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Fuzzy Soft Lattice Theory

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ABSTRACT

Soft set theory was introduced by Molodtsov in 1999 as a general mathematical tool for dealing with problems that contain uncertainty. In this paper, we define concept of fuzzy soft lattice, fuzzy soft sublattice, complete fuzzy soft lattice, modular fuzzy soft lattice, distributive fuzzy soft lattice, fuzzy soft chain and study their basic properties.

Keywords: *Fuzzy soft sets, Fuzzy soft lattices, Fuzzy soft sublattices, Complete fuzzy soft lattices, Modular fuzzy soft lattices, Distributive fuzzy soft lattices.*

1. INTRODUCTION

To solve complex problems in economy, engineering, environmental science and social science, the methods in classical mathematics may not be succesfully modeled because of various types of uncertainties. There are some mathematical theories for dealing with uncertainties such as; fuzzy set theory [28], soft set theory [20], fuzzy soft set theory [17] and so on.

Soft set theory [20] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainty. At present, research works on the soft set theory and its applications are making progress rapidly. The operations of soft sets are defined in [1, 5, 18, 21]. The algebraic structures of soft sets have been studied by some authors [3, 8, 11, 12, 13, 14, 15]. By embedding the ideasof fuzzy sets, many interesting applications of soft set theory have been expanded [2, 6, 7, 9, 17, 21, 23, 24, 25, 26]. and also algebraic structures of fuzzy soft sets have been studied [4, 10, 16, 19, 27, 22].

In this paper, we first define concept of fuzzy soft lattice. We then study fuzzy soft sublattice, complete fuzzy soft lattice, modular fuzzy soft lattice, distributive fuzzy soft lattice, fuzzy soft chain with their basic properties and give several illustrative examples.

2. PRELIMINARIES

In this section, we have presented the basic definition and results of fuzzy sets, soft sets and fuzzy soft sets theory which are useful for subsequent discussions.

Definition 1:

[28]Let E be a crisp set. Then a fuzzy set μ over E is a function from E into [0, 1].

Definition 2:

[5]Let U be an initial universe, P(U) be the power set of U, E be a set of all parameters and. Then, a soft set $f_A$over U is a function from E into P(U) such that $f_A(x) = \emptyset$, $x \notin A$.

Where f_A is called approximate function of the soft set $f_A$and the value $f_A(x)$is a set called x-element of the soft set for all x 2 E.

Definition 3:

[7] Let U be an initial universe, F(U) be the set of all fuzzy sets over U, E be a set of parameters and $A \subset E$. Then, a fuzzy soft set (f,A)over U is a function from E into F(U).

Definition 4:

[7] Let f_A and $f_B$be two fuzzy soft sets. Then, f_A is a fuzzy soft subset of f_B, denoted by $f_A \subseteq f_B$, if $\mu_A \subseteq \mu_B$and $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 5:

[7] Let f_A and f_B be two fuzzy soft sets. Then, union of f_A andf_B, denoted by $f_A \cup f_B$, is defined by its fuzzy approximate function

$$f_{A\cup B}(x) = f_A(x) \cap f_B(x), \text{ for all } x \in E$$

Definition 6:

[7] Let $f_A$and f_B be two fuzzy soft sets. Then, intersection of$f_A$and f_B, denoted by $f_A \cap f_B$, is defined by its approximate function

$$f_{A\cap B}(x) = f_A(x) \cap f_B(x), \text{ for all } x \in E .$$

Definition 7:

[7] Let f_A be a fuzzy soft set over U.Then, the complement $f_A{}^c$off_A is an fuzzy soft set such that

$$f_{A^c} = f^c{}_A(x), \text{ for all } x \in E$$

where $f^c{}_A(x)$ is complement of the set $f_A(x)$.

3. LATTICESTRUCTURESOFFUZZYSOFTSETS

In this section, the notion of fuzzy soft lattices is defined and several related properties are investigated.

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Definition 8:

Let f_L be a fuzzy soft set over U, and $\curlyvee$ and $\curlywedge$ be two binary operation on f_L. If, elements of f_L are equipped with two commutative and associative binary operations $\curlyvee$ and $\curlywedge$ which are connected by the absorbtionlaw, then algebraic structure $(f_L, \curlyvee, \curlywedge)$ is called a fuzzy soft lattice.

Example1.Let U ={x_1,x_2,x_3,x_4} be a universe set and L ={e_0,e_1,e_2,e_3,e_4,e_5}be parameter set. Assume that,

$f_L(e_5)$ = {0.3/x_1,0.4/x_2, 0.5/x_3,0.6=x_4}
$f_L(e_4)$ = {0.2/x_1,0.4/x_2, 0.4/x_3}
$f_L(e_3)$ = {0.3/x_1,0.3/x_2, 0.5/x_3,0.6=x_4}
$f_L(e_2)$ = {0.2/x_1, 0.3/x_2, 0.4/x_4}
$f_L(e_1)$ = {0.1/x_1,0.4/x_2}
$f_L(e_0)$ = {0.1/x_1,0.3/x_2}

Then, $(f_L, \cup, \cap)$ is a fuzzy soft lattice. Here binary operations are fuzzy unionand fuzzy intersection. Hasse diagram of f_L appear in Figure 1.

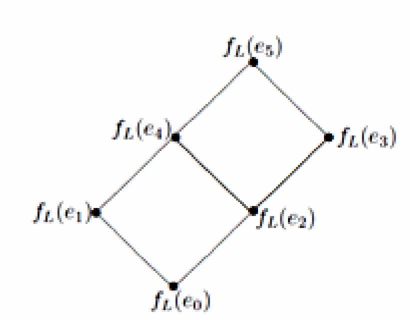

Fig 1: A Fuzzy soft lattice structure

Theorem 1:

Let $(f_L, \curlyvee, \curlywedge)$ be a fuzzy soft lattice. Then

$$f_L(x) \curlywedge f_L(y) = f_L(x),\ f_L(x) \curlyvee f_L(y) = f_L(y)$$

Proof:

$f_L(x) \curlyvee f_L(y) = (f_L(x) \curlywedge f_L(y)) \curlyvee f_L(y)$
$= f_L(y) \curlyvee (f_L(x) \curlywedge f_L(y))$
$= f_L(y) \curlyvee (f_L(y) \curlywedge f_L(x))$
$= f_L(y)$

Conversely, $f_L(x) \curlywedge f_L(y) = f_L(x) \curlywedge (f_L(x) \curlyvee f_L(y))$
$= f_L(x)$.

Theorem 2:

Let $(f_L, \curlyvee, \curlywedge)$ be a fuzzy soft lattice and $\preccurlyeq$ be a relation is denotedby

$f_L(x) \preccurlyeq f_L(y)$, $f_L(x) \curlywedge f_L(y) = f_L(x)$ or $f_L(x) \curlyvee f_L(y) = f_L(y)$

the relation is an ordering relation on f_L.

Proof:

For all $f_L(x)$, $f_L(y)$ and $f_L(z) \in f_L$,

(i) $\preccurlyeq$ is reflexive. $f_L(x) \preccurlyeq f_L(y)$, $f_L(x) \curlywedge f_L(x) = f_L(x)$ by Theorem 1 andDefinition 8.

(ii) $\preccurlyeq$ is antisymmetric. Let be $f_L(x) \preccurlyeq f_L(y)$ and $f_L(y) \preccurlyeq f_L(x)$. Then by Definition 8 and Theorem 1.
$f_L(x) = f_L(x) \curlywedge f_L(y)$
$= f_L(y) \curlywedge f_L(x)$
$= f_L(y)$.

(iii) $\preccurlyeq$ is transitive. Let be $f_L(x) \preccurlyeq f_L(y)$ and $f_L(y) \preccurlyeq f_L(x)$. Then
$f_L(x) \curlywedge f_L(z) = (f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z)$
$= f_L(x) \curlywedge (f_L(y) \curlywedge f_L(z))$
$= f_L(x) \curlywedge f_L(y)$
$= f_L(x)$
from Theorem 1, $f_L(x) \preccurlyeq$ fL(z).

Theorem 3:

Let $(f_L, \curlyvee, \curlywedge)$ be a fuzzy soft lattice. Then,

(i) $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(x)$ and $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(y)$

(ii) $f_L(x) \preccurlyeq f_L(x) \curlyvee f_L(y)$ and $f_L(y) \preccurlyeq f_L(x) \curlyvee f_L(y)$

Proof:

(i) By Definition 8, $(f_L(x) \curlywedge f_L(y)) \curlyvee f_L(x) = f_L(x) \curlyvee (f_L(x) \curlywedge f_L(y))$ =fL(x). From Theorem 1, we get $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(x)$. It can be show that$f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(y)$.

The proof ii. can be made similarly.

Theorem 4:

Let $(f_L, \curlyvee, \curlywedge)$ be a fuzzy soft lattice. Then,
$f_L(x) \preccurlyeq f_L(y)$ and $f_L(z) \preccurlyeq f_L(t))$
$\Rightarrow f_L(x) \curlywedge f_L(z) \preccurlyeq f_L(y) \curlywedge f_L(t)$

Proof:

From hypothesis and Theorem 1,
$f_L(x) \curlywedge f_L(y) = f_L(x)$ and $f_L(z) \curlyvee f_L(t) = f_L(z)$
$(f_L(x) \curlywedge f_L(z)) \curlywedge (f_L(y) \curlywedge f_L(t)) =$
$[(f_L(x) \curlywedge f_L(z)) \curlywedge f_L(y)] \curlywedge f_L(t)$
$= [f_L(x) \curlywedge (f_L(z) \curlywedge f_L(y))] \curlywedge f_L(t)$
$= [f_L(x) \curlywedge (f_L(y) \curlywedge f_L(z))] \curlywedge f_L(t)$
$= [(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z)] \curlywedge f_L(t)$
$= (f_L(x) \curlywedge f_L(y)) \curlywedge (f_L(z) \curlywedge f_L(t))$
$= f_L(x) \curlywedge f_L(z)$

Then, from Theorem 2, $f_L(x) \curlywedge f_L(z) \preccurlyeq f_L(y) \curlywedge f_L(t)$.

Theorem 5:

Let (f_L, $\curlyvee$, $\curlywedge$) be a fuzzy soft lattice. Then, $f_L(x) \preccurlyeq f_L(y)$ and $f_L(z) \preccurlyeq f_L(t)$) $f_L(x) \curlywedge f_L(z) \preccurlyeq f_L(y) \curlywedge f_L(t)$

Proof:

Proof is similarly to Theorem 4.

Example 2:

Consider the lattice f_L in Example 1. Since $f_L(e_4) \subseteq f_L(e_5)$ and$f_L(e_2) \subseteq f_L(e_3)$, then $f_L(e_4) \cap f_L(e_2) \subseteq f_L(e_5) \cap f_L(e_3)$, and $f_L(e_4) \cup f_L(e_2) \subseteq f_L(e_2) \cup f_L(e_5)$.

Lemma 6:

Let (f_L, $\curlyvee$, $\curlywedge$) be a fuzzy soft lattice Then, $f_L(x)$ g $f_L(y)$ and$f_L(x) \curlywedge f_L(y)$ are the least upper bound and the greatest lower bound of $f_L(x)$and $f_L(y)$, respectively.

Proof:

From Theorem 3, $f_L(x) \curlywedge f_L(y)$ and $f_L(x) \curlyvee f_L(y)$ are lower bound andupper bound $f_L(x)$ and $f_L(y)$, respectively. Assume that, $f_L(x) \curlywedge f_L(y)$ is nota greatest lower bound of $f_L(x)$ and $f_L(y)$. Then $f_L(z) \in f_L$ is exist, such that

$f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(z) \preccurlyeq f_L(x)$ and $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(z) \preccurlyeq f_L(y)$. Hence, byTheorem 4, $f_L(z) \curlywedge f_L(z) \preccurlyeq f_L(x) \curlywedge f_L(y)$.

Thus $f_L(z) \preccurlyeq f_L(x) \curlywedge f_L(y)$. Thatis $f_L(z) = f_L(x) \curlywedge f_L(y)$. This is a contradiction.For $f_L(x) \curlyvee f_L(y)$ the proof can be made similarly.

Theorem 7:

A fuzzy soft lattice is a poset.

Proof:

The proof is obviously, from Lemma 6.Example 3. Let U $=\{x_1,x_2,x_3\}$ be a universe set and L = $\{e_0,e_1,e_2,e_3,e_4\}$ be parameter set. Assume that,

$f_L(e_0) = \{0.1/x_1, 0.1/x_2, 0.15/x_3\}$
$f_L(e_1) = \{0.3/x_1, 0.1/x_2, 0.15/x_3\}$
$f_L(e_2) = \{0.2/x_1, 0.4/x_2, 0.15/x_3\}$
$f_L(e_3) = \{0.1/x_1, 0.2/x_2, 0.3/x_3\}$
$f_L(e_4) = \{0.6/x_1, 0.8/x_2, 0.6/x_3\}$

and $\ll$ is a partial ordering relation on $f_L$defined by "$f_L(e_i)(x)$ divides $f_L(e_j)(x)$"$\forall$ i, j$\in$ I and x $\in$ U. Hasse diagram of f_L appears in Figure.2. Thus, f_L is afuzzy soft lattice.

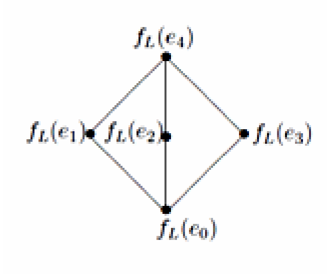

Fig 2: A soft lattice structure

Theorem 8:

Let f_L be a fuzzy soft set. Then, an algebraic structure (f_L, $\curlyvee$, $\curlywedge$, $\preccurlyeq$)is a fuzzy soft lattice.

Proof:

For all $f_L(x)$, $f_L(y)$, $f_L(z) \in f_L$,

(i) From Lemma 6,
$f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(x)$ and $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(y)$.
FromTheorem 3, $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(y) \curlywedge f_L(x)$.
Similarly,
$f_L(y) \curlywedge f_L(x) = f_L(x) \curlywedge f_L(y)$then, $f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(y) \curlywedge f_L(x)$. By the same way, the proof of $f_L(x) \curlyvee f_L(y) = f_L(y) \curlyvee f_L(x)$ can be made.

(ii) From Lemma 6, $(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) \preccurlyeq (f_L(x) \curlywedge f_L(y)) \preccurlyeq f_L(y)$ and$(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) \preccurlyeq f_L(z)$, and from Theorem 3,
$(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) \preccurlyeq f_L(y) \curlywedge f_L(z)$ (1)

Also
$(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) \preccurlyeq f_L(x) \curlywedge f_L(y) \preccurlyeq f_L(x)$.(2)

From (1) and (2), $(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) \preccurlyeq f_L(x) \curlywedge (f_L(y) \curlywedge f_L(z))$.

Similarly,$f_L(x) \curlywedge (f_L(y) \curlywedge f_L(z)) \preccurlyeq (f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z)$.
Then, $(f_L(x) \curlywedge f_L(y)) \curlywedge f_L(z) = f_L(x) \curlywedge (f_L(y) \curlywedge f_L(z))$.
By the same way, the proof of $(f_L(x) \curlyvee f_L(y)) \curlyvee f_L(z) = f_L(x) \curlyvee (f_L(y) \curlyvee f_L(z))$ can be made.From Theorem 3,
$f_L(x) \preccurlyeq (f_L(x) \curlyvee f_L(y))$ and $f_L(x) \preccurlyeq f_L(x)$.
From Theorem4, $f_L(x) \preccurlyeq (f_L(x) \curlyvee f_L(y)) \curlyvee f_L(x)$.
Similarly,$(f_L(x) \curlyvee f_L(y)) \curlywedge f_L(x) \preccurlyeq f_L(x)$.
Then, $f_L(x) \curlywedge (f_L(x) \curlyvee f_L(y)) = f_L(x)$.By the same way, the proof of $f_L(x) \curlyvee (f_L(x) \curlyvee f_L(y)) = f_L(x)$ can be made

Note 1:

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According to Theorem 8, a fuzzy soft lattice ($f_L, \curlyvee, \curlywedge$) has the samecharacter with ($f_L, \curlyvee, \curlywedge, \preccurlyeq$). Therefore, we shall identify any fuzzy soft lattice($f_L, \curlyvee, \curlywedge$) with ($f_L, \curlyvee, \curlywedge, \preccurlyeq$)and use these two concept as interchangeable.

Lemma 9:

Let ($f_L, \cup, \cap$) be a fuzzy soft set and $\subseteq$ be a relation is definedby$f_L(x) \subseteq f_L(y)$, $f_L(x) \cup f_L(y) = f_L(y)$ or $f_L(x) \cap f_L(y) = f_L(x)$for all $f_L(x)$, $f_L(y) \in f_L$, is an ordering relation on f_L.

Proof:

The proof is straightforward.

Corollary 1:

Let ($f_L, \cup, \cap, \subseteq$) is a fuzzy soft lattice.

Definition 9:

Let ($f_L, \curlyvee, \curlywedge, \preccurlyeq$)be a fuzzy soft lattice,
If $f_L(x) \preccurlyeq f_L(y)$ for all $y \in L$, then $f_L(x)$ is called the minimum element off$_L$.If $f_L(y) \preccurlyeq f_L(x)$ for all $y \in L$, then $f_L(x)$ is called the maximum element off$_L$.

Definition 10:

Let ($f_L, \curlyvee, \curlywedge, \preccurlyeq$)be a fuzzy soft lattice. Then, f_L is called asoft chain, If $f_L(y) \preccurlyeq f_L(x)$ or $f_L(x) \preccurlyeq f_L(y)$ for all $f_L(x)$, $f_L(y) \in f_L$.

Example 4:

Consider the fuzzy soft lattice f_L in Example 1, a fuzzy soft subsetf$_S$= $\{f_L(e_0), f_L(e_1), f_L(e_4), f_L(e_5)\}$ of f_L is a fuzzy soft chain. But ($f_L, \cup, \cap, \subseteq$) is not fuzzy soft chain because $f_L(e_1)$ and $f_L(e_2)$ can not be comparable.

Definition 11:

Let ($f_L, \curlyvee, \curlywedge, \preccurlyeq$)be a fuzzy soft lattice. If, every subsets off$_L$ have both a greatest lower bound and a least upper bound, then it is calledcomplete fuzzy soft lattice.

Example 5:

Let $U = \{x_1, x_2, x_3, x_4\}$ and $L = \{e_0, e_1, e_2, e_3\}$ such that,

$f_L(e_0) = \{0.1/x_1, 0.3/x_2\}$
$f_L(e_1) = \{0.2/x_1, 0.3/x_2, 0.4/x_4\}$
$f_L(e_2) = \{0.1/x_1, 0.4/x_2\}$
$f_L(e_3) = \{0.2/x_1, 0.4/x_2, 0.4/x_3\}$

Then ($f_L, \cup, \cap, \subseteq$) is a complete fuzzy soft lattice. Because every finite subsetsof f_L have a greatest lower bound and a least upper bound.

Definition 12:

Let ($f_L, \curlyvee, \curlywedge, \preccurlyeq$)be a fuzzy soft lattice and $f_M \subseteq f_L$. If f_M isa fuzzy soft lattice with the operations of f_L, then f_M is called a fuzzy softsublattice of f_L.

Example 6:

Let f_L be a fuzzy soft lattice as in Figure 1. It is clear that

f_{L_1}= {(e$_0$,{0.09/x$_1$,0.29/x$_2$}),(e$_1$,{0.09/x$_1$,0.39/x$_2$}), (e$_2$, {0.19/x$_1$,0.29/x$_2$,0.39/x$_4$}), (e$_4$, {0.19/x1, 0.39/x$_2$; 0.49/x$_4$})}

And

f_{L_2}= {(e$_0$, {0.08=x$_1$, 0.28/x$_2$}),(e$_2$, {0.18/x$_1$,0.28/x$_2$, 0.38/x$_4$}), (e$_3$,{0.28/x$_1$,0.28/x$_2$, 0.48=x$_3$, 0.55/x$_4$})} are both fuzzy soft sublattices of f_L.

Proposition 1:

Let f_{L_1} and f_{L_2}be two fuzzy soft sublattices of fL. Then,$f_{L_1} \cap f_{L_2}$ is a fuzzy soft lattice with operations of f_L.

The following example shows Proposition 1 doesn't hold in general withrespect to the soft union.

Example 7:

Let $f_{L_1}$and f_{L_2}be two fuzzy soft sublattices of f_L as in Figure1, such that

f_{L_1}= {(e$_0$,{0.09/x$_1$, 0.29/x$_2$}),(e$_2$, {0.19/x$_1$, 0.29/x2,0.39/x$_4$}), (e$_3$,{0.29/x$_1$, 0.39/x$_2$,0.49/x$_3$, 0.54/x$_4$}),(e$_5$, 0.29/x$_1$,0.39/x$_2$, 0.49/x$_3$,0.55/x$_4$)})}

And

f_{L_2} = {(e$_0$, {0.08/x$_1$,0.28/x$_2$}),(e$_2$, {0.18/x$_1$,0.28/ x$_2$,.38/x$_4$}),(e$_4$, {0.18/x$_1$,0.38/ x$_2$,0.54/x$_4$}),(e5, {0.28/x$_1$,0.38/ x$_2$,0.48/x$_3$,0.54/x$_4$})}.
It is easly verifed that
$f_{L_1} \cap f_{L_2}$ = {(e$_0$, {0.08/x$_1$,0.28/ x$_2$}),(e$_2$, {0.18/x$_1$,0.28/ x$_2$,0.38/x$_4$}), (e$_5$, {0.28/x$_1$,0.38/ x$_2$,0.48/x$_3$,0.54/x$_4$})}
is a fuzzy soft lattice. But
$f_{L_1} \cup f_{L_2}$ = f(e$_0$, {0.09/x$_1$,0.29/ x$_2$}),(e$_2$, {0.19/x$_1$, 0.29/ x$_2$,0.39/x$_4$}),(e$_3$, {0.29/x$_1$,0.29/ x$_2$,0.49/x$_3$,0.54/x$_4$}),(e$_4$, 0.18/x$_1$,0.38/x$_2$,0.54/x$_4$}),(e$_5$, {0.29/x$_1$,0.39/x$_2$,0.49/x$_3$,0.55/x$_4$})}
isn't a fuzzy soft sublattice of f_L.

Corollary 2:

Every fuzzy soft chain is a fuzzy soft sublattice.

Proof:

Let $f_C$be a fuzzy soft chain. Since any two elements of $f_C$is comparable, and for all $f_C(x)$, $f_C(y) \in f_C$, $f_C(x) \curlywedge f_C(y)$ and $f_C(x) \curlyvee f_C(y) \in f_C$.

Corollary 3:

Every fuzzy soft lattice is fuzzy soft sublattice of itself.

Definition 13:

Let (f_L,∨ ,∧,≤)be a fuzzy soft lattice. Then, f_L is calleddistributive fuzzy soft lattice. If

$f_L(x) \wedge (f_L(y) \vee f_L(z)) = (f_L(x) \wedge f_L(y)) \vee (f_L(x) \wedge f_L(z))$

$f_L(x) \vee (f_L(y) \wedge f_L(z)) = (f_L(x) \vee f_L(y)) \wedge (f_L(x) \vee f_L(z))$.

Theorem 10:

Let f_L be a distributive fuzzy soft lattice. Then every fuzzy softsublattice of f_L is distributive.

Proof:

It is straightforward.

Theorem 11:

Every fuzzy soft chain is a distributive fuzzy soft lattice.

Proof:

It is immediate by Defnition 5 and 13.

Definition 14:

Let (f_L,∨ ,∧,≤)be a fuzzy soft lattice. Then, f_L is calledmodular fuzzy soft lattice. If it satisfies the following axiom:

$f_L(z) \leq f_L(x) \Rightarrow f_L(x) \wedge (f_L(y) \vee f_L(z)) = (f_L(x) \wedge f_L(y)) \vee f_L(z)$.

Theorem 12:

If (f_L,∨ ,∧,≤)be a distributive fuzzy soft lattice, then (f_L,∨ ,∧,≤)is a fuzzy soft modular lattice.

Proof:

The proof is clearly.

Example 8:

Consider the fuzzy soft lattice f_L in Example 3, f_L is a modularfuzzy soft lattice.

Note 2:

Modular fuzzy soft lattice may not be a distributive fuzzy soft lattice.

Example 9:

Consider the fuzzy soft lattice f_L in Example 3. Since $f_L(e_1) \wedge (f_L(e_2) \vee f_L(e_3)) \neq (f_L(e_1) \wedge f_L(e_2)) \vee f_L(e_3)$, although ($f_L$,∨ ,∧,≪)is amodular fuzzy soft lattice, it is not distributive soft lattice.

Theorem 13:

Let (f_L,∨ ,∧,≤)be a fuzzy soft lattice. Then, $f_L(x) \leq f_L(y) \Rightarrow f_L(x) \leq f_L(y) \wedge (f_L(x) \vee f_L(z))$

Proof:

The proof is clearly.

Example 10:

The fuzzy soft lattice (f_L,∪ ,∩,⊆), is an example for Theorem13.

Theorem 14:

Every fuzzy soft chain is a modular fuzzy soft lattice.

Proof:

It is immediate by Defnition 5 and 14.

4. CONCLUSION

The fuzzy soft set theory has been applied to many fields from theoreticalto practical. In this study, we first define concept of fuzzy soft lattice. Wethen study fuzzy soft sublattice,complete fuzzy soft lattice, modular fuzzysoft lattice, distributive fuzzy soft lattice, fuzzy soft chain with their basic properties and give several illustrative examples. Moreover, based on thispaper, we can investigated other properties of fuzzy soft lattices.

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