VOL. 3, NO. 2, February 2013 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

An Analysis of Centrally Symmetric Gravitational Field

[1] Mohammad Ali [2] Abul Kalam Azad
[1] Dept. of Mathematics, Chittagong University of Engineering, and Technology (CUET), Chittagong-4349, Bangladesh
[2] Dept. of Mathematics, Chittagong University, Chittagong-4331, Bangladesh

ABSTRACT

In this paper we have considered a centrally symmetric metric through which we have tried to solve some quantities λ, μ and ν which are functions of R and τ , where R and τ are the radial and time coordinates and also discuss the physical significance of the result. It is noted that the analysis give us a number of cosmological solutions, i.e. the centrally symmetric gravitational field with constant density is automatically takes the shape of the steady state like universe.

Keywords: *Tensor, Ricci Tensor, General Relativity, Einstein field equations and cosmology*

1. INTRODUCTION

We consider a centrally symmetric gravitational field [5]. Such a field can be produced by any centrally symmetric distribution of matter. The centrally symmetry of the field means that the space time metric, that is, the expression for the interval ds, must be the same for all points located at the same distance from the center[6]. In Euclidean space this distance is equal to the radius vector; in a non- Euclidean space, such as we have in the presence of a gravitational field, there is no quantity which has all the properties of the Euclidean radius vector (for example to be equal both to the distance from the centre and to the length of the circumference divided by 2π). Therefore the choice of a 'radius vector' is now arbitrary. In this paper, we have considered a centrally symmetric metric trying to solve some quantities λ, μ and ν which are functions of R and τ , where R and τ are the radial and time coordinates and also discuss the physical significance of the result. It is noted that the Eq.21 and Eq.20 gives us a number of cosmological solutions. The centrally symmetric gravitational field with constant density is automatically takes the shape of the steady state like Universe. This centrally symmetric gravitational field also satisfies the rigorous theorem known as the Birkhoff's theorem [3], which state that "any spherically symmetric vacuum solution of Einstein equation is necessarily Schwarzschild solution that is, static". This theorem implies that if a spherically symmetric source like a star undergoes pulsations or changes its shape, while maintaining the spherically symmetry, it cannot radiate any disturbances in the exterior such as the Schwarzschild exterior solution can be used to describe the outside metric for several situations such as spherically symmetric star which is either static or which undergoes radial spherically symmetric gravitational collapse [4].

2. METHODOLOGY

Let us consider "spherical" space coordinates r, θ, φ then the most of the general centrally symmetric expression for ds^2 is [1]

$$ds^2 = h(r,t)dr^2 + k(r,t)(\sin^2\theta d\varphi^2 + d\theta^2) + l(r,t)dt^2 + a(r,t)drdt$$

where a, h, k, l are certain functions of the radius vector 'r' and the time't'. But because of the arbitrariness in the choice of a reference system in the general theory of relativity, we can still subject the coordinates to any transformation which does not destroy the central symmetry of ds^2; this means that we can transform the coordinates r and t according to the formula $r = f_1(r', t')$ and $t = f_2(r', t')$, where f_1, f_2 are functions of the new coordinates r' and t'. We make use of the two possible transformation of the co ordinate's r, t in the element of interval in order to, first, make the coefficient $a(r,t)$ of $drdt$ vanish and second, to make the radial velocity of the matter vanish at each point (because of the central symmetry the other components are not present).After this is done, r and t can still be subjected to an arbitrary transformation of the form $r = r(r')$ and $t = t(t')$. We denote the radial coordinate and time selected in this way by R and τ and the coefficients h, k, l by $-e^{-\lambda}, -e^{\mu}, e^{\nu}$ respectively [Where $\lambda, \mu,$ and ν are functions of R and τ]. We then have the form as follows

$$ds^2 = c^2 e^{\nu} d\tau^2 - e^{\lambda} dR^2 - e^{\mu}(d\theta^2 + \sin^2\theta d\varphi^2) \quad (1)$$

The energy momentum tensor for perfect fluid is as follows: $T_{ij} = (P+\varepsilon)u_i u_j - Pg_{ij}$, Where P is the pressure and ε is the energy density.

In the commoving reference system the components of the energy-momentum tensor are:

$$T_0^0 = \varepsilon, T_1^1 = T_2^2 = T_3^3 = -p .$$

From Eq.1, we get the non-zero metric tensor are

$$\begin{aligned} g_{00} &= c^2 e^{\nu} & g^{00} &= c^{-2} e^{-\nu} \\ g_{11} &= -e^{\lambda} & g^{11} &= -e^{-\lambda} \\ g_{22} &= -e^{\mu} & g^{22} &= -e^{-\mu} \\ g_{33} &= -e^{\mu}\sin^2\theta & g^{33} &= -e^{-\mu}\sin^{-2}\theta \end{aligned} \quad (2)$$

Now we will find out the non -zero values of 'Γ' are as follows:

We have $\Gamma^{i}_{jk} = \frac{1}{2} g^{il}\left(g_{il,k} + g_{kl,j} - g_{jk,l}\right)$ (3)

Put i = j = k = 0 in Eq.3, we obtain

$\Gamma^{0}_{00} = \frac{\dot{\nu} c}{2}$ [By using Eq.2]

Similarly by applying Eq.2 and Eq.3, we get

$$\Gamma^{0}_{10} = \Gamma^{0}_{01} = \frac{\nu'}{2}, \Gamma^{1}_{01} = \Gamma^{1}_{10} = \frac{\dot{\lambda} c}{2}$$
$$\Gamma^{0}_{11} = \frac{c^{-1} e^{\lambda-\nu}\dot{\lambda}}{2}, \Gamma^{0}_{22} = \frac{c^{-1} e^{\mu-\nu}\dot{\mu}}{2},$$
$$\Gamma^{0}_{33} = \frac{c^{-1} e^{\mu-\nu}\dot{\mu}}{2}\sin^2\theta \quad (4)$$
$$\Gamma^{1}_{11} = \frac{\lambda'}{2}, \Gamma^{1}_{22} = -\frac{e^{\mu-\lambda}\mu'}{2},$$
$$\Gamma^{1}_{33} = -\frac{e^{\mu-\lambda}\mu'}{2}\sin^2\theta$$
$$\Gamma^{2}_{02} = \Gamma^{2}_{20} = \frac{\dot{\mu} c}{2}, \Gamma^{2}_{12} = \Gamma^{2}_{21} = \frac{\mu'}{2}$$
$$\Gamma^{3}_{03} = \Gamma^{3}_{30} = \frac{\dot{\mu} c}{2}, \Gamma^{3}_{31} = \Gamma^{3}_{13} = \frac{\mu'}{2} \quad (5)$$
$$\Gamma^{3}_{23} = \Gamma^{3}_{32} = \cot\theta, \Gamma^{2}_{33} = -\sin\theta\cos\theta,$$
$$\Gamma^{1}_{00} = \frac{c^2 e^{\nu-\lambda}\nu'}{2}$$

We have the Ricci tensor

$$R_{jk} = -\Gamma^{i}_{jk,i} + \Gamma^{i}_{ji,k} - \Gamma^{m}_{jk}\Gamma^{i}_{mi} + \Gamma^{m}_{ji}\Gamma^{i}_{mk} \quad (6)$$

We have the Ricci Scalar $R = g^{ii} R_{ii}$ [i=0,1,2,3]

Therefore, from Eq.6 [after putting k =j= 0], we get [By using Eq.5 and .Eq. 4]

$$R_{00} = \frac{c^2\dot{\nu}^2}{4} + \frac{c^2\dot{\lambda}^2}{4} + \frac{c^2\dot{\mu}^2}{2} - \frac{c^2 e^{\nu-\lambda}\nu'\mu'}{2} + \frac{c^2 e^{\nu-\lambda}\nu'\lambda'}{4} - \frac{c^2 e^{\nu-\lambda}\nu'^2}{4} - \frac{c^2 e^{\nu-\lambda}\nu''}{2} + \frac{c^2\ddot{\lambda}}{2} - \frac{c^2\dot{\nu}^2}{4} - \frac{c^2\dot{\nu}\dot{\lambda}}{4} - \frac{c^2\dot{\nu}\dot{\mu}}{2} + c^2\ddot{\mu} \quad (7)$$

Similarly, we get

$$R_{11} = -\frac{e^{\lambda-\nu}\ddot{\lambda}}{2} - \frac{e^{\lambda-\nu}\dot{\lambda}^2}{4} - \frac{e^{\lambda-\nu}\dot{\lambda}\dot{\mu}}{2} + \frac{e^{\lambda-\nu}\dot{\lambda}\dot{\nu}}{4} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{\lambda'\mu'}{2} + \frac{\mu'^2}{2} + \frac{\nu''}{2} + \mu'' \quad (8)$$

$$R_{22} = -\frac{e^{\mu-\nu}\ddot{\mu}}{2} - \frac{e^{\mu-\nu}\dot{\mu}^2}{2} + \frac{e^{\mu-\lambda}\mu''}{2} - \frac{e^{\mu-\lambda}\mu'^2}{2} + \frac{e^{\mu-\nu}\dot{\mu}\dot{\nu}}{4} - \frac{e^{\mu-\nu}\dot{\mu}\dot{\lambda}}{4} + \frac{e^{\mu-\lambda}\mu'\lambda'}{4} - \frac{e^{\mu-\lambda}\mu'\nu'}{4} - 1 \quad (9)$$

$$R_{33} = -\frac{e^{\mu-\nu}\ddot{\mu}\sin^2\theta}{2} + \frac{e^{\mu-\lambda}\mu''\sin^2\theta}{2} - \frac{e^{\mu-\nu}\dot{\mu}\dot{\nu}\sin^2\theta}{4} - \frac{e^{\mu-\nu}\dot{\mu}\dot{\lambda}\sin^2\theta}{4} - \frac{e^{\mu-\nu}\dot{\mu}^2\sin^2\theta}{2} + \frac{e^{\mu-\lambda}\mu'\nu'\sin^2\theta}{4} - \frac{e^{\mu-\lambda}\mu'\lambda'\sin^2\theta}{4} + \frac{e^{\mu-\lambda}\mu'^2\sin^2\theta}{2} - \sin^2\theta \quad (10)$$

In Eq.6 put j = 1, k = 0, we get

$$R_{10} = c\dot{\mu}' - \frac{c\dot{\mu}\nu'}{2} - \frac{c\dot{\lambda}\mu'}{2} + \frac{c\dot{\mu}\mu'}{2}$$

$$R = -e^{-\lambda}\nu'' + e^{-\nu}\ddot{\lambda} + 2e^{-\nu}\ddot{\mu} + \frac{e^{-\lambda}\lambda'\nu'}{2} - \frac{e^{-\nu}\dot{\lambda}\dot{\nu}}{2} - \frac{e^{-\lambda}\nu'^2}{2} + \frac{e^{-\nu}\dot{\lambda}^2}{2} + \frac{3e^{-\nu}\dot{\mu}^2}{2} - \frac{3e^{-\lambda}\mu'^2}{2} - e^{-\nu}\dot{\mu}\dot{\nu} - 2e^{-\lambda}\mu'' + e^{-\nu}\dot{\mu}\dot{\lambda} + e^{-\lambda}\mu'\lambda' - e^{-\lambda}\mu'\nu' + 2e^{-\mu} \quad (11)$$

We have, the Einstein field equation

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{-8\pi k}{c^4} T_{ij} \quad (12)$$

In Eq.12, put i = j = 0, we get

$$R_{00} - \frac{1}{2} g_{00} R = \frac{-8\pi k}{c^4} T_{00}$$

$$\Rightarrow e^{-\nu}\left[\frac{\dot{\mu}^2}{4} + \frac{1}{2}\dot{\lambda}\dot{\mu} + e^{-\mu}\right] - e^{-\lambda}\left[\mu'' + \frac{3\mu'^2}{4} - \frac{1}{2}\lambda'\mu'\right] = \frac{8\pi k\varepsilon}{c^4}$$

In Eq.12, put i = j = 1, we get

$$R_{11}-\frac{1}{2}g_{11}R=\frac{-8\pi k}{c^4}T_{11}$$

$$\Rightarrow e^{-\lambda}\left[\frac{\mu''}{2}+\frac{\nu''}{2}+\frac{\mu'^2}{4}+\frac{\nu'^2}{4}-\frac{\lambda'\mu'}{4}+\frac{\mu'\nu'}{4}-\frac{\lambda'\nu'}{4}\right]$$
$$+e^{-\nu}\left[-\frac{\ddot\mu}{2}-\frac{\ddot\lambda}{2}-\frac{\dot\mu^2}{4}-\frac{\dot\lambda^2}{4}-\frac{\dot\mu\dot\lambda}{4}+\frac{\dot\mu\dot\nu}{4}+\frac{\dot\lambda\dot\nu}{4}\right]$$
$$=\frac{8\pi k}{c^4}p$$

In Eq.12, put i = j = 2, we get

$$R_{10}-\frac{1}{2}g_{10}R=\frac{-8\pi k}{c^4}T_{10}$$
$$\Rightarrow 2\dot\mu-\dot\mu\nu'-\dot\lambda\mu'+\dot\mu\mu'=0 \qquad \text{as } e^{-\lambda}\neq 0$$

Therefore the calculation gives the following field equations:

$$2\dot\mu-\dot\mu\nu'-\dot\lambda\mu'+\dot\mu\mu'=0,\ \text{as } e^{-\lambda}\neq 0 \qquad (13)$$

$$e^{-\nu}\left[\frac{\dot\mu^2}{4}+\frac{1}{2}\dot\lambda\dot\mu+e^{-\mu}\right]-e^{-\lambda}\left[\mu''+\frac{3\mu'^2}{4}-\frac{1}{2}\lambda'\mu'\right]$$
$$=\frac{8\pi k\varepsilon}{c^4} \qquad (14)$$

$$R_{22}-\frac{1}{2}g_{22}R=\frac{-8\pi k}{c^4}T_{22}$$
$$\Rightarrow e^{-\nu}\left[\frac{\ddot\mu}{2}+\frac{\dot\mu^2}{2}-\frac{\dot\mu\dot\nu}{4}+\frac{\dot\mu\dot\lambda}{4}\right]$$
$$-e^{-\lambda}\left[\frac{\mu''}{2}-\frac{\mu'^2}{2}+\frac{\mu'\lambda'}{4}-\frac{\mu'\nu'}{4}\right]+e^{-\mu}-\frac{1}{2}=\frac{8\pi k}{c^4}p$$

$$e^{-\lambda}\left[\frac{\mu'^2}{4}+\frac{1}{2}\mu'\nu'\right]-e^{-\nu}\left[\ddot\mu+\frac{3\dot\mu^2}{4}-\frac{1}{2}\dot\mu\dot\nu\right]-e^{-\mu}$$
$$=\frac{-8\pi k}{c^4}p \qquad (15)$$

$$e^{-\lambda}\left[\frac{\mu''}{2}+\frac{\nu''}{2}+\frac{\mu'^2}{4}+\frac{\nu'^2}{4}-\frac{\lambda'\mu'}{4}+\frac{\mu'\nu'}{4}-\frac{\lambda'\nu'}{4}\right]$$
$$+e^{-\nu}\left[-\frac{\ddot\mu}{2}-\frac{\ddot\lambda}{2}-\frac{\dot\mu^2}{4}-\frac{\dot\lambda^2}{4}-\frac{\dot\mu\dot\lambda}{4}+\frac{\dot\mu\dot\nu}{4}+\frac{\dot\lambda\dot\nu}{4}\right]$$
$$=\frac{8\pi k}{c^4}p \qquad (16)$$

Where the prime denotes differentiation with respect to R and the dot with respect to τ.

Now we have the Einstein's field equation for the metric g_{ij} in the presence of matter:
In Eq.12, put i = 1, j = 0, we get

$$R_{ij}-\frac{1}{2}g_{ij}R=\frac{-8\pi k}{c^4}T_{ij}$$
$$\Rightarrow R^{ij}-\frac{1}{2}g^{ij}R=\frac{-8\pi k}{c^4}T^{ij}$$
$$\Rightarrow G^{ij}=\frac{-8\pi k}{c^4}T^{ij}$$
$$\Rightarrow G^i_j=\frac{-8\pi k}{c^4}T^i_j$$

Differentiate this with respect to i, we get [12]

$$G^i_{j,i}=\frac{-8\pi k}{c^4}T^i_{j,i}\Rightarrow 0=\frac{-8\pi k}{c^4}T^i_{j,i}$$

For conservation we can write the following relation:

$$\Rightarrow T^i_{j,i}=0\Rightarrow T^k_{i,k}=0$$
$$\Rightarrow \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}(\sqrt{-g}T^k_i)-\frac{1}{2}\frac{\partial g_{lk}}{\partial x^i}T^{kl}=0 \qquad (17)$$

Therefore first part of Eq.17 [Take k=0 only, i=0, 1, 2, 3] is $\frac{c}{2}(\dot\lambda+2\dot\mu+\dot\nu)\varepsilon+2\dot\varepsilon$ and 2nd part of Eq.17 is $-\frac{c}{2}\left[\dot\nu\varepsilon-\dot\lambda p-2\dot\mu p\right]$.

Therefore from these we can write

$$\Rightarrow \dot\lambda+2\dot\mu=\frac{-2\dot\varepsilon}{p+\varepsilon} \qquad (18)$$

From the Eq.18 if we consider the density is constant everywhere i.e. ε = constant so that $\dot\varepsilon=0$, then the Eq.18 becomes

$$\dot\lambda+2\dot\mu=0\Rightarrow \lambda+2\mu=f(t) \qquad (19)$$

But in the beginning of the universe at time t = 0, then the Eq.19 becomes $\lambda+2\mu=0$.Again, we have the first part of

Eq.17 is [Take k = 1only] $-\frac{1}{2}\left[(\lambda'+2\mu'+\nu')p+2p'\right]$

And the 2nd part of the Eq.17 can be written as $-\frac{1}{2}\left[\nu'\varepsilon-\lambda'p-2\mu'p\right]$.Therefore from Eq.17, for k = 1, we get the following result:

$$\therefore \nu' = \frac{2p'}{p+\varepsilon} \qquad (20)$$

If p is known as a function of ε, Eq.18 can be integrated in the form

$$\lambda+2\mu=-2\int\frac{d\varepsilon}{p+\varepsilon}+f_1(R) \qquad (21)$$

Where the functions $f_1(R)$ can be chosen arbitrary in view of the possibility mentioned above of making arbitrary transformations of the form R = R(R'). To solve the equation Eq.21 at the initial moment t = 0, then $f_1(R) = 0$.

3. RESULTS AND DISCUSSION

We need one more equation, which is provided by the equation of state, P = P (ε), in which the pressure is given as a function of the mass-energy density. This equation of state is as follows: $P=(\gamma-1)\varepsilon$,(22) where $1\le\gamma\le 2$

From Eq.21 by applying Eq.22 we get the following table:

Table 1: Various values of ε, and P for various values of γ

Case	γ	P	ε
I	1	0	1
II	4/3	$\varepsilon/3$	1
III	3/2	$\varepsilon/2$	1
IV	2	ε	1

The Table:1 shows that for various values of γ ($1\le\gamma\le 2$) we getε = constant i.e. the centrally symmetric gravitational field with constant density is automatically takes the shape of the steady state like universe. Similar case will be happened in the Eq.20.

4. CONCLUSIONS

We choose the interval ds^2 in the form of Eq.1, there still remained the possibility of an arbitrary transformation of the time of the formt = f(t) like Eq.19. Such a transformation is equivalent to adding to ν an arbitrary function of the time, and with its aid we can always make f(t) in Eq.19 vanish if we consider t = 0.

And so without any loss in generality, we can get $\lambda+2\mu=0$. Note that the centrally symmetric gravitational field with constant density in each solutions [Table:1] automatically takes the shape of the steady state Universe. Since the Universe is expanding, the principle demands that new matter must be created to maintain a constant density of the Universe.

The most remarkable feature of the theory is that the new matter (believed to be hydrogen atom) is supposed to be created out of nothing in a creation field called the C field. Matter, therefore, requires to be continuously created in the Universe according to this theory. [2]

REFERENCES

[1] Ali, M. Studies on Global Aspects in Gravitation and Gravitational Waves. M.Sc. Thesis Department of Mathematics, University of Chittagong. 2002.

[2] Basu, B. An introduction to Astrophysics. Prentice Hall of India Private limited New Delhi-110001. 2006.

[3] Birkhoff, G.D. Relativity and Modern Physics. Harvard University Press. Cambridge M.A. 1923.

[4] Joshi, P.S. Global Aspects in Gravitation and Cosmology. Oxford University Press lnc. New York. 1993.

[5] LANDAU, L.D. and LIFTSHITZ, E.M. “The classical theory of Fields” Peragamon press, [unsolved problem P. No. 304]. 1975.

[6] Lyer, B.R.;Mukunda, N. and Vishvershwasa , C.V. Gravitation, Gauge theories and the Eashy Universe. Kluwer Academic Publishers, the Netherlands. P.P. 31-42. 1988.

[7] N.P.Konopleva, Relativistic physics as geometry, Proc. of XVISHEPP, JINR, Dubna, 2001, p.77-86

[8] N.P.Konopleva, General relativity and gauge gravity theories of higher order, Proc. Of the Int. Sem. on Math. Cosmology, Potsdam, Germany, 1999.

[10] P. Fiziev, “The Gravitational Field of Massive Non-charged Point Source in General Relativity,” gr_ qc/0412131V1 (2004).

[11] S. S. Gershtein, A. A. Logunov, and M. A. Mestvirish_ vili, “General Relativity and the Schwarzschild Singularity,” Fiz.Elem.Chastits At.Yadra 39, 81 (2008) [Phys. Part.Nucl.39, 39 (2008)].

ARPN Journal of Science and Technology

http://www.ejournalofscience.org

[12] STEVEN WEINBERG, "Gravitation and Cosmology"; Principles and applications of the general theory of Relativity. John Wiley and Sons, 1912, P-333

[13] V. V. Kiselev, A. A. Logunov, and M. A. Mestvirishvili, "Black Holes: Theoretical Prediction or Fantasy?," Fiz. Elem. Chastits At.Yadra 37, 597 (2006) [Phys. Part.Nucl.37, 317 (2006)].