

Forecasting Daily Bangladeshi Exchange Rate Series based on Markov Model, Neuro Fuzzy Model and Conditional Heteroskedastic Model

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ABSTRACT

Prediction of exchange rate is very important for many international agents e.g. investors, money managers, investment banks, funds makers and others. We forecasted the daily Bangladeshi exchange rate series for the period of January 1992 to March 2009 using popular non-linear forecasting models, namely Markov switching autoregressive (MS_AR) model, fuzzy extension of artificial neural network model (ANFIS) and generalized autoregressive conditional heteroscedastic (GARCH) model. Our target is to investigate whether selected models can serve as useful forecasting models to find volatile and non-linear behaviours of the considered series. By most commonly used statistical measures: mean absolute percentage error, root mean square error and coefficient of determination, we found that ANFIS is a superior predictor than other two selected predictors. We believe findings of this paper will be helpful to make a wide range of policies for multinational companies who are involved with various international business activities.

Keywords: Forecasting, Markov model, non-linearity, artificial neural network models, fuzzy logic, heteroscedasticity, time series model.

1. INTRODUCTION

Exchange rate is the value of domestic currency against foreign currency, which often moves drastically because of demand and supply sides in the foreign exchange market. This generally affects multinational companies' (e.g. investors, money managers, investment banks, hedge funds and others) profit when companies involve various international business activities. For example, these types of companies have a variety of foreign-currency denominated payables, receivables, credit purchases, credit sales and others. All of these expose multinational companies to exchange rate risks and push companies to hedge against potential losses. Thus, understanding and forecasting exchange rate movements are important to a wide range of decision problems for these companies. A large amount of research ([1-6] and others) has been published in recent times and is continuing to find an optimal (or nearly optimal) prediction model for the exchange rate series. Many forecasting research ([2, 7-9] and others) have shown that the behaviour of exchange rate series cannot be modelled solely by linear time series models (e.g. regression model, AR(p), ARIMA(p,q) and others) because exchange rate nature is mostly complex (non-linear) and volatile. Therefore, developing a model for forecasting requires an iterative process of knowledge discovery, system improvement through data mining as well as error and trial experimentation. To overcome this problem, in recent years ([2,4,6,9,11] and others), we have noticed an increasing interest in modelling data as nonlinear models. This is due to the realization that these studies have revealed significant non-linear behaviours in time series data.

Various nonlinear models have been considered as alternatives to the widely used linear models. These

are: (i) artificial intelligence (AI) models, including artificial neural network (ANN) model, fuzzy logic model, genetic algorithm model, hybridization of ANN and fuzzy system model (known as adaptive neuro fuzzy inference system (ANFIS)), (ii) Markov switching (MS) model, (iii) conditional heteroskedastic (CH) models and others. There is a growing interest in using AI models [1, 3-6, 8-9], MS model [2,10-11,15] and CH models [12-14] to forecast exchange rate series. The reason for this rising popularity is that these models pay particular attention to non-linearities and learning processes both of which can help to improve predictions for complex variables. For our study, we have used various models (e.g. MS model, AI model and CH model) to predict Bangladeshi exchange rates (BEXR) series in order to see whether selected models can help to raise predictive power. By analyzing applied models validity and precision, our plan is to find which model is the best to predict BEXR series.

The MS model is one of the most popular nonlinear time series model in literature, which involves multiple structures that can characterize time series behaviours in different regimes. By permitting switching between these structures, this model is able to capture more complex dynamic patterns. A novel feature of this model is that the switching mechanism is controlled by an unobservable state variable that follows a first-order Markov chain. In particular, the Markovian property regulates that the current value of the state variable depends on its immediate past value. As such, a structure may prevail for a random period of time and it will be replaced by another structure when a switching takes place. The most common AI model e.g. fuzzy extension of ANN, namely ANFIS is particularly useful for future predictions for variables, which is subject to non-linearities. Although ANN based models have been found to perform better compared to conventional statistical models, the main drawback of ANN models is their prediction capabilities deteriorate over a short period of

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time especially when data are very much chaotic. ANFIS has been proposed by many authors ([4-5, 8-9] and many others) to overcome this drawback and develop a reliable prediction of time series data. Thus, we have chosen ANFIS as an AI model to predict BEXR values.

The most successful CH models are the autoregressive conditional heteroskedasticity (ARCH) model introduced by Engle [13] and the generalized ARCH (known as GARCH) model extended by Bollerslev [14]. Engle won the Nobel Prize in 2003 for his contribution of modelling volatility in the financial time series. These models success stems from their ability to capture time-varying volatility and volatility clustering stylized facts of time series data. A time series is said to have ARCH or GARCH effect, the series is known as heteroskedastic (i.e. it's variance vary with time) otherwise homoskedastic. These types of models are widely used in empirical economics and finance. In this paper, we will investigate whether the selected models are useful tools to describe the behaviour of BEXR series more efficiently. To our knowledge, forecasting daily BEXR series under the powerful nonlinear models yet not considered in literature. We have considered this project in this paper, thus we believe findings of the paper will be useful for those who are interested to make wise policies about the complex variable BEXR. The paper is organized as follows. Section 2 describes about the data set with numerical properties. The theoretical framework of considered models is reviewed in section 3. Section 4 contains experimental designs and results of the developed models. We end with some concluding remarks and some future research plans in final section.

2. DATA

The data under investigation are the daily BEXR series, over the period of January 1992 to March 2009 for a total of 6300 observations. The exchange rates are the local currency (TK) against the US dollar, collected from <http://ia.ita.doc.gov/exchange/bangladesh.txt>. The training and testing data sets are shown in Fig-1 and for understanding of changes, return series is depicted in Fig-2. To understand behaviours of the BEXR variable, summary statistics are reported in Table 1. It is noticed that the minimum and maximum rates are 35.88 TK and 70.62 TK respectively. Mean rate is found to be 52.61 TK and SD rate is found to be 10.93 TK, which means that all times BEXR was not observed 52.61 TK (expected range: 41.68 TK to 63.54 TK). Skewed (Sk) and kurtosis (Kur) measures indicate that exchange rate patterns do not

follow the normal distribution. More clearly, Sk 0.15 indicates us most of days exchange rates are observed below 52.61 TK. Kur 1.62 tells us exchange rates are spread in a wider fashion than the normal distribution, which means that fewer BEXR cluster near to 52.61 TK and more BEXR populate the extremes either far above or far below 52.61 TK as compared to the normal curve.

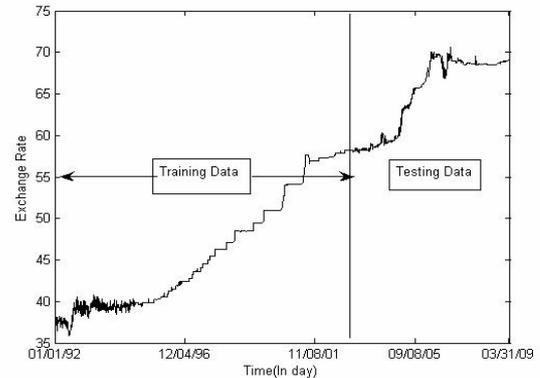


Fig 1: Rates Bangladeshi taka

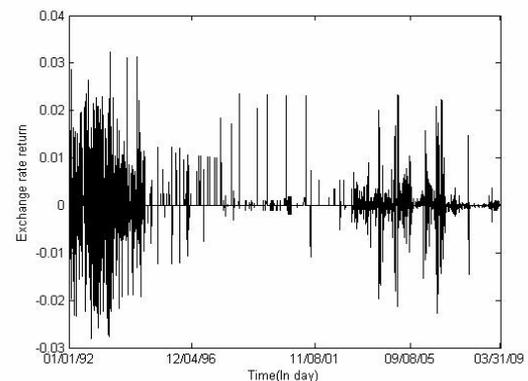


Fig 2: The return series of BEXR

3. FORECASTING MODELS USED TO PREDICT DAILY BEXR SERIES

3.1 MS Model

In real world, changes in regime happen quite suddenly. For example, exchange rate appears to follow long swings.

Table 1: Descriptive Statistics of BEXR

Statistical measures	n	Min	Max	Mean	SD	Sk	Kur
	6300	35.88	70.62	52.61	10.93	0.15	1.62

It means that rate drifts upward for a considerable period of time and then switches to a long period with downward drift. To model this dramatic change, a more practical and realistic model is the Markov switching autoregressive (MS_AR) model,

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developed by Hamilton [15]. This is a mixed model developed based on AR time series model and Markov model principles, which assume that regimes switching are exogenous (unknown) and there is a fixed probability for each regime changes. To understand the model clearly, consider the daily BEXR series at time t and S_t is an unobservable discrete state variable that takes values of 1 (appreciation period- an increase in the value of domestic currency relative to foreign currency) or 2 (depreciation period- a decrease in the value of domestic currency relative to foreign currency). A MS_AR model with two possible states is defined as follows:

$$\text{BEXR}_t = \alpha_{S_t} X_t + e_t, S_t \in \{1, 2\}, t = 1, 2, \dots, n$$

$$e_t \sim N(0, \sigma_{S_t}^2)$$

where X_t includes an intercept (denoted by $\mu_i, i=1,2$) and lags of the dependent variable BEXR_t , $\alpha_{S_t}, S_t \in \{1, 2\}$ are the parameters of lags of dependent variable BEXR_t (details see section 4.1.1) and a random variable e_t with a state dependent variance $\sigma_{S_t}^2$. The changes in states are rules by transition probabilities, which is governed by a first order Markov process as follows:

$$P(S_t = 1 | S_{t-1} = 1) = p_{11}, P(S_t = 1 | S_{t-1} = 2) = p_{12},$$

$$P(S_t = 2 | S_{t-1} = 1) = p_{21}, P(S_t = 2 | S_{t-1} = 2) = p_{22}$$

with $p_{11} + p_{21} = 1$ and $p_{12} + p_{22} = 1$, where p_{ij} ($i=1,2$ and $j=1,2$) are the transition probabilities for switching from one state to other state. The advantage of these transition probabilities is that they allow the series to tell the nature and incidence of significant changes. As S_t is unobserved, the parameter vector (say) $\theta = (\alpha_{S_t}, \sigma_1, \sigma_2, p_{11}, p_{12}, p_{21}, p_{22})$ is estimated by maximum likelihood method using EM algorithm developed by Hamilton [15]. Here fitted BEXR series will be calculated by the probability of $S_t=1$ or 2 based on the observed BEXR series.

3.2 ANFIS Model

Based on the fuzzy logic, Jang [8] introduced this model in computing literature, which is a combination of two intelligence systems: (i) neural network (NN) system and (ii) fuzzy inference system (FIS), where NN learning algorithm is used to determine parameters of FIS. NNs are non-linear statistical data modeling tools, which can capture and model any input-output relationships. FIS is the process of formulating the mapping from a given input to an output using the fuzzy logic. The process of FIS involves: i) membership functions (mfs) (ii) fuzzy logic operators and (iii) if-then-rules, where the above mapping provides a basis from which decisions can be made or patterns can be discerned. The structure of ANFIS has 5 layers: (i) 1 input layer (ii) 3 hidden layers that represents mfs and fuzzy rules and (iii) 1 output layer. The learning algorithm of ANFIS is a hybrid algorithm, which

combines the gradient descent (GD) method and the least square estimation (LSE) for an effective search of parameters. ANFIS uses a two pass of learning algorithm to reduce error: (i) forward pass and (ii) backward pass. The hidden layer is computed by the GD method of the feedback structure and the final output is estimated by the LSE method (details, see [8-9]).

3.3 CH Model

As mentioned before, ARCH and GARCH are the most commonly used CH models to model financial time series data that exhibit time varying volatility clustering. To understand an ARCH model clearly, consider an AR(1) model:

$$\text{BEXR}_t = \text{Constant} + \text{BEXR}_{t-1} + e_t, t = 1, 2, \dots, n$$

Suppose error term $e_t = \varepsilon_t \sigma_t$, where

$$\sigma_t = \beta_0 + \sum_{i=1}^q \beta_i e_{t-i}^2 \text{ with } \beta_0 > 0 \text{ and } \beta_i \geq 0 (i = 1, 2, \dots,$$

$q)$ and $\varepsilon_t \sim N(0,1)$, known as ARCH process of order q .

In 1986, Bollerslev [14] improved the ARCH models by inventing the GARCH model, where the current volatility depends not only on the past errors, but also on the past volatilities. To understand it clearly, consider an AR(1) model:

$$\text{BEXR}_t = \text{Constant} + \text{BEXR}_{t-1} + e_t, t = 1, 2, \dots, n$$

Suppose $e_t = \varepsilon_t \sigma_t$, where

$$\sigma_t = \beta_0 + \sum_{i=1}^q \beta_i e_{t-i}^2 + \sum_{j=1}^p \gamma_j \sigma_{t-j}^2 \text{ with } \beta_0 > 0, \beta_i \geq 0 (i$$

$= 1, 2, \dots, q), \gamma_j \geq 0 (j = 1, 2, \dots, p)$ and $\varepsilon_t \sim N(0,1)$, known as an GARCH process of orders p and q .

We have chosen GARCH as a CH model instead of ARCH because of ability of GARCH to deal with more variations than ARCH.

4. RESULTS AND DISCUSSION

The first 4000 observations (63%) for daily BEXR series are used as the training period and the rest as the testing period (see Fig-1). All computational works were carried out using the programming code of MATLAB (version 7.0).

4.1 Results

4.1.1 MS_AR Model

We begin by calculating the Bayesian Information Criteria (BIC) (other information criterions e.g. AIC, SIC and others can also be used) to find the number of lags to be used in the AR process for BEXR series. We found that the number of lags to be used is 4 for BEXR series. Based on this information, a MS with AR(4) model is considered and all parameters are estimated using the maximum likelihood method, which are reported in Tables 2a-2c. These parameters estimates are used to predict daily BEXR series.

Table 2a: Parameters Estimates for the MS_AR(4) Model for State 1

Parameters estimates	μ_1	σ_1	α_{11}	α_{12}	α_{13}	α_{14}
	0.0004 (0.0001)	0.0063 (0.0001)	-0.2062 (0.0269)	-0.1419 (0.0291)	-0.1462 (0.0296)	-0.1576 (0.0347)

Note: Standard errors are in parenthesis

Table 2b: Parameters Estimates for the MS_AR(4) Model for State 2

Parameters estimates	μ_2	σ_2	α_{21}	α_{22}	α_{23}	α_{24}
	0.0005 (0.008)	0.0118 (0.0003)	0.1143 (0.0274)	-0.0800 (0.0402)	-0.1045 (0.0694)	0.0029 (0.0610)

Note: Standard errors are in parenthesis

Table 2c: Transition Probability Matrix

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.66 & 0.14 \\ 0.34 & 0.86 \end{bmatrix}$$

4.1.2 ANFIS Model

A trial and error approach is used to design the topology of ANFIS. Initially, a number of networks are trained and the error gradient was observed. Training algorithm is used to update the mfs parameters of FIS, is a hybrid rule. As a result, a decreased training error throughout the learning process is obtained. The performance of the network is evaluated by decreasing or increasing the number of inputs and the premise rules. Best performance is obtained by a network consists of: 4 inputs with 2 mfs (type Gaussian-shaped) with each input, 8 if-then fuzzy rules were learned, total parameters (44) = premise parameters (12) + consequent parameters (32),

where premise parameters is calculated by # of inputs × # of mfs × # of parameters of Gaussian distribution and consequent parameters is calculated by # of mfs × # of parameters of Gaussian distribution × # of fuzzy rules. The training data was used with the MATLAB command Genfis1 in order to create a FIS (see Fig-3 for mfs for first, second, third and fourth inputs respectively). Thus, the following ANFIS forecasting model was selected to predict daily BEXR values:

$$P_{5th\ day_BEXR} = f(1st\ day_BEXR, 2nd\ day_BEXR, 3rd\ day_BEXR, 4th\ day_BEXR)$$

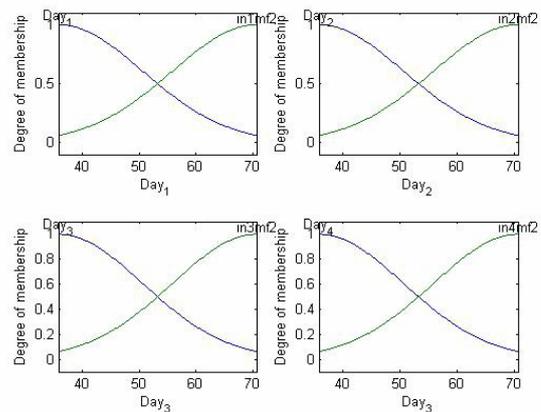


Fig 3: Membership functions for first, second, third and fourth inputs

4.1.3 GARCH Model

Higher order GARCH is seldom used. After having gone through pre and post analyses (using ACF, PACF, ARCH test and likelihood ratio test (estimation results are available upon request)), it was estimated that it would be good to try GARCH(2,1) model. The MATLAB output for GARCH (2,1) model is reported in Table 3. Thus, the GARCH(2,1) model was used to predict BEXR series.

4.2 Discussion of Results

Forecasting performance are evaluated against three widely used statistical metrics, namely, Mean absolute percentage error (MAPE) =

$$\frac{1}{n} \sum_{i=1}^n \left| \frac{\text{Actual BEXR}_i - \text{Predicted BEXR}_i}{\text{Actual BEXR}_i} \right|,$$

Root mean square error (RMSE)=

$$\sqrt{\frac{1}{n} \sum_{i=1}^n (\text{Actual BEXR}_i - \text{Predicted BEXR}_i)^2}$$

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coefficient of determination $R^2 = 1 - (ESS/TSS)$, where ESS is the sum of squares of differences between actual and predicted BEXR rates and TSS is sum of squares of actual BEXR rates. MAPE/RMSE is used to measure the accuracy of prediction through representing the degree of scatter. R^2 is a measure of the accuracy of prediction of the trained network models. Smaller values of MAPE and RMSE metrics indicate higher accuracy in forecasting. Higher R^2 values indicate better prediction. After a model is built using the training data, BEXR rate is forecasted over the test data. To compare forecasted and actual exchange rates, prediction performance is measured in terms of considered statistical metrics MAPE, RMSE and R^2 over the training and testing data. Fig-4a to Fig-6b presents the training and testing performance metrics graphically. In order to see how well our considered models fitted to the actual data, Fig-7 was added, which shows the one- day ahead predicted BEXR values over our considered data periods. From figures 4a, 5a and 6a, it is clear that ANFIS forecasting model is better in terms of all metrics (MAPE, RMSE and R^2), followed by the GARCH forecasting model, then the MS_AR model. In our experiment, this is consistently observed in case of testing

Table 3: The GARCH(2,1) Model Estimation Results

Number of Model Parameters Estimated: 5			
Parameter	Value	Standard Error	T Statistic
Constant	1.5079e-005	1.073e-005	1.4053
β_0	2.3335e-007	2.7331e-009	85.380
β_1	0.16597	0.0036654	45.281
γ_1	0.42427	0.00072366	86.282
γ_2	0.40976	0.0012077	39.298

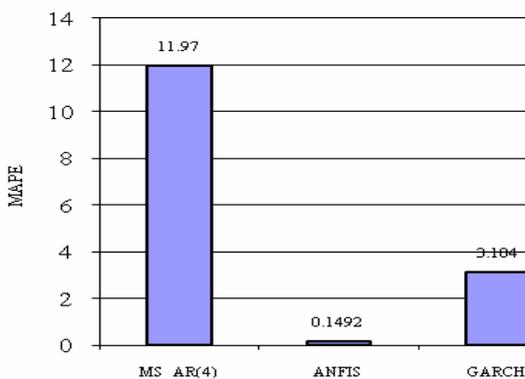


Fig 4a: MAPE for Training Data

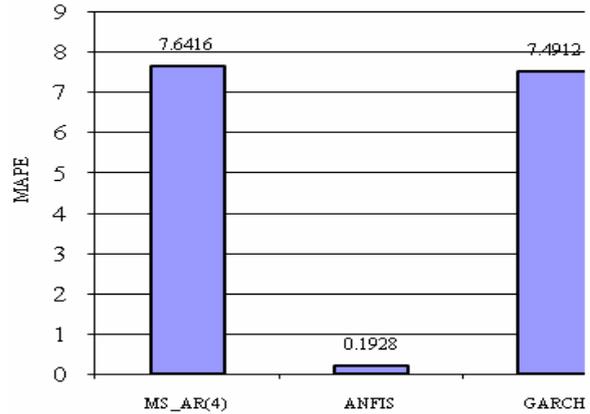


Fig 4b: MAPE for Training Data

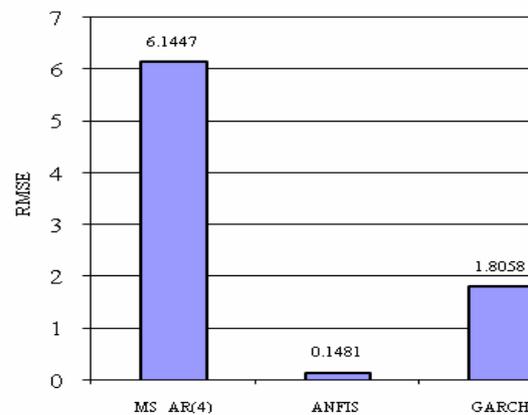


Fig 5a: RMSE for Training Data

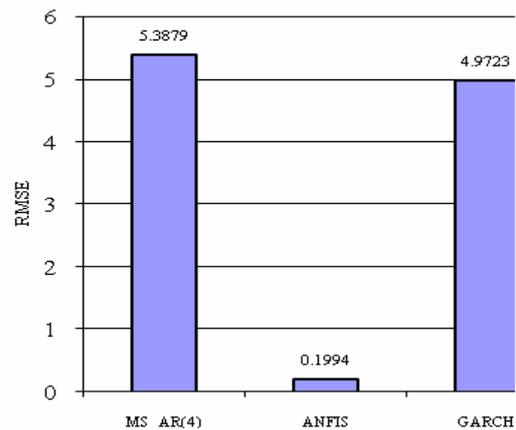


Fig 5b: RMSE for Training Data

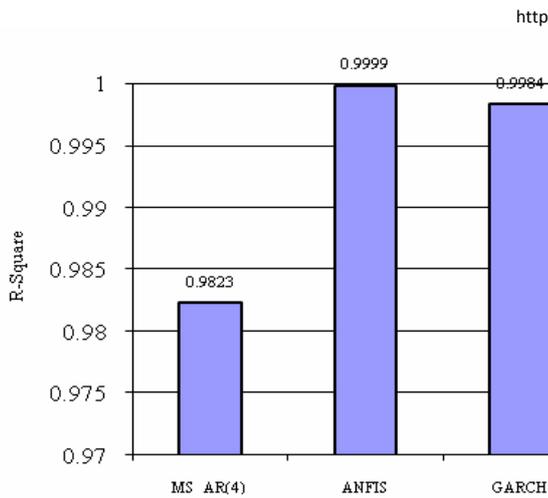


Fig 6a: R-Square Values for Training Data

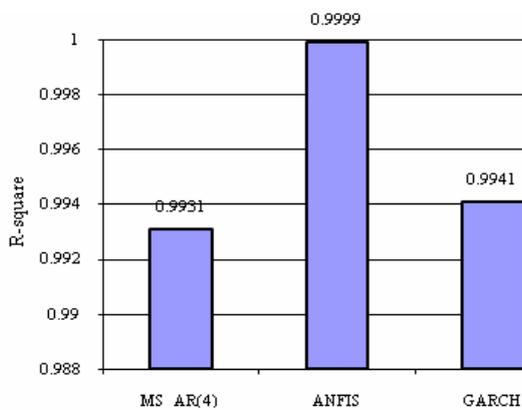


Fig 6b: R-Square Values for Training Data

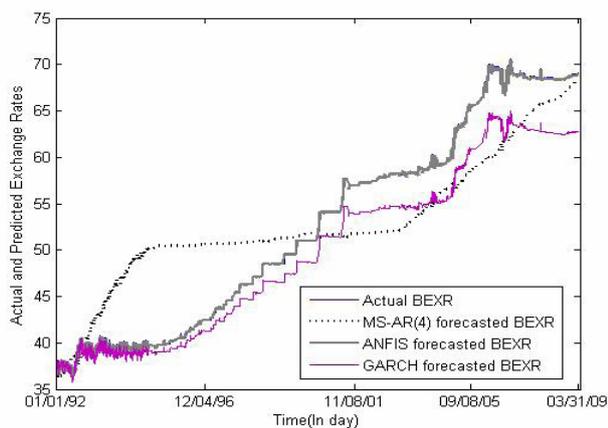


Fig 7: Actual and predicted BEXR rates for 3 forecasting models

data also (see Fig-4b, Fig-5b and Fig-6b). Thus, our findings indicate us the ANFIS forecasting model is more suitable for BEXR series modelling than others. The daily actual and forecasted BEXR series for all considered models for the time period January 1992 to March 2009

are shown in Fig-7. From this figure, one can again easily imagine the superiority of the ANFIS forecasting model over GARCH and MS_AR forecasting models.

5. CONCLUSION

In this paper, we investigated three popular forecasting models (MS_AR model, ANFIS model and GARCH model) to predict BEXR series. By nature, exchange rate is volatile and non-linear. Literature suggests that the above-mentioned popular models pay particular attention to non-linearities, which helps to improve complex data predictions. We used daily BEXR series for the period from January 1992 to March 2009. The forecasting performances of selected models are measured by commonly used measures MAPE, RMSE and R^2 . Our findings suggest that the ANFIS forecasting model can forecast the daily BEXR series closely as compare to other two selected forecasting models, followed by GARCH. We believe our findings will be useful to researchers who are planning to make wise decisions with this complex variable. Our next step is to improve forecasting results using other forecasting models such as fuzzy extension of genetic algorithm model, MS-GARCH model etc.

ACKNOWLEDGEMENT

We greatly acknowledge comments and very useful suggestions from anonymous referees. We are also very grateful to the Editor (publications), Journal of Science and Technology for his very valuable cooperation. An earlier version of this paper was presented in the International Conference on Computer and Information Technology (ICCIT-2009) jointly organized by School of Engineering and Computer Science, Independent University, Bangladesh and Department of Computer Science and Engineering, Military Institute of Science and Engineering, Bangladesh held on 21-23 December.

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