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The Solution of Lagrange Interpolation Approaches with Maple 6 on the Experimental Data

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ABSTRACT

In this study, Lagrange interpolation technique which could be an alternative modeling passing to exactly all data points with respect to linear or nonlinear models was applied to experimental data by using Maple 6 packaged software. Since this model is a polynomial function, researcher could easily find extremum points and turning points (if exist) or he/she could easily find the result values of the interpolation values of a factor or the interpolation values of the result values of a factor. The standard errors of the result values mentioned above could be found with Lagrange interpolation method. Confidence bounds were determined according to statistical significance level. For that reason, density of the sowing of cotton variety Ersan-92 belonging to the first year of the experiment data is used.

Keywords: *Lagrange Interpolation, Newton Interpolation, Experimental Data*

1. INTRODUCTION

In this study, Lagrange interpolation, an alternative modeling passing to exactly all data points with respect to linear or nonlinear models applied to experimental data will be discussed and this method estimates will be presented with the maple 6 package program. The reason of presentation of Lagrange interpolation is quite successful for estimating and has easy calculation [1] for investigators since Lagrange interpolation is a polynomial function. So it is useful to use instead of more general and complicate functions which do not pass exactly through the data points.

In this study, interpolation approaches and their solutions presented with maple 6 package program were applied on real research data. Classical statistical method widely used in this way and the results obtained by analyzing what can be added as new information are presented. Conventional methods can not be applied to obtain such information as the result values of any intermediate levels, the absolute and local maximum / minimum, turning point, a desired outcome at any level or intermediate level of a factor, the standard errors of all estimated values. This information used by researchers to support traditional statistical analysis, strengthen quality is thought to be useful as additional information [2].

2. MATERIALS AND METHODS

2.1 Materials

Ersan-92 cotton cultivar and row space between different order to yield the effect in Kahramanmaras in 1992 on the grounds of the Institute for Agricultural Research was conducted in split plot design. Row of 60 cm, 70 cm, 80 cm and 10 cm in row, 20 cm and 30 cm was taken as. Experimenting is with four replicates [3].

2.2 Methods

2.2.1 Interpolation Methods

There is the interpolation polynomial $P_n(x)$ according to $x_0, x_1, x_2, ..., x_n$ provide the conditions

$$a_0 + a_1 x_i + a_2 x_i^2 + ... + a_n x_i^n = f(x_i) = P_n(x_i)\ ,\ i = 0,1,2,..,n \quad \textbf{(3.1)}$$

given n +1 corresponding points $f(x_0), f(x_1), f(x_2), ..., f(x_n)$ for $x_0, x_1, x_2, ..., x_n$ in the range of $[a,b]$. There is only one polynomial $P_n(x)$ which degree is not greater than n [4,5].

2.2.1.1 Lagrange Interpolation

The most important and practical method obtained polynomial interpolation from without solving the equation system (2.1) is Lagrange interpolation. This method does not need to be in the range of data equal [6].

nth degree Lagrange interpolation polynomial for n+1 data points $(x_i, y_i),\ i = 0,1,2,...,n$ can be obtained as follows.

$$f(x) = y = \sum_{i=0}^{n} y_i \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

2.2.1.2 Continuity of Lagrange Interpolation

Lagrange interpolation function is continuous because it is an polynomial function. As known if a function is continuous, then it's intermediate values can be found.

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2.2.2 Multivariate Interpolation

Lagrange interpolation method can be used in multivariate [7]. For example, for two and three independent variables;

$$f_{pq}(x,y)=\sum_{i=0}^{p}\sum_{j=0}^{q}L_{x_i}(x)L_{y_i}(y)\,f(x_i,y_j)$$

$$f_{pqr}(x,y,z)=\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{r}L_{x_i}(x)L_{y_j}(y)L_{z_k}(z)\,f(x_i,y_j,z_k)$$

can be written [8] where

$$L_{x_i}(x)=\prod_{\substack{l=0\\ l\neq i}}^{p}\frac{x-x_l}{x_i-x_l}\ ,\ L_{y_j}(y)=\prod_{\substack{l=0\\ l\neq j}}^{q}\frac{y-y_l}{y_j-y_l}\ ,\ L_{z_k}(z)=\prod_{\substack{l=0\\ l\neq k}}^{r}\frac{z-z_l}{z_k-z_l}$$

3. RESULTS AND DISCUSSION

3.1 Results Belonging to Sowing Frequency of Cotton

To examine the effect of the seed cotton yield of distance on and among the rows, Table 1 is given for the cotton sowing in the first year of the experiment.

Table 1: Values of the cotton yield (kg/da, 1992)

Cotton Yield (kg/da)		Distance on the row (cm)		
		10	20	30
Distance among the rows (cm)	60	400	341.66	391.66
		383.33	383.33	350.00
		358.33	341.66	450.00
		408.33	341.66	316.66
	70	307.14	350.00	335.71
		407.14	328.57	364.29
		314.29	385.71	314.29
		342.86	350.00	321.43
	80	331.25	375.00	356.25
		400.00	400.00	331.25
		381.25	387.50	337.50
		312.50	375.00	368.75

With making the interpolation research, additional information has been obtained and some of this information will be presented in the tables below.

Command syntax (syntax) and the result expressions will be seen in bold colors and light color, respectively. Firstly, the general software of Lagrange interpolation functions with one independent variable will be created. For that reason, the syntax for command will be written as follows, and can be obtained under the image. However, some image which is too long to show will not be presented.

With a fixed distance on the row y=10 cm, Lagrange interpolation function was obtained below.

```
> f(x,10) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,10)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,10)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,10);
```

$$f(x,10):=\frac{(x-x1)(x-x2)\,f(x0,10)}{(x0-x1)(x0-x2)}+\frac{(x-x0)(x-x2)\,f(x1,10)}{(x1-x0)(x1-x2)}+\frac{(x-x0)(x-x1)\,f(x2,10)}{(x2-x0)(x2-x1)}$$

Then the level of factors and their values will be written. So the command syntax for Lagrange interpolation functions with one variable is shown below.

>f(x,10):=eval(f(x,10),[x0=60,x1=70,x2=80,f(x0,10)=(400.00+383.33+358.33+408.33)/4,f(x1,10)=(307.14+407.14+314.29+342.86)/4,f(x2,10)=(331.25+400.00+381.25+312.50)/4]);

$$f(x, 10) := 1.937487500\,(x-70)(x-80) - 3.428575000\,(x-60)(x-80) + 1.781250000\,(x-60)(x-70)$$

Similarly, with a fixed distance on the rows y=20 cm, and y=30 cm Lagrange interpolation functions were obtained below,
respectively.

> f(x,20) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,20)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,20)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,20);

$$f(x, 20) := \frac{(x-x0)(x-x2)\,f(x0, 20)}{(x0-x1)(x0-x2)} + \frac{(x-x0)(x-x2)\,f(x1, 20)}{(x1-x0)(x1-x2)} + \frac{(x-x0)(x-x1)\,f(x2, 20)}{(x2-x0)(x2-x1)}$$

>f(x,20):=eval(f(x,20),[x0=60,x1=70,x2=80,f(x0,20)=(341.66+383.33+341.66+341.66)/4,f(x1,20)=(350.00+328.57+385.71+350.00)/4,f(x2,20)=(375.00+400.00+387.50+375.00)/4]);

$$f(x, 20) := 1.760387500\,(x-70)(x-80) - 3.535700000\,(x-60)(x-80) + 1.921875000\,(x-60)(x-70)$$

> f(x,30) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,30)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,30)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,30);

$$f(x, 30) := \frac{(x-x1)(x-x2)\,f(x0, 30)}{(x0-x1)(x0-x2)} + \frac{(x-x0)(x-x2)\,f(x1, 30)}{(x1-x0)(x1-x2)} + \frac{(x-x0)(x-x1)\,f(x2, 30)}{(x2-x0)(x2-x1)}$$

>f(x,30):=eval(f(x,30),[x0=60,x1=70,x2=80,f(x0,30)=(391.66+350.00+450.00+316.66)/4,f(x1,30)=(335.71+364.29+314.29+321.43)/4,f(x2,30)=(356.25+331.25+337.50+368.75)/4]);

$$f(x, 30) := 1.885400000\,(x-70)(x-80) - 3.339300000\,(x-60)(x-80) + 1.742187500\,(x-60)(x-70)$$

Firstly, for y = 10 cm, to find the distance among the rows which has minimum or maximum yield, and the yields in the dose, taking the derivative of the above equation equal to zero, and then the distance among the rows having the maximum and (or) minimum called critical points will be estimated below.

> z1:=diff(f(x,10),x);

$$z1 := .580325000\,x - 42.1851250$$

$$f(72.69224142, 10) := 340.7543547$$

> solve(z1=0);

72.69224142

When y = 10 cm (distance on the row), this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(72.69224142,10):=eval(f(x,10),[x=72.69224142]);

Distance among the rows which has a minimum yield, and yield can be seen from the graphics (minimum yield is the amount 340.7543547).

With Lagrange interpolation function, for the distance among the rows such as 65 cm and 74 cm yields will be estimated, respectively. Then, for getting 350 kg / da cotton yield, the distance among the rows could be estimated.

> f(65,10):=eval(f(x,10),[x=65]);
f (65 , 10) := 357.9234375

> f(74,10):=eval(f(x,10),[x=74]);
f (74 , 10) := 341.2506000

> solve(f(x,10)=350);
78.33703380 , 67.04744904

Now below for y = 20cm (distance on the row), to find the distance among the rows which has minimum or maximum yield, and the yields in the dose, the process above is repeated.

> z2:=diff(f(x,20),x);
$z2 := .293125000\ x - 18.9038750$

> solve(z2=0);
64.49083156

When y = 20 cm (distance on the row), this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(64.49083156,20):=eval(f(x,20),[x=64.49083156]);

f (64.49083156 , 20) := 349.1216908

Distance among the rows which has a minimum yield and yield can be seen in the graphics (minimum yield is the amount 349.1216908).

With Lagrange interpolation function, for the distance among the rows such as 65 cm and 74 cm yields will be estimated, respectively. Then, for getting 350 kg / da cotton yield, the distance among the rows could be estimated.

> f(65,20):=eval(f(x,20),[x=65]);
f (65 , 20) := 349.1596875

> f(74,20):=eval(f(x,20),[x=74]);
f (74 , 20) := 362.3745000

> solve(f(x,20)=350);
66.93883646 , 62.04282666

Now below for y = 30cm (distance on the row), to find the distance among the rows which has minimum or maximum yield, and the yields in the dose, the process above is repeated.

> z1:=diff(f(x,30),x);
$z1 := .576575000\ x - 41.7923750$

> solve(z1=0);
72.48384859

When y = 30 cm (distance on the row), this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(72.48384859,30):=eval(f(x,30),[x=72.48384859]);
f (72.48384859 , 30) := 332.1514092

Distance among the rows which has a minimum yield and yield can be seen in the graphics (minimum yield is the amount 332.1514092).

With Lagrange interpolation function, for the distance among the rows such as 65 cm and 74 cm yields will be estimated, respectively. Then, for getting 350 kg / da cotton yield, the distance among the rows could be estimated.

> f(65,30):=eval(f(x,30),[x=65]);
f (65 , 30) := 348.2978125

> f(74,30):=eval(f(x,30),[x=74]);
f (74 , 30) := 332.8141000

> solve(f(x,30)=350);
80.35229626 , 64.61540092

Now when y = 10, 20, 30 cm (distances on the row), estimated exchange graphs with in the graph by marking the observed and predicted extreme values, will be shown on the graph. The following commands are written for it.

```
> with(plots) :
>pic1:=pointplot({[72.69224142,340.7543547],[60,(400.00+383.33+358.33+408.33)/4],[70,(307.14+407.14+314.29+342.86)/4],[80,(331.25+400.00+381.25+312.50)/4]},color=red);

> pic2 :=plot(f(x,10), x = 60..80,color=red) ;
>pic3:=pointplot({[64.49083156,349.1216908],[60,(341.66+383.33+341.66+341.66)/4],[70,(350.00+328.57+385.71+350.00)/4],[80,(375.00+400.00+387.50+375.00)/4]},color=blue);

> pic4 :=plot(f(x,20), x = 60..80,color=blue) ;
```

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```
>pic5:=pointplot({[72.48384859,332.1514092],[60,(391.66+350.00+450.00+316.66)/4],[70,(335.71+364.29+314.29+321.43)/4],[80,(356.25+331.25+337.50+368.75)/4]},color=green);

> pic6 :=plot(f(x,30), x = 60..80,color=green) ;

> display(pic1, pic2,pic3, pic4,pic5, pic6) ;
```

With last command above, for each level of factors estimated graphs were drawn within range of the other factors and presented Figure 1.

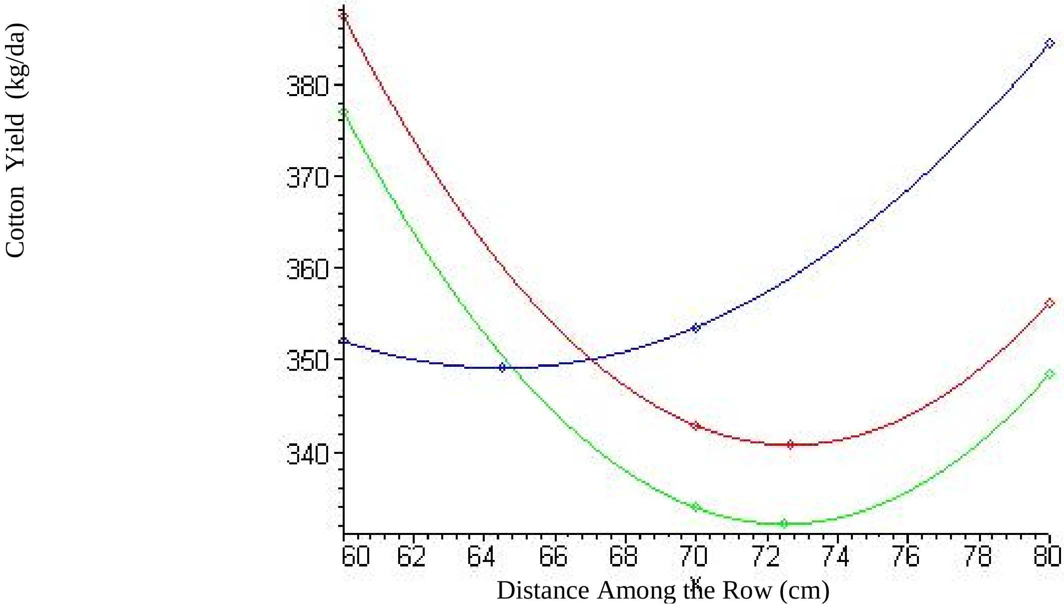

Fig 1: Lagrange interpolation for distances on the rows 10 cm ( ), 20 cm ( ) and 30 cm ( ) $(60 \le x \le 80)$

With a fixed distance among the row y=60 cm, Lagrange interpolation function was obtained below.

```
> f(x,60) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,60)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,60)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,60);
```

$$f(x, 60) := \frac{(x - x1)(x - x2)\, f(x0, 60)}{(x0 - x1)(x0 - x2)} + \frac{(x - x0)(x - x2)\, f(x1, 60)}{(x1 - x0)(x1 - x2)} + \frac{(x - x0)(x - x1)\, f(x2, 60)}{(x2 - x0)(x2 - x1)}$$

Then the level of factors and their values will be written. So for Lagrange interpolation functions with one variable, the command syntax is shown below.

```
>f(x,60):=eval(f(x,60),[x0=10,x1=20,x2=30,f(x0,60)=(400.00+383.33+358.33+408.33)/4,f(x1,60)=(341.66+383.33+341.66+341.66)/4,f(x2,60)=(391.66+350.00+450.00+316.66
)/4]);
```

$$f(x, 60) := 1.937487500\,(x - 20)(x - 30) - 3.520775000\,(x - 10)(x - 30) + 1.885400000\,(x - 10)(x - 20)$$

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Similarly, with a fixed distance among the rows y=70 cm and y=80 cm Lagrange interpolation functions were obtained below, respectively.

> f(x,70) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,70)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,70)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,70);

$$f(x, 70) := \frac{(x - x0)(x - x2)\, f(x0, 70)}{(x0 - x1)(x0 - x2)} + \frac{(x - x0)(x - x2)\, f(x1, 70)}{(x1 - x0)(x1 - x2)} + \frac{(x - x0)(x - x1)\, f(x2, 70)}{(x2 - x0)(x2 - x1)}$$

>f(x,70):=eval(f(x,70),[x0=10,x1=20,x2=30,f(x0,70)=(307.14+407.14+314.29+342.86)/4,f(x1,70)=(350.00+328.57+385.71+350.00)/4,f(x2,70)=(335.71+364.29+314.29+321.43)/4]);

$$f(x, 70) := 1.714287500\ (x\quad 20)(x\quad 30)$$
$$3.535700000\ (x\quad 10)(x\quad 30)$$
$$1.669650000\ (x\quad 10)(x\quad 20)$$

> f(x,80) := (x-x1)*(x-x2)/((x0-x1)*(x0-x2))*f(x0,80)+(x-x0)*(x-x2)/((x1-x0)*(x1-x2))*f(x1,80)+(x-x0)*(x-x1)/((x2-x0)*(x2-x1))*f(x2,80);

$$f(x, 80) := \frac{(x - x1)(x - x2)\, f(x0, 80)}{(x0 - x1)(x0 - x2)} + \frac{(x - x0)(x - x2)\, f(x1, 80)}{(x1 - x0)(x1 - x2)} + \frac{(x - x0)(x - x1)\, f(x2, 80)}{(x2 - x0)(x2 - x1)}$$

> f(x,80):=eval(f(x,80),[x0=10,x1=20,x2=30,f(x0,80)=(331.25+400.00+381.25+312.50)/4,f(x1,80)=(375.00+400.00+387.50+375.00)/4,f(x2,80)=(356.25+331.25+337.50+368.75)/4]);

$$f(x, 80) := 1.781250000\ (x - 20)(x - 30) - 3.843750000\ (x - 10)(x - 30) + 1.742187500\ (x - 10)(x - 20)$$

Firstly, for y = 60 cm (distance among the rows), to find the distance on the rows which has minimum or maximum yield, and the yields in the dose, taking the derivative of the above equation equal to zero, then, and so distance on the rows having the maximum and (or) minimum called critical points will be estimated below.

> z1:=diff(f(x,60),x);

$$z1 := .604225000\ x - 12.60537500$$

> solve(z1=0);

$$20.86205470$$

When y = 60 cm, this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(20.86205470,60):=eval(f(x,60),[x=20.86205470]);

$$f(20.86205470, 10) := 351.8529886$$

Distance on the rows which has a minimum yield and yield can be seen in the graphics (minimum yield is the amount 351.8529886).

With Lagrange interpolation function, for the distance on the rows such as 65 cm and 74 cm yields will be estimated, respectively.

> f(15,60):=eval(f(x,60),[x=15]);

$$f(15, 60) := 362.2346875$$

> f(25,60):=eval(f(x,60),[x=25]);

$$f(25, 60) := 357.0259375$$

Now below for y = 70cm (distance among the rows), to find the distance on the rows which has minimum or maximum yield, and the yields in the dose, the process above is repeated.

> z2:=diff(f(x,70),x);

$$z2 := -.303525000\ x + 5.62412500$$

> solve(z2=0);
18.52936331

When y = 70 cm, this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(18.52936331,70):=eval(f(x,70),[x=18.52936331]);

f (18.52936331 , 70) := 353.8982277

Distance on the rows which has a maximum yield and yield can be seen in the graphics (maximum yield is the amount 353.8982277).

With Lagrange interpolation function, for the distance on the rows such as 15 cm and 25 cm yields will be estimated, respectively. Then, for getting 350 kg / da cotton yield, the distance on the rows could be estimated.

> f(15,70):=eval(f(x,70),[x=15]);
f (15 , 70) := 352.0078125

> f(25,70):=eval(f(x,70),[x=25]);
f (25 , 70) := 347.5440625

> solve(f(x,70)=350);
13.46119106 , 23.59753557

Firstly, for y = 80 cm (distance among the rows), to find the distance on the rows which has minimum or maximum yield, and the yields in the dose, taking the derivative of the above equation equal to zero, then, and so distance on the rows having the maximum and (or) minimum called critical points will be estimated below.

> z1:=diff(f(x,80),x);
$z1 := -.640625000\, x + 12.42187500$

> solve(z1=0);
19.39024390

When y = 80 cm, this value which reaches the minimum yield is value of x and below this minimum value was calculated. There is no maximum value here.

> f(19.39024390,80):=eval(f(x,80),[x=19.39024390]);
f (19.39024390 , 80) := 384.4940929

Distance on the rows which has a maximum yield and yield can be seen in the graphics (maximum yield is the amount 384.4940929).

With Lagrange interpolation function, for the distance on the rows such as 15 cm and 25 cm yields will be estimated, respectively. Then, for getting 350 kg / da cotton yield, the distance on the rows could be estimated.

> f(15,80):=eval(f(x,80),[x=15]);
f (15 , 80) := 378.3203125

> f(25,80):=eval(f(x,80),[x=25]);
f (25 , 80) := 374.4140625

> solve(f(x,80)=350);
9.012918830 , 29.76756898

Now when y = 10, 20, 30 cm (distances among the rows), estimated exchange graphs with in the graph by marking the observed and predicted extreme values, will be shown on the graph. The following commands are written for it.

> with(plots) :
>pic1:=pointplot({[20.86205470,351.8529886],[10,(400.00+383.33+358.33+408.33)/4],[20,(341.66+383.33+341.66+341.66)/4],[30,(391.66+350.00+450.00+316.66
)/4]},color=red);

> pic2 :=plot(f(x,60), x = 10..30,color=red) ;

>pic3:=pointplot({[18.52936331,353.8982277],[10,(307.14+407.14+314.29+342.86)/4],[20,(350.00+328.57+385.71+350.00)/4],[30,(335.71+364.29+314.29+321.43)/4]},color=blue);

> pic4 :=plot(f(x,70), x = 10..30,color=blue) ;

> pic5:=pointplot({[19.39024390,384.4940929],[10,(331.25+400.00+381.25+312.50)/4],[20,(375.00+400.00+387.50+375.00)/4],[30,(356.25+331.25+337.50+368.75)/4]},color=green);

> pic6 :=plot(f(x,80), x = 10..30,color=green) ;

> display(pic1, pic2,pic3, pic4,pic5, pic6) ;

With the last command above, for each level of factors estimated graphs were drawn within range of the other factors and presented Figure 2.

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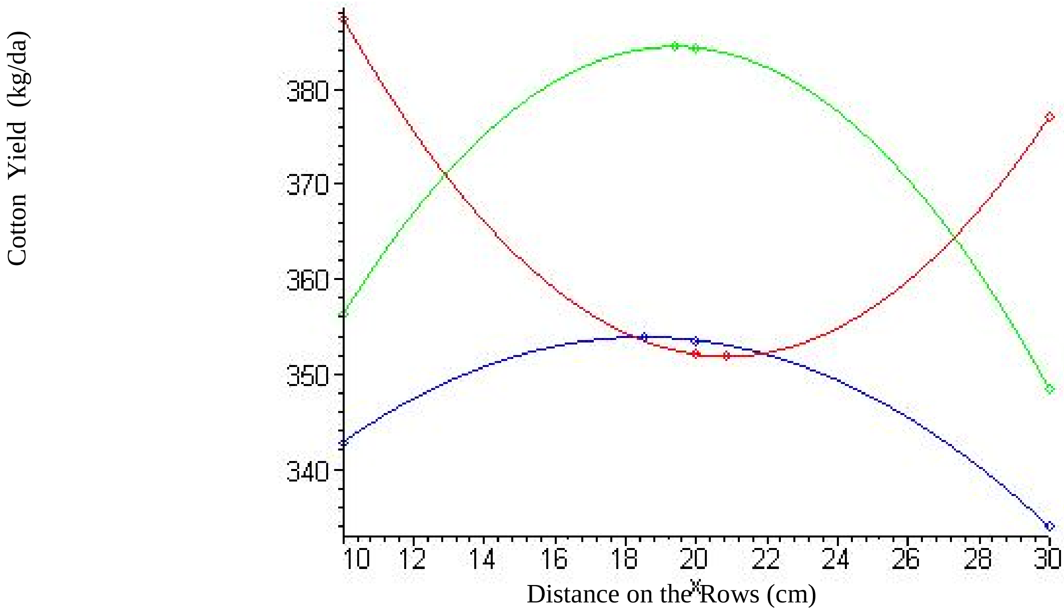

Figure 2. Lagrange interpolation for distances among the rows 60 cm ( ), 70 cm ( ) and 80 cm ( ) $(10 \le x \le 30)$

The standard error for the average yield value can be calculated as well. As known, experimentation has been made in four replicates and the tables below have been created using the average yield value. The tables have observed and estimated values mentioned above.

The standard errors for the average yields estimated are obtained below by Lagrange interpolation. Here, only the above mentioned factors, taking into account the level and intermediate levels are estimated standard errors. Firstly, command syntax (syntax) for getting the standard errors of the observed values is given below.

```
> O11:=(400+383.33+358.33+408.33)/4;
 O11    :=    387.4975000

> sh11:=root((((400-O11)^2+(383.33-O11)^2+(358.33-O11)^2+(408.33-O11)^2)/3)/4,2);
 sh11    :=    11.02427879

> O21:=(307.14+407.14+314.29+342.86)/4;
 O21    :=    342.8575000

> sh21:=root((((307.14-O21)^2+(407.14-O21)^2+(314.29-O21)^2+(342.86-O21)^2)/3)/4,2);
 sh21    :=    22.77440390

> O31:=(331.25+400.00+381.25+312.50)/4;
 O31    :=    356.2500000

> sh31:=root((((331.25-O31)^2+(400.00-O31)^2+(381.25-O31)^2+(312.50-O31)^2)/3)/4,2);
 sh31    :=    20.57126839

> O12:=(341.66+383.33+341.66+341.66)/4;
 O12    :=    352.0775000

> sh12:=root((((341.66-O12)^2+(383.33-O12)^2+(341.66-O12)^2+(341.66-O12)^2)/3)/4,2);
 sh12    :=    10.41750000

> O22:=(350.00+328.57+385.71+350.00)/4;
 O22    :=    353.5700000

> sh22:=root((((350.00-O22)^2+(328.57-O22)^2+(385.71-O22)^2+(350.00-O22)^2)/3)/4,2);
 sh22    :=    11.84437068
```

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```
> O32:=(375.00+400.00+387.50+375.00)/4;
O32 := 384.3750000

> sh32:=root((((375.00-O32)^2+(400.00-O32)^2+(387.50-O32)^2+(375.00-O32)^2)/3)/4,2);
sh32 := 5.983919423

> O13:=(391.66+350.00+450.00+316.66)/4;
O13 := 377.0800000

> sh13:=root((((391.66-O13)^2+(350.00-O13)^2+(450.00-O13)^2+(316.66-O13)^2)/3)/4,2);
sh13 := 28.74283331

> O23:=(335.71+364.29+314.29+321.43)/4;
O23 := 333.9300000

> sh23:=root((((335.71-O23)^2+(364.29-O23)^2+(314.29-O23)^2+(321.43-O23)^2)/3)/4,2);
sh23 := 11.05621092

> O33:=(356.25+331.25+337.50+368.75)/4;
O33 := 348.4375000

> sh33:=root((((356.25-O33)^2+(331.25-O23)^2+(337.50-O33)^2+(368.75-O33)^2)/3)/4,2);
sh33 := 7.073684322
```

Secondly, general command syntax of Lagrange interpolation function with two independent variables (3x3) is given below.

```
> s := (x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh11+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh12+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh13+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh21+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh22+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh23+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh31+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh32+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh33;
```

Where x is the distance among the rows, y is the distance on the rows and shij are the standard error values of cotton yield for the observed values (i, the distance among the rows and j, the distance among the rows)

```
> sl:=eval(s,[x1=60,x2=70,x3=80,y1=10,y2=20,y3=30]);

> eval(sl,[x=65,y=10]);
18.64349892

> eval(sl,[x=74,y=10]);
23.56754097

> eval(sl,[x=65,y=20]);
12.04185058

> eval(sl,[x=74,y=20]);
10.37466881

> eval(sl,[x=65,y=30]);
18.18651014

> eval(sl,[x=74,y=30]);
7.818708786

> eval(sl,[x=60,y=15]);
8.354375386

> eval(sl,[x=60,y=25]);
17.21365264

> eval(sl,[x=70,y=15]);
16.04165310

> eval(sl,[x=70,y=25]);
10.18255662

> eval(sl,[x=80,y=15]);
11.31795468

> eval(sl,[x=80,y=25]);
4.569162639

> eval(sl,[x=72.69224142,y=10]);
23.55386635

> eval(sl,[x=64.49083156,y=20]);
11.95975255

> eval(sl,[x=72.48384859,y=30]);
8.787804023

> eval(sl,[x=60,y=20.86205470]);
11.25156405

> eval(sl,[x=70,y=18.52936331]);
12.81570372

> eval(sl,[x=80,y=19.39024390]);
6.424575085

> eval(sl,[x=67.04744904,y=10]);
20.75681350

> eval(sl,[x=78.33703380,y=10]);
21.90489674

> eval(sl,[x=62.04282666,y=20]);
11.30126662

> eval(sl,[x=66.93883646,y=20]);
12.18152909
```

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> eval(sl,[x=64.61540092,y=30]);
18.87687132

> eval(sl,[x=80.35229626,y=30]);
7.183280742

> eval(sl,[x=70,y=13.46119106]);
17.84365279

> eval(sl,[x=70,y=23.59753557]);
10.39283380

> eval(sl,[x=80,y=9.012918830]);
22.86126070

> eval(sl,[x=80,y=29.76756898]);
6.870397141

Table 2 shows that factors which can be obtained for each level of the maximum or minimum level cotton yield can be achieved among other factors. Selected intermediate values for each factor are completely optional. The researchers now could estimate the yield values with standard error for all levels of the other factors. For example, when distance among the rows is 70 cm and the distance on the rows is 18.53, 353.90 kg / da maximum seed cotton yield with 12.82 kg / da standard error was estimated.

Table 2: Intermediate values having ekstremum levels and their cotton yield and standard error estimates (kg / da)

Cotton Yield (kg/da)		Distance on the rows (cm)					
		10	**18.53**	**19.39**	**20**	**20.86**	**30**
Distance among the rows (cm)	60	387.50 11.02			352.08 10.42	351.85 11.25	377.08 28.74
	64.49				349.12 11.96		
	70	342.86 22.77	353.90 12.82		353.57 11.84		333.93 11.06
	72.48						332.15 8.79
	72.69	340.75 23.55					
	80	356.25 20.57		384.49 6.42	384.38 5.98		348.44 7.07

In addition the maximum or minimum values of a desired intermediate levels could be estimated.

Table 3: For some intermediate levels, cotton yield and standard error estimates (kg/da)

Cotton Yield (kg/da)		Distance on the rows (cm)				
		10	**15**	**20**	**25**	**30**
Distance among the rows (cm)	60	387.50 11.02	362.23 8.35	352.08 10.42	357.03 17.21	377.08 28.74
	65	357.92 18.64		349.16 12.04		348.30 18.19
	70	342.86 22.77	352.01 16.04	353.57 11.84	347.54 10.18	333.93 11.06
	74	341.25 23.57		362.38 10.37		332.81 7.82
	80	356.25 20.57	378.32 11.32	384.38 5.98	374.41 4.57	348.44 7.07

Table 3 is given for some intermediate-level cotton yield estimates (kg / da). For example, when distance among the rows is 65 cm and the distance on the rows is 10, 20, 30 cm, the cotton yields with their standard errors are estimated 357.92 and 18.64, 349.16 and 12.04 and 348.30 and 18.19 kg / da, respectively.

The distance among or on the rows of the desired cotton yield estimated and presented in the Table 4. For example, when distance among the rows is 70 cm, for getting 350 kg / da cotton yield the distance on the rows should be 13.46 or 23.6 cm.

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Table 4: For the desired cotton yield (350 kg / da), estimations of the distances among or on the rows and their standard errors (kg/da)

Cotton Yield (kg/da)		**Distance on the rows (cm)**						
		9.01	**10**	**13.46**	**20**	**23.60**	**29.77**	**30**
Distance among the rows (cm)	60		387.50 11.02		352.08 10.42			377.08 28.74
	62.04				350 11.30			
	64.62							350 18.87
	66.94				350 12.18			
	67.05		350 20.76					
	70		342.86 22.77	350 17.84	353.57 11.84	350 10.39		333.93 11.06
	78.34		350 21.90					
	80	350 22.86	356.25 20.57		384.38 5.98		350 6.87	348.44 7.07
	80.35							350 7.18

In Table 4, the amounts of the standard errors which extend to the right and left of the average value (350 kg / da) are given. For example, while the distance on the rows is 20 cm, when the amount of average yield is 350 kg / da, the estimated distance among the rows is 62.04 cm.

In addition, estimation for any intermediate level of any factor to the desired cotton yield could be made. For example, when the distance among the rows is 65 cm, to getting 350 kg/da cotton yield the investigator can find what distance on the rows should be.

Because the level of factor number is three, there is no turning point here. Now the graph of the prediction obtained for each level of factors will be drawn within the range of the other factor.

Command syntax of Langrange interpolation function (3x3) with two independent values is given below.

```
> f := (x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*f11+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*f12+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*f13+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*f21+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*f22+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*f23+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*f31+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*f32+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*f33;
```

Where x is the distance among the rows, y is the distance on the rows and fij are the standard error values of cotton yield for the observed values (i, the distance among the rows and j, the distance among the rows)

```
>fl:=eval(f,[x1=60,x2=70,x3=80,y1=10,y2=20,y3=30,f11=(400.00+383.33+358.33+408.33)/4,f12=(341.66+383.33+341.66+341.66)/4,f13=(391.66+350.00+450.00+316.66)/4,f21=(307.14+407.14+314.29+342.86)/4,f22=(350.00+328.57+385.71+350.00)/4,f23=(335.71+364.29+314.29+321.43)/4,f31=(331.25+400.00+381.25+312.50)/4,f32=(375.00+400.00+387.50+375.00)/4,f33=(356.25+331.25+337.50+368.75)/4]);
```

If the partial derivatives of the function above are taken by firstly x and then y,

```
> flx:=diff(fl,x);

> fly:=diff(fl,y);
```

are obtained. Whether the partial derivatives equal to zero

```
> eqns := {flx=0,fly=0};
```

then is resolved.

> sols := solve(eqns);

$x = 67.18529585$ and $y = 15.59758556$ are obtained. Then If the partial derivatives of the function taken by x and y are taken by x and y, again.

> fxx:=diff(flx,x);

> fxy:=diff(flx,y);

> fyy:=diff(fly,y);

are obtained.

> delta:=fxx*fyy-fxy^2;

the delta equation is obtained. If the estimated critical values $x = 67.18529585$ ve $y = 15.59758556$ are written in the equations of partial derivatives taken by x and y and then delta equation, respectively,

> eval(flx,[x=67.18529585,y=15.59758556]);

$.3\ 10^{-8}$

> eval(fly,[x=67.18529585,y=15.59758556]);

0.

> delta:=eval(delta,[x=67.18529585,y=15.59758556]);

$\delta := -.1606665656$

The obtained value of delta is negative. So the estimated critical values $x = 67.18529585$ ve $y = 15.59758556$ are saddle point. That's this point is not an extreme point.

With Lagrange interpolation function, for the distance among and on the rows such as 65 cm and 15 cm yields will be estimated, respectively. With two variables, this method is very practical according to the Lagrange interpolation applied as shown separately.

> eval(fl,[x=65,y=15]);

352.5538281

> eval(fl,[x=65,y=10]);

357.9234375

By using command syntax with two variables Lagrange interpolation, Figure 3 is drawn.

> plot3d(fl,x=60..80,y=10..30,axes=boxed,color=x);

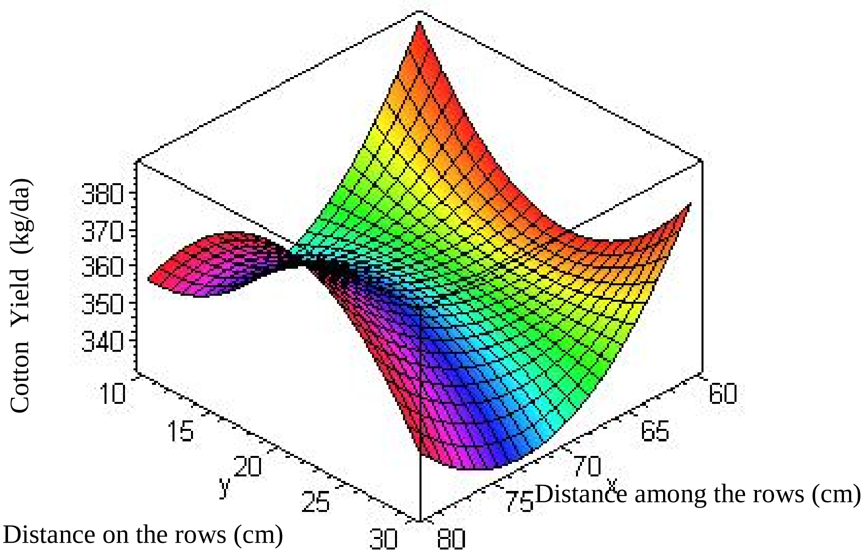

Fig 3: With three dimensionally Lagrange interpolation graph according to cotton yield for the distance among and on the rows $(10 \le y \le 30,\ 60 \le x \le 80)$

Moreover, in this study the mean of the distance among the rows 60 cm, 70 cm and 80 cm and the distance on the rows 10 cm, 20 cm and 30 were found no difference for 95% level of statistical significance, respectively.

Command syntax of Langrange interpolation function (3x3) with two independent values is given below.

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> f := (s-s2)*(s-s3)/((s1-s2)*(s1-s3))*(k-k2)*(k-k3)/((k1-k2)*(k1-k3))*s11+(s-s2)*(s-s3)/((s1-s2)*(s1-s3))*(k-k1)*(k-k3)/((k2-k1)*(k2-k3))*s12+(s-s2)*(s-s3)/((s1-s2)*(s1-s3))*(k-k1)*(k-k2)/((k3-k1)*(k3-k2))*s13+(s-s1)*(s-s3)/((s2-s1)*(s2-s3))*(k-k2)*(k-k3)/((k1-k2)*(k1-k3))*s21+(s-s1)*(s-s3)/((s2-s1)*(s2-s3))*(k-k1)*(k-k3)/((k2-k1)*(k2-k3))*s22+(s-s1)*(s-s3)/((s2-s1)*(s2-s3))*(k-k1)*(k-k2)/((k3-k1)*(k3-k2))*s23+(s-s1)*(s-s2)/((s3-s1)*(s3-s2))*(k-k2)*(k-k3)/((k1-k2)*(k1-k3))*s31+(s-s1)*(s-s2)/((s3-s1)*(s3-s2))*(k-k1)*(k-k3)/((k2-k1)*(k2-k3))*s32+(s-s1)*(s-s2)/((s3-s1)*(s3-s2))*(k-k1)*(k-k2)/((k3-k1)*(k3-k2))*s33;

Where s is the distance among the rows, k is the distance on the rows and sij are the standard error values of cotton yield for the observed values (i, the distance among the rows and j, the distance among the rows)

>fl:=eval(f,[s1=60,s2=70,s3=80,k1=10,k2=20,k3=30,s11=(400.00+383.33+358.33+408.33)/4,s12=(341.66+383.33+341.66+341.66)/4,s13=(391.66+350.00+450.00+316.66)/4,s21=(307.14+407.14+314.29+342.86)/4,s22=(350.00+328.57+385.71+350.00)/4,s23=(335.71+364.29+314.29+321.43)/4,s31=(331.25+400.00+381.25+312.50)/4,s32=(375.00+400.00+387.50+375.00)/4,s33=(356.25+331.25+337.50+368.75)/4]);

> K2:=eval(fl,[s=m,k=20]);

$$K2 := 1.760387500\,(m-70)(m-80) - 3.535700000\,(m-60)(m-80) + 1.921875000\,(m-60)(m-70)$$

> s := (x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh11+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh12+(x-x2)*(x-x3)/((x1-x2)*(x1-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh13+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh21+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh22+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh23+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y2)*(y-y3)/((y1-y2)*(y1-y3))*sh31+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y3)/((y2-y1)*(y2-y3))*sh32+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*(y-y1)*(y-y2)/((y3-y1)*(y3-y2))*sh33;

> sl:=eval(s,[x1=60,x2=70,x3=80,y1=10,y2=20,y3=30]);

> S2:=eval(sl,[x=m,y=20]);

$$S2 := .05208750000\,(m-70)(m-80) - .1184437068\,(m-60)(m-80) + .02991959712\,(m-60)(m-70)$$

> T22:=K2+2*S2;

$$T22 := 1.864562500\,(m-70)(m-80) - 3.772587414\,(m-60)(m-80) + 1.981714194\,(m-60)(m-70)$$

> T21:=K2-2*S2;

$$T21 := 1.656212500\,(m-70)(m-80) - 3.298812586\,(m-60)(m-80) + 1.862035806\,(m-60)(m-70)$$

> plot([T22,K2,T21],m=60..80, color=[blue,red,black]);

With the last command above for the distance on the rows 20 cm in 95% significance level, Langange interpolation and its confidence level in the range of other factor within the boundaries were drawn and presented Figure 4.

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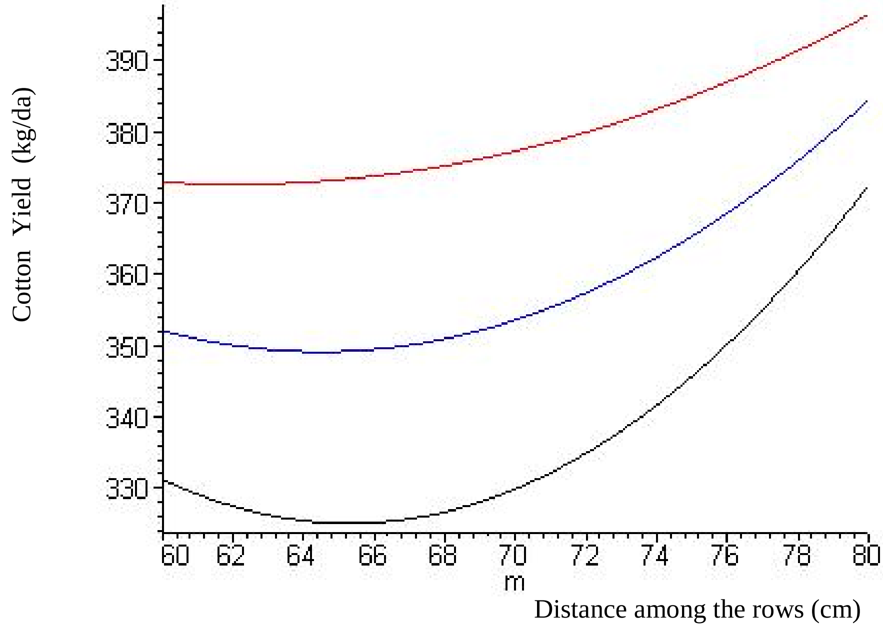

Fig 4: Lagrange interpolasyon with its confidence level for the distance on the rows 20 cm $(60 \le m \le 80)$

> eval(T21,[m=60]);
331.2425000

> eval(T22,[m=60]);
372.9125000

> eval(T21,[m=70]);
329.8812586

> eval(T22,[m=70]);
377.2587414

> eval(T21,[m=80]);
372.4071612

> eval(T22,[m=80]);
396.3428388

It seems that the mean of the distance among the rows 60 cm, 70 cm and 80 cm are found no difference for 95% level of statistical significance.

Now if the same procedure is performed below for the distance among the rows 70 cm,

> L2:=eval(fl,[s=70,k=n]);

$$L2 := 1.714287500\,(n-20)(n-30) - 3.535700000\,(n-10)(n-30) + 1.669650000\,(n-10)(n-20)$$

> S2:=eval(sl,[x=70,y=n]);

$$S2 := .1138720195\,(n-20)(n-30) - .1184437068\,(n-10)(n-30) + .05528105460\,(n-10)(n-20)$$

> T22:=L2+2*S2;

$$T22 := 1.942031539\,(n-20)(n-30) - 3.772587414\,(n-10)(n-30) + 1.780212109\,(n-10)(n-20)$$

> T21:=L2-2*S2;

$$T21 := 1.486543461\,(n-20)(n-30) - 3.298812586\,(n-10)(n-30) + 1.559087891\,(n-10)(n-20)$$

is obtained.

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> plot([T22,L2,T21],n=10..30, color=[blue,red,black]);

With the last command above for the distance among the rows 70 cm in 95% significance level, Langange interpolation and its confidence level in the range of other factor within the boundaries were drawn and presented Figure 5.

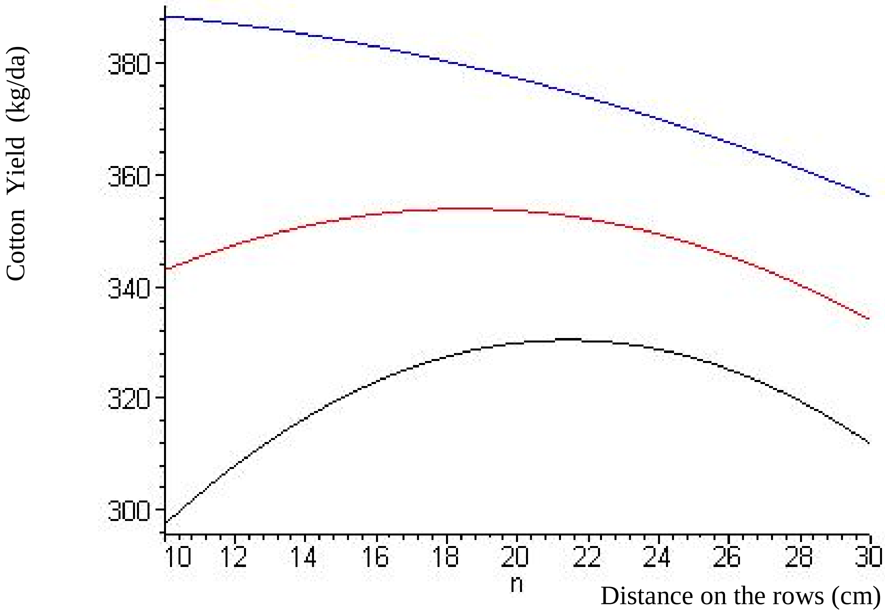

Fig 5: Lagrange interpolasyon with its confidence level for the distance among the rows 70 cm $(10 \le n \le 30)$

> eval(T21,[n=10]);
297.3086922

> eval(T22,[n=10]);
388.4063078

> eval(T21,[n=20]);
329.8812586

> eval(T22,[n=20]);
377.2587414

> eval(T21,[n=30]);
311.8175782

> eval(T22,[n=30]);
356.0424218

It seems that the mean of the distance on the rows 60 cm, 70 cm and 80 cm are found no difference for 95% level of statistical significance.

Now average distance among the rows is examined below. For that reason, command syntax (syntax) and its images were presented Figure 6.

> f := (s-s2)*(s-s3)/((s1-s2)*(s1-s3))*f1+(s-s1)*(s-s3)/((s2-s1)*(s2-s3))*f2+(s-s1)*(s-s2)/((s3-s1)*(s3-s2))*f3;

$$f := \frac{(s-s2)(s-s3)\,f1}{(s1-s2)(s1-s3)} + \frac{(s-s1)(s-s3)\,f2}{(s2-s1)(s2-s3)} + \frac{(s-s1)(s-s2)\,f3}{(s3-s1)(s3-s2)}$$

>fl:=eval(f,[s1=60,s2=70,s3=80,f1=(400.00+383.33+358.33+408.33+341.66+383.33+341.66+341.66+391.66+350.00+450.00+316.66)/12,f2=(307.14+407.14+314.29+342.86+350.00+328.57+385.71+350.00+335.71+364.29+314.29+321.43)/12, f3=(331.25+400.00+381.25+312.50+375.00+400.00+387.50+375.00+356.25+331.25+337.50+368.75)/12]);

$$fl := 1.861091666\,(s-70)(s-80) - 3.434525000\,(s-60)(s-80) + 1.815104166\,(s-60)(s-70)$$

> K:=eval(fl,[s=m]);

$$K := 1.861091666\,(m-70)(m-80) - 3.434525000\,(m-60)(m-80) + 1.815104166\,(m-60)(m-70)$$

>O1:=(400.00+383.33+358.33+408.33+341.66+383.33+341.66+341.66+391.66+350.00+450.00+316.66)/12;

$$O1 := 372.2183333$$

> sh1:=root((((400-O1)^2+(383.33-O1)^2+(358.33-O1)^2+(408.33-O1)^2+(341.66-O1)^2+(383.33-O1)^2+(341.66-O1)^2+(341.66-O1)^2+(391.66-O1)^2+(350.00-O1)^2+(450.00-O1)^2+(316.66-O1)^2)/11)/12,2);

$$sh1 := 10.77506998$$

>O2:=(307.14+407.14+314.29+342.86+350.00+328.57+385.71+350.00+335.71+364.29+314.29+321.43)/12;

$$O2 := 343.4525000$$

> sh2:=root((((307.14-O2)^2+(407.14-O2)^2+(314.29-O2)^2+(342.86-O2)^2+(350.00-O2)^2+(328.57-O2)^2+(385.71-O2)^2+(350.00-O2)^2+(335.71-O2)^2+(364.29-O2)^2+(314.29-O2)^2+(321.43-O2)^2)/11)/12,2);

$$sh2 := 8.768060160$$

>O3:=(331.25+400.00+381.25+312.50+375.00+400.00+387.50+375.00+356.25+331.25+337.50+368.75)/12;

$$O3 := 363.0208333$$

> sh3:=root((((331.25-O3)^2+(400.00-O3)^2+(381.25-O3)^2+(312.50-O3)^2+(375.0-O3)^2+(400.0-O3)^2+(387.5-O3)^2+(375.0-O3)^2+(356.25-O3)^2+(331.25-O3)^2+(337.50-O3)^2+(368.75-O3)^2)/11)/12,2);

$$sh3 := 8.373188313$$

> s := (x-x2)*(x-x3)/((x1-x2)*(x1-x3))*sh1+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*sh2+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*sh3;

$$s := \frac{10.77506998\,(x-x2)(x-x3)}{(x1-x2)(x1-x3)} + \frac{8.768060160\,(x-x1)(x-x3)}{(x2-x1)(x2-x3)} + \frac{8.373188313\,(x-x1)(x-x2)}{(x3-x1)(x3-x2)}$$

> sl:=eval(s,[x1=60,x2=70,x3=80]);

$$sl := .05387534990\,(x-70)(x-80) - .08768060160\,(x-60)(x-80) + .04186594156\,(x-60)(x-70)$$

> S:=eval(sl,[x=m]);

$$S := .05387534990\,(m-70)(m-80) - .08768060160\,(m-60)(m-80) + .04186594156\,(m-60)(m-70)$$

> T2:=K+2*S;

$$T2 := 1.968842366\,(m-70)(m-80) - 3.609886203\,(m-60)(m-80) + 1.898836049\,(m-60)(m-70)$$

> T1:=K-2*S;

$$T1 := 1.753340966\,(m-70)(m-80) - 3.259163797\,(m-60)(m-80) + 1.731372283\,(m-60)(m-70)$$

> plot([T2,K,T1],m=60..80, color=[blue,red,black]);

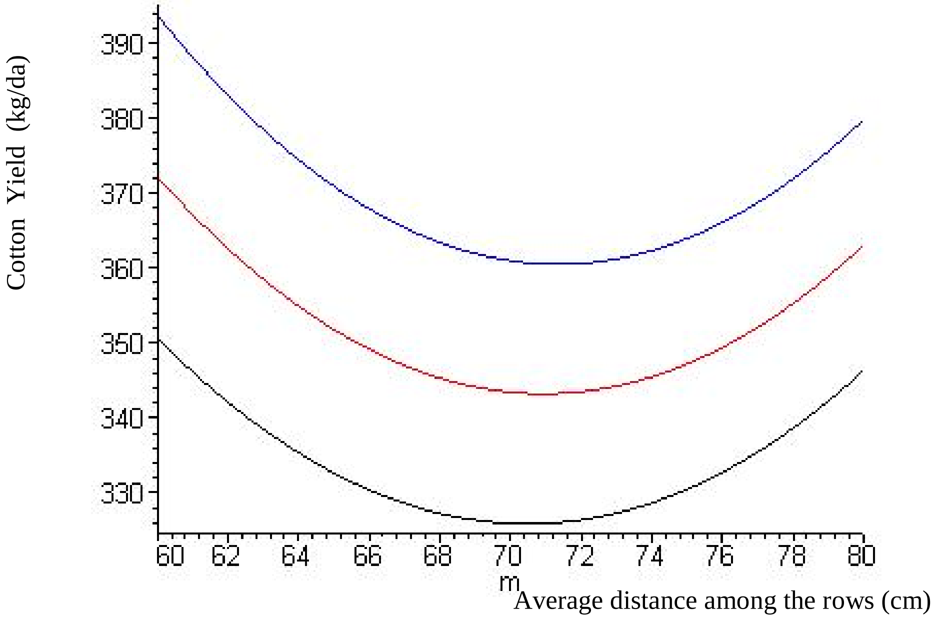

Fig 6: Lagrange interpolation with with its confidence level for average distance among the rows $(60 \le m \le 80)$

> eval(T1,[m=60]);
350.6681932

> eval(T2,[m=60]);
393.7684732

> eval(T1,[m=70]);
325.9163797

> eval(T2,[m=70]);
360.9886203

> eval(T1,[m=80]);
346.2744566

> eval(T2,[m=80]);
379.7672098

The mean of the average distance among rows are found no difference for 95% level of statistical significance.

Now average distance on the rows is examined below. For that reason, command syntax (syntax) and its images were presented Figure 7.

> f := (s-s2)*(s-s3)/((s1-s2)*(s1-s3))*f1+(s-s1)*(s-s3)/((s2-s1)*(s2-s3))*f2+(s-s1)*(s-s2)/((s3-s1)*(s3-s2))*f3;

$$f := \frac{(s - s2)(s - s3)\, f1}{(s1 - s2)(s1 - s3)} + \frac{(s - s1)(s - s3)\, f2}{(s2 - s1)(s2 - s3)} + \frac{(s - s1)(s - s2)\, f3}{(s3 - s1)(s3 - s2)}$$

>fl:=eval(f,[s1=10,s2=20,s3=30,f1=(400.00+383.33+358.33+408.33+307.14+407.14+314.29+342.86+331.25+400+381.25+312.5)/12,f2=(341.66+383.33+341.66+341.66+350+328.57+385.71+350+375+400+387.5+375)/12,f3=(391.66+350+450+316.66+335.71+364.29+314.29+321.43+356.25+331.25+337.5+368.75)/12]);

$$fl := 1.811008334\,(s - 20)(s - 30) - 3.633408333\,(s - 10)(s - 30) + 1.765745834\,(s - 10)(s - 20)$$

> K:=eval(fl,[s=m]);

$$K := 1.811008334\,(m - 20)(m - 30) - 3.633408333\,(m - 10)(m - 30) + 1.765745834\,(m - 10)(m - 20)$$

>O1:=(400.00+383.33+358.33+408.33+307.14+407.14+314.29+342.86+331.25+400+381.25+312.5)/12;

O1 := 362.2016667

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> sh1:=root((((400-O1)^2+(383.33-O1)^2+(358.33-O1)^2+(408.33-O1)^2+(307.14-O1)^2+(407.14-O1)^2+(314.29-O1)^2+(342.86-O1)^2+(331.25-O1)^2+(400-O1)^2+(381.25-O1)^2+(312.5-O1)^2)/11)/12,2);

$sh1 := 11.33465341$

>O2:=(341.66+383.33+341.66+341.66+350+328.57+385.71+350+375+400+387.5+375)/12;

$O2 := 363.3408333$

> sh2:=root((((341.66-O2)^2+(383.33-O2)^2+(341.66-O2)^2+(341.66-O2)^2+(350.00-O2)^2+(328.57-O2)^2+(385.71-O2)^2+(350.00-O2)^2+(375-O2)^2+(400-O2)^2+(387.5-O2)^2+(375-O2)^2)/11)/12,2);

$sh2 := 6.783737623$

>O3:=(391.66+350+450+316.66+335.71+364.29+314.29+321.43+356.25+331.25+337.5+368.75)/12;

$O3 := 353.1491667$

> sh3:=root((((391.66-O3)^2+(350-O3)^2+(450-O3)^2+(316.66-O3)^2+(335.71-O3)^2+(364.29-O3)^2+(314.29-O3)^2+(321.43-O3)^2+(356.25-O3)^2+(331.25-O3)^2+(337.50-O3)^2+(368.75-O3)^2)/11)/12,2);

$sh3 := 11.05303979$

> s := (x-x2)*(x-x3)/((x1-x2)*(x1-x3))*sh1+(x-x1)*(x-x3)/((x2-x1)*(x2-x3))*sh2+(x-x1)*(x-x2)/((x3-x1)*(x3-x2))*sh3;

$$s := 11.33465341\,\frac{(x-x2)(x-x3)}{(x1-x2)(x1-x3)} + \frac{6.783737623\,(x-x1)(x-x3)}{(x2-x1)(x2-x3)} + \frac{11.05303979\,(x-x1)(x-x2)}{(x3-x1)(x3-x2)}$$

> sl:=eval(s,[x1=10,x2=20,x3=30]);

$$sl := .05667326705\,(x-20)(x-30) - .06783737623\,(x-10)(x-30) + .05526519895\,(x-10)(x-20)$$

> S:=eval(sl,[x=m]);

$$S := .05667326705\,(m-20)(m-30) - .06783737623\,(m-10)(m-30) + .05526519895\,(m-10)(m-20)$$

> T2:=K+2*S;

$$T2 := 1.924354868\,(m-20)(m-30) - 3.769083086\,(m-10)(m-30) + 1.876276232\,(m-10)(m-20)$$

> T1:=K-2*S;

$$T1 := 1.697661800\,(m-20)(m-30) - 3.497733580\,(m-10)(m-30) + 1.655215436\,(m-10)(m-20)$$

> plot([T2,K,T1],m=10..30, color=[blue,red,black]);

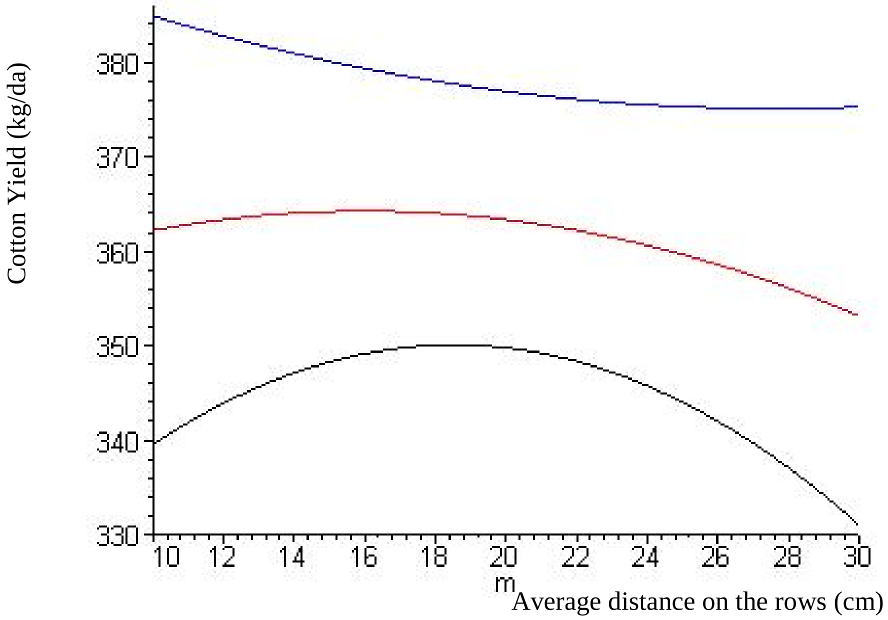

Fig 7: Lagrange interpolation with its confidence level for average distance on the rows $(10 \le m \le 30)$

> eval(T1,[m=10]);
339.5323600

> eval(T2,[m=10]);
384.8709736

> eval(T1,[m=20]);
349.7733580

> eval(T2,[m=20]);
376.9083086

> eval(T1,[m=30]);
331.0430872

> eval(T2,[m=30]);
375.2552464

The mean of the average distance on the rows is found no difference for 95% level of statistical significance.

Here Figure 6 seems biologically meaningless. With Lagrange and Newton interpolation for writing accurate and biologically meaningful interpretation experimental data should be obtained as accurate and biologically meaningful. These graphs show the overall trend of the data obtained and biological data obtained in a meaningful way to reveal whether or not to help investigators. So researchers can have ideas for whether to trust their data or not.

4. CONCLUSIONS AND RECOMMENDATIONS

A factor at any level or among levels belonging to the maximum or minimum result value or the turning point value of other factor or factors which call levels to be reached Lagrange interpolation techniques applied to maple 6 were estimated.

The standard error of any intermediate level of a factor was also estimated by using the Lagrange interpolation techniques.

The graph of Lagrange interpolation with its confidence level was drawn according to the level of statistical significance for the range of factors and in this way, it was provided to examine narrowing or expanding ranges of confidence levels visually.

Since interpolation is based on the average observed value, interpolation polynomial graph is exactly passing through the observed value. So observed values as accurate and biologically significant must be obtained. Otherwise, with errors of observations to be made, the interpolation estimates will lead investigators to the wrong interpretation. If the average observed values are wrong or meaningless as biological research, this situation may be referred investigators to a different direction. So reliability of the average observed values is important. Experiment should be conducted properly from its establishment to its termination.

The graphs about interpolation show the overall trend of the data obtained, and help investigators to reveal about the data whether in a biologically meaningful or not. So researchers can have an idea whether to trust their data or not.

Response surfaces with a combination of two factors are presented the variation of the result as a visual tracking. Three-dimensional planar surface with filling the space among all lines instead of lines until the number of levels of factors can be seen in the graph and about presence of exchange interaction visually with more opportunity to get new ideas are put forward.

Intermediate values of Lagrange interpolation applied separately for a single variable, and two or more variables, are exactly the same. Application of both Lagrange interpolation polynomials is rather significant. With Lagrange interpolation applied for a single variable, the extreme points and turning points can be easily understood but the information about intermediate values is limited, however, the desired value of Lagrange interpolation with two or more variables could be easily reached.

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