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Ordinary Matter as Black Holes

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ABSTRACT

According to the theories of gravity, we know that if an ordinary matter is compressed within its Schwarzschild radius, it will become black hole. According to the theory of relativity, there is possibility that a moving mass may collapse to black hole at the certain velocity v which we can say as the critical velocity. We can calculate de Broglie wavelength of the moving body for different masses. We can also find new modified de-Sitter solution of Einstein's field equations for this case.

Keywords: *Special relativity; General relativity; Black hole; De Broglie wavelength; Compton wavelength: Planck mass*

1. INTRODUCTION

There is possibility that the size of the moving body may reduce to the Schwarzschild radius and mass of the moving body may increase to such extent that it becomes black hole. The velocity at which the moving mass behaves as black hole can be said as the critical velocity. The Compton wavelength of the moving mass is constant at any velocity but there is variation in the de Broglie wavelength of the moving mass at different velocities. On the basis of de Broglie wavelength we can do analysis whether the moving mass is outside the black hole, is the black hole or inside the black. The value of $\mathrm{v}^2/\mathrm{c}^2$ so obtained can be used in the Schwarzschild solution of Einstein's field equations for centrally symmetric metric [1]. For $\mathrm{v}=0$, the Schwarzschild solution will be for static body but for $\mathrm{v}\neq 0$, the Schwarzschild solution will be for dynamic body. The value of v^2 so obtained can be used in de-Sitter space-time to obtain new results.

This paper is organized as follows: in Section 2, using theory of relativity we consider there is possibility that the size of the moving body may reduce to the Schwarzschild radius and mass of the moving body may increase to such extent that it becomes black hole. Then we have calculated de Broglie wavelength in terms of Compton wavelength for different cases. Also we have done the analysis of de Broglie wavelength of moving mass when its size is greater than its Schwarzschild radius, equal to Schwarzschild radius and smaller than Schwarzschild radius. Then the modified de-Sitter solution of Einstein's field equations is obtained. Then we present our conclusions in Section 3.

2. ORDINARY MATTER AS BLACK HOLES

According to special theory of relativity, there is length contraction and increment in mass when a massive body moves with velocity v. The relationship between moving length and rest length and moving mass and rest mass can be written as [8]:

$$L = L_0\sqrt{1-\frac{\mathrm{v}^2}{c^2}}\,, \tag{1}$$

$$M = \frac{M_0}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} \tag{2}$$

where L and L_0 are moving length and rest length of the body and M and M_0 are moving mass and rest mass of the body respectively.

There is the possibility that the size of the moving body may reduce to the Schwarzschild radius and mass of the moving body may increase to such extent that it becomes black hole which can be written as:

$$L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{2G_N M_0}{c^2\sqrt{1-\frac{v^2}{c^2}}}\,. \tag{3}$$

Solving equation (3), we can calculate the velocity at which the moving body becomes black hole which we call as the critical velocity.

$$L_0\left(1-\frac{\mathrm{v}^2}{c^2}\right) = 1-\frac{2G_N M_0}{c^2}\;;\;\frac{\mathrm{v}^2}{c^2} = 1-\frac{2G_N M_0}{c^2 L_0}\;;$$

$$\mathrm{v} = c\sqrt{1-\frac{2G_N M_0}{c^2 L_0}} \tag{4}$$

We can calculate the de Broglie wavelength of the moving mass using equation (4) as given below:

$$\lambda_d = \frac{h}{M_0 v} = \frac{h}{M_0 c\sqrt{1-\frac{2GM_0}{c^2 L_0}}} = \frac{\lambda_C}{\sqrt{1-\frac{2GM_0}{c^2 L_0}}} = \frac{\lambda_C}{\sqrt{1-\frac{R_S}{L_0}}} \quad (5)$$

In equation (5), λ_d is the de Broglie wavelength of the moving body and $\lambda c = \frac{h}{M_0 c}$ is the Compton wavelength of the moving mass. $R_S = \frac{2GM_0}{c^2}$ is the Schwarzschild radius of the mass. According to the well known condition for $M_0 \le M_{Pl}$, $\lambda c \ge l_{Pl}$. For $M_0 > M_{Pl}$, $\lambda c < l_{Pl}$.

CASE I:

$$1-\frac{R_S}{L_0} > 0 \text{ i.e. } L_0 > R_S.$$

This is the case in which the size of the moving mass is greater than its Schwarzschild radius. In this case

if $0 < \sqrt{1-\frac{R_S}{L_0}} < 1$, $\lambda_d > \lambda_C$ and if $\sqrt{1-\frac{R_S}{L_0}} > 1$,

then $\lambda_d < \lambda_C$. For $M_0 < M_{Pl}$, $\lambda_d > l_{Pl}$.

CASE II:

$$1-\frac{R_S}{L_0} = 0 \text{ i.e. } L_0 = R_S, \ \lambda_d = \infty.$$

This is the case in which the size of the moving mass is equal to its Schwarzschild radius. This means it is a singular point in the space-time graph i.e it is a black hole.

CASE III:

$$1-\frac{R_S}{L_0} < 0 \text{ i.e. } L_0 < R_S.$$

In this case λ_d is imaginary. This is the case in which we can consider the motion of moving mass inside the black hole which has imaginary de Broglie wavelength but real Compton wavelength. Inside the black hole magnitude of the imaginary de Broglie wavelength depends upon the density of the black hole. For more dense black hole, its magnitude will be less but for less dense black hole, its magnitude should be greater.

The expression $\frac{\text{v}^2}{c^2} = 1-\frac{2G_N M_0}{c^2 L_0}$ is used in the Schwarzschild solution of Einstein's field equations for centrally symmetric metric which is given by [1];

$$ds^2 = c^2\left[1-\frac{2G_N M_0}{rc^2}\right](dt)^2 - \left[1-\frac{2G_N M_0}{rc^2}\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{………. (6)}$$

From equation (6), it is clear that the value of $\frac{\text{v}^2}{c^2}$ and $\frac{\text{c}^2}{\text{v}^2}$ have been used as coefficients of two terms on the right hand side.

The de-Sitter solution of Einstein's modified field equations can be written as

$$ds^2 = c^2 dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right) \quad (7)$$

Differentiating equation (7) with respect to t, we get

$$\left(\frac{ds}{dt}\right)^2 = c^2 - e^{2Ht}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] \quad \text{………………………….(8)}$$

Putting $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = \text{v}^2$ in equation (8), we will get new equation as:

$$\left(\frac{ds}{dt}\right)^2 = c^2 - e^{2Ht}\ \text{v}^2 \quad (9)$$

Putting, $\text{v}^2 = c^2\left(1-\frac{2G_N M_0}{rc^2}\right)$ in equation (9), we get

$$\left(\frac{ds}{dt}\right)^2 = c^2 - e^{2Ht} c^2\left(1-\frac{2G_N M_0}{rc^2}\right) = c^2\left[1-e^{2Ht}\left(1-\frac{2G_N M_0}{rc^2}\right)\right] \quad (10)$$

Solving equation (10), we get

$$ds^2 = c^2 dt^2 \left[1 - e^{2Ht}\left(1 - \frac{2G_N M_0}{rc^2} \right) \right] \qquad (11)$$

or,

$$ds = cdt\sqrt{1 - e^{2Ht}\left(1 - \frac{2G_N M_0}{rc^2} \right)} \qquad (12)$$

Equation (11) or (12) represents the new form of the metric.

3. CONCLUSION

There is possibility that the size of the moving body may reduce to the Schwarzschild radius and mass of the moving body may increase to such extent that it becomes black hole. Using theory of relativity we show that ordinary matter behaves as black hole at a certain velocity v due to gravity which we can say as the critical velocity. We can calculate the de Broglie wavelength of the moving body for different masses i.e. for mass less than Planck mass, mass greater than Planck mass and mass equal to the Planck mass. Finally, we get new different modified de-Sitter solution of Einstein's field equations for this case.

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