
- [21] I. Yavuz, I.Tarhan, “Some string cosmological models in Bianchi type I space time” *Astrophysics and space Science* , 240, 45 (1996).
- [22] S. K. Tripathi, S.K. Sahu, T.R. Routray, “String Cloud Cosmologies for Bianchi type III with electro magnetic field” *Astrophysics & Space Science*, 315, 105 (2008)
- [23] D.R. K. Reddy, “Plane Symmetric Cosmic strings in Lyra manifold” *Astrophysics & Space Sci.* 300 , 381(2005).
- [24] M. Giovanini,“Magnetic field, strings and Cosmology” *lecture notes in Phys.* 737, 863 (2008)
- [25] Kanti Jotania , Padmini Yadav, S.A. Faruki. “Magnetized String Cosmology in Anisotropic Bianchi-II space time with variable Cosmological term $-\Lambda$ ” *IJTP*, 50, 1424 (2011)
- [26] Bijan Saha et. al , “Magnetized string Cosmology in anisotropic Bianchi II models with variable ‘ Λ ’” *IJTP*, 49, 1411 (2010)
- [27] G. P. Singh & T. Singh, “String Cosmological models with magnetic field” *Gen. Rel. Grav.* 31, 371 (1999).
- [28] D.D. Pawar, S.W. Bhaware , A.G. Deshmukh, “Bulk Viscous Fluid Plane symmetric string dust Magnetised Cosmological model in General relativity” *IJTP*,47, 599 (2008)
- [29] Bali & Deo Karan Singh, “LRS Bianchi type V bulk viscous fluid string dust Cosmological model in general relativity” *Astro. & Space Science*, 288, 415, (2003)
- [30] Xing-Xang Wang, “Bianchi type I Cosmological model with Bulk viscous fluid” *Astro. & Sp. Sci.*, 293, 433, (2003)
- [31] T. Kaluza, “Zum Unitatsproblem der Physik”, *Sitz, Press. Akad. Wiss. Phys. math. kl*, 966 (1921)
- [32] O. Klein, “Quantentheorie und fundimensionale Relativistattheorie” *Zeitzs. Phys.* 37, 895 (1926)
- [33] Paul Wesson , “Astrophysical Consequences of extended Cosmology” *Astron. Astrophys.* 441, 41 (2005)
- [34] S. Perlmutter et. al “Measurements* of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at $z \geq 0.35$ ”, *Astrophys. J.* 483 (1997) 565.

- [35] Reiss et.al , “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant” *Astron. J.* 116 (1998) 1009.
- [36] Overduin and Cooperstock, “Evolution of the scale factor with a variable cosmological term” *Phys. Rev. D* 58, 043506 (1998)
- [37] Varun Sahani, A. Starobinsky, “The Case for a Positive Cosmological Λ -Term” *Int. J. Mod. Phys. D* 9, 373 (2000)
- [38] T. Padmanabhan , “Cosmological Constant : The weight of the Vacuum” arXiv : hep-th 0212290v2 (2003)
- [39] Arbab I. Arbab , “Cosmological consequences of a built-in cosmological constant Model” *JCAP* 05, 008 (2003)
- [40] A.Pradhan, “ Magnetized string Cosmological model in cylindrically symmetric inhomogeneous Universe with cosmological term Λ ” *Fizika* **B16**, 205 (2007)
- [41] R.K. Tiwari, Farook Rahman, Saibal Ray, “ Five dimensional Cosmological models in General Relativity” arXiv : 1007.0479v2 [gr-qc] (2010)
- [42] H. Amir, Abbasi, H.Rajmi, “ Cosmological Constant influence on cosmic string space time” *Phy. Rev. D*, 67, 103504 (2003)
- [43] Anil K.Yadav, Vineet Kumar Yadav, Lallen Yadav “ Bianchi type V string Cosmological models in General relativity” arXiv: 0912.0464v2 (gr-qc) (2009)
- [44] Ray , Rahman & Mukhopadhyay , “Scenarios of cosmic string with variable cosmological constant” arXiv : Astro-ph / 0610519v2 (2006)
- [45] Chen & Wu, “ Implications of Cosmological constant varying as R^{-2} ” *Phys. Rev. D*, 41, 695 (1990).
- [46] Carvalho ,Lima , Waga, “ Cosmological Consequences of time dependent Λ term” *Physics Review D* 46, 2404 (1992).
- [47] . Yilmiz , String cloud, Domain walls with quark matter in 5-D Kaluza Klein Cosmological model” *Gen. Rel. Grav.* **38**, 1397 (2006).
- [48] S. Katore, “ Strange quark matter attached to String Cosmological in FRW Space-time” *IJTP*, 51, 1881, (2012)
- [49] Ozel et. al , “ Kaluza Klein Cosmological model with strange quark matter” *Adv. Studies, Theo. Phys.* Vol.4 , 117 (2010)