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A Note on the Variance Estimation of Ratio Estimator

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ABSTRACT
The present paper is concerned with the estimation of variance of the ratio estimator under simple random sampling. Following techniques adopted earlier for the estimation of finite population variance under classical as well as predictive approach, we formulate some new estimators for the approximate variance of the classical ratio estimator. It is assumed that the population mean and variance of the auxiliary variable are known prior to sampling. A simulation study has been undertaken for evaluating relative performance of the suggested variance estimators in respect of efficiency, coverage rate based on 95% confidence interval and stability.

Keywords: *Auxiliary variable, confidence interval, efficiency, prediction approach, ratio estimator, stability.*

1. INTRODUCTION

Consider a finite population U of N units. Let y_i and x_i be the values of the study variable y and an auxiliary variable x on the ith unit $(i = 1, 2, \dots, N)$. Define $\bar{Y} = \sum_{i=1}^{N} y_i/N$ and $\bar{X} = \sum_{i=1}^{N} x_i/N$ as the population means of y and x, $S_y^2 = \sum_{i=1}^{N}(y_i - \bar{Y})^2/(N-1)$ and $S_x^2 = \sum_{i=1}^{N}(x_i - \bar{X})^2/(N-1)$ as the population variances of y and x, and $S_{yx} = \sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})/(N-1)$ as the population covariance between y and x. Assume that a sample s of n units is drawn from the population according to simple random sampling without replacement (SRSWOR) for estimating the unknown mean $\bar{Y}$. Let $\bar{y} = \sum_{i \epsilon s} y_i/n$ and $\bar{x} = \sum_{i \epsilon s} x_i/n$ be the sample means, $s_y^2 = \sum_{i \epsilon s}(y_i - \bar{y})^2/(n-1)$ and $s_x^2 = \sum_{i \epsilon s}(x_i - \bar{x})^2/(n-1)$ the sample variances, and $s_{yx} = \sum_{i \epsilon s}(y_i - \bar{y})(x_i - \bar{x})/(n-1)$ be the sample covariance.

Survey sampling literature gives considerable attention towards the estimation of population mean $\bar{Y}$ using information on the auxiliary variable x. But, one of the most widely used estimators in this context that utilizes known value of $\bar{X}$ is the classical ratio estimator, defined by $t_R = \bar{y}\bar{X}/\bar{x}$. In the past more attention has been given to this estimator because of its computational ease and applicability for general sampling designs. This estimator also performs well when the correlation coefficient between y and x has a high positive value. There is no closed form for the mean square or variance of t_R. Both can be approximated by the approximate variance [cf., Cochran [1], p.155]

$$V(t_R) = \frac{N-n}{Nn}\left(S_y^2 - 2RS_{yx} + R^2 S_x^2\right), \qquad (1.1)$$

where $R = \bar{Y}/\bar{X}$.

Because of its popularity and simplicity, the ratio estimator has been studied from a variety of angles. Variance estimation is one of these aspects. An estimate of the variance of t_R calculated from the survey data is needed to provide a measure of the error in estimation of the mean $\bar{Y}$. A variance estimate is often used to construct a confidence interval for the unknown mean. The most standard estimator of $V(t_R)$ is its sample analogue

$$\hat{\mathcal{V}}_0 = \frac{N-n}{Nn}\left(s_y^2 - 2rs_{yx} + r^2 s_x^2\right), \qquad (1.2)$$

where $r = \bar{y}/\bar{x}$. However, following Scott and Wu [2] and Wu [3] we may consider two alternative estimators defined by

$$\hat{\mathcal{V}}_1 = \left(\frac{\bar{X}}{\bar{x}}\right)\hat{\mathcal{V}}_0 \text{ and } \hat{\mathcal{V}}_2 = \left(\frac{\bar{X}}{\bar{x}}\right)^2\hat{\mathcal{V}}_0.$$

An extensive comparative study of the variance estimators $\hat{\mathcal{V}}_0$, $\hat{\mathcal{V}}_1$ and $\hat{\mathcal{V}}_2$ carried out by Wu and Deng [4] shows that $\hat{\mathcal{V}}_2$ is better than $\hat{\mathcal{V}}_0$ and $\hat{\mathcal{V}}_1$ on several desirable aspects.

In this paper, our principal objective is an estimation of $V(t_R)$ with concentration on the estimation of S_y^2 motivated by the assumption that both $\bar{X}$ and S_x^2 are known accurately.

2. PROPOSED ESTIMATION METHODOLOGY

Let us now express $V(t_R)$ in the following alternative form:

$$V(t_R) = \frac{N-n}{Nn}\left[S_y^2 - 2RS_x^2\left(\beta - \frac{1}{2}R\right)\right], \qquad (2.1)$$

where $\beta = S_{yx}/S_x^2$ is the population regression coefficient of y on x. This equation clearly indicates that t_R is more efficient than the simple expansion estimator $\bar{y}$ when $\beta \geq \frac{1}{2}R$.

Survey sampling literature suggests a number of successful methods for estimating unknown variance S_y^2 using auxiliary variable x either based on the classical approach [cf., Das and Tripathi [5], Isaki [6], Kadilar and Cingi [7], Grover [8], Yadav [9]] or on the predictive approach [cf., Biradar and Singh [10], Nayak and Sahoo [11]]. However, the sample regression coefficient $b = s_{yx}/s_x^2$ (an OLS estimator) always remains as the most discussed estimator of β. On the other hand, under the condition $\beta \geq \frac{1}{2}R$, r is always more efficient than $\bar{y}/\bar{X}$ for estimating the ratio R. Hence, our mechanism of estimating $V(t_R)$ in this work consists of selecting some alternative estimators of S_y^2 in place of s_y^2 that incorporates available auxiliary information on x, and selecting b and r as the estimators of β and R respectively in the usual way. This means that, our present work makes an attempt on the construction of an estimator of $V(t_R)$ having the following generalized form:

$$\hat{V}(t_R) = \frac{N-n}{Nn}\left[\hat{S}_y^2 - 2rS_x^2\left(b - \frac{1}{2}r\right)\right]$$
$$= \frac{N-n}{Nn}\left[\hat{S}_y^2 - \hat{\eta}S_x^2\right], \qquad (2.2)$$

where $\hat{S}_y^2$ is an estimator of S_y^2 and $\hat{\eta} = 2r\left(b - \frac{1}{2}r\right)$.

3. FORMULATION OF THE ESTIMATORS

It is easily understood that for different selections of $\hat{S}_y^2$, the general class of estimators $\hat{V}(t_R)$ defined in (2.2) can provide us a number of estimators for $V(t_R)$. This can be done by considering estimators from the variety of finite population variance estimation methods available in the literature. But, here we exclude an estimator of S_y^2 for which $\hat{V}(t_R)$ achieves negative values very frequently under repeated sampling from a given population. For example, based on this consideration we do not consider $\hat{S}_y^2 = s_y^2$ and $\hat{S}_y^2 = s_y^2\bar{X}/\bar{x}$ [cf., Das and Tripathi [5]]. Now we present a review of the estimators of S_y^2 those are taken into account for our research.

A well known estimator of S_y^2 under classical approach was initially considered by Isaki [6]. This estimator is defined by

$$v_3 = s_y^2 S_x^2 / s_x^2.$$

Motivated by Bolfarine and Zacks [12], emphasis has also been given on the prediction of the population variance using auxiliary information. The mechanics of estimating S_y^2 under this approach give rise to many estimators. But, here we report below four estimators *viz.*, v_4, v_5 suggested by Biradar and Singh [10], and v_6, v_7 suggested by Nayak and Sahoo [11]:

$$v_4 = \frac{s_y^2}{s_x^2}S_x^2 + \frac{nN(\bar{x}-\bar{X})^2}{(N-n)(N-1)}\left(r^2 - \frac{s_y^2}{s_x^2}\right)$$
$$v_5 = \frac{s_y^2}{s_x^2}S_x^2 + \frac{nN(\bar{x}-\bar{X})^2}{(N-n)(N-1)}\left(b^2 - \frac{s_y^2}{s_x^2}\right)$$
$$v_6 = \left(\frac{n-1}{N-1}\right)\left[s_y^2 + r^2\left(\frac{N-1}{n-1}S_x^2 - s_x^2\right)\right]$$
$$v_7 = \left(\frac{n-1}{N-1}\right)\left[s_y^2 + b^2\left(\frac{N-1}{n-1}S_x^2 - s_x^2\right)\right].$$

After selecting five estimators $v_i, i = 3, 4, \dots, 7$ for S_y^2, we then utilize equation (2.2) to produce five new estimators of $V(t_R)$ by substituting an estimator v_i for $\hat{S}_y^2$. This operation generates five new estimators corresponding to different choices of $\hat{S}_y^2$ as shown in Table 3.1. To save space, the detail expressions of proposed estimators are not given.

Table 3.1: Proposed Estimators of $V(t_R)$

Selection of $\hat{S}_y^2$	v_3	v_4	v_5	v_6	v_7
$\hat{V}(t_R) = \frac{N-n}{Nn}\left[\hat{S}_y^2 - \hat{\eta}S_x^2\right]$	$\hat{\mathcal{V}}_3$	$\hat{\mathcal{V}}_4$	$\hat{\mathcal{V}}_5$	$\hat{\mathcal{V}}_6$	$\hat{\mathcal{V}}_7$

4. PERFORMANCE OF THE PROPOSED ESTIMATORS

To illustrate the performances of five variance estimators $\hat{\mathcal{V}}_3$, $\hat{\mathcal{V}}_4$, $\hat{\mathcal{V}}_5$, $\hat{\mathcal{V}}_6$ and $\hat{\mathcal{V}}_7$ built from $\hat{V}(t_R)$ compared to the traditional estimators $\hat{\mathcal{V}}_0$, $\hat{\mathcal{V}}_1$ and $\hat{\mathcal{V}}_2$, we carry out a Monte Carlo simulation in which 5000 independent samples for $n = 6, 8$ and 10 are drawn from 22 natural populations available in text books and research papers. The following performance measures of an estimator $\hat{\mathcal{V}}_i$ $(i = 0, 1, 2, \dots, 7)$ are taken into consideration:

(i) *Standard Error (SE)*: This performance measure of $\hat{\mathcal{V}}_i$ is defined by

$$SE(\hat{\mathcal{V}}_i) = +\sqrt{E[\hat{\mathcal{V}}_i]^2 - [E(\hat{\mathcal{V}}_i)]^2},$$

which is a convenient and widely used indicator of the precision or efficiency attained by the estimator.

(ii) *Coverage Rate (CR) Based on 95% Confidence Interval for Estimating* $\bar{Y}$: We consider an approximate 95% confidence interval for $\bar{Y}$ based on t_R and its variance estimator $\hat{\mathcal{V}}_i$ defined by $t_R \pm 1.96\sqrt{\hat{\mathcal{V}}_i}$ under the assumption that the sampling distribution of t_R is approximately a normal distribution. This performance measure gives us an idea about which percentage of the so constructed confidence intervals based on the variance estimator covers the true value of $\bar{Y}$ under repeated draws of independent samples from a population.

(iii) *Coefficient of Variation (CV)*: Coefficient of variation is the most appropriate measure of the stability of a variance estimator. This performance measure has been extensively used by Rao [13], Rao and Bayless [14], Bayless and Rao [15] among others. For the estimator $\hat{\mathcal{V}}_i$, CV is defined by

$$CV(\hat{\mathcal{V}}_i) = 100 \times \sqrt{\frac{E[\hat{\mathcal{V}}_i - V(t_R)]^2}{[V(t_R)]^2}}.$$

5. DESCRIPTION OF THE SIMULATION STUDY

Our simulation study involves repeated draws of simple random without replacement samples from 22 natural populations. Table 5.1 presents the source, size (N), definitions of the variables y and x, and the correlation coefficient between y and x (ρ) in respect of these populations. 5,000 independent samples, for $n = 6, 8$ and 10, were selected from each population and for each sample numerical values of the comparable estimators were calculated. Then, considering 5,000 such combinations, values of the performance measures *viz*., SE, CR and CV were computed and summarized in Tables 5.2, 5.3 and 5.4. Results for $n = 8$ are not shown, as they confirm more or less the tendencies found in the cases of $n = 6$ and 10. Major findings of the study are discussed in subsections 5.1, 5.2 and 5.3.

5.1 Results Based on the Standard Error

Numerical values on the SE are given in Table 5.2. Judging efficiency of an estimator is inversely proportional to its SE, we see that $\hat{\mathcal{V}}_3$ is straightforwardly the most inefficient estimator among all. On the other hand, the performance of $\hat{\mathcal{V}}_2$ is better than $\hat{\mathcal{V}}_0$ and $\hat{\mathcal{V}}_1$, and the performance of $\hat{\mathcal{V}}_4$ or $\hat{\mathcal{V}}_5$ is highly unsatisfactory. Results on the SE of the competing estimators show that the estimator $\hat{\mathcal{V}}_6$ is decidedly the best performer as it is the most efficient in 16 and 17 populations for $n = 6$, and 10 respectively, and ranked second in 3 and 4 populations for $n = 6$, and 10 respectively. $\hat{\mathcal{V}}_7$ can be regarded as the second best performer being ranked respectively as the first and second in 6 and 13 populations for $n = 6$, and in 6 and 12 populations for $n = 10$.

Table 5.1: Description of the Populations

Pop. No.	Source	N	y	x	ρ
1	Cochran (1977) p.152	49	no. of inhabitants in 1930	no. of inhabitants in 1920	0.9817
2	Sukhatme and Sukhatme (1977) p.185	34	area under wheat in1937	area under wheat in1936	0.9299
3	Sukhatme and Sukhatme (1977) p.185	34	area under wheat in1937	area under wheat in1931	0.8993
4	Samford (1962) p.61	35	acreage under oats in 1957	acreage of crops and grass in 1947	0.8381
5	Wetherill (1981) p.104	32	percent yield of petroleum	petroleum fraction end point	0.2463
6	Murthy (1967) p.398	43	no. of absentees	no, of workers	0.6608
7	Murthy (1967) p.399	34	area under wheat in 1964	cultivated area in 1961	0.9043
8	Murthy (1967) p.399	34	area under wheat in 1964	area under wheat in 1963	0.9801
9	Steel and Torrie (1960) p.282	30	leaf burn in secs.	percentage of potassium	0.1794
10	Shukla (1966)	50	fiber yield	height of plant	0.5403
11	Shukla (1966)	50	fiber yield	base diameter	0.5316
12	Dobson (1990) p.83	30	cholesterol	age in years	0.6029
13	Dobson (1990) p.83	30	cholesterol	body mass	0.5353
14	Yates (1960) p.159	25	measured volume of timber	eye estimated volume of timber	0.3346
15	Yates (1960) p.159	43	no. of absentees	total no. of persons	0.6656
16	Panse and Sukhatme (1985) p.118	25	progeny mean	parental plant value	0.6624
17	Panse and Sukhatme (1985) p.118	25	progeny mean	parental plot mean	0.4475
18	Dobson (1990) p.69	20	total calories from	calories as protein	0.4629

			carbohydrate		
19	Horvitz and Thompson (1952)	20	actual no. of households	eye estimated no. of households	0.8662
20	Murthy(1967)p.422	24	no. of cattle (survey)	no. of cattle (census)	0.9589
21	Singh and Choudhury (1986) p.176	20	no. of cows giving milk (survey)	no. of cows giving milk (census)	0.8744
22	Singh and Choudhury (1986) p.279	20	dry yield of paddy (kg)	harvest yield of paddy (kg)	0.9968

Table 5.2: Standard Error of the Estimators

n	Pop. No.	$\hat{\mathcal{V}}_0$	$\hat{\mathcal{V}}_1$	$\hat{\mathcal{V}}_2$	$\hat{\mathcal{V}}_3$	$\hat{\mathcal{V}}_4$	$\hat{\mathcal{V}}_5$	$\hat{\mathcal{V}}_6$	$\hat{\mathcal{V}}_7$
6	1	264.02	283.98	244.45	412.98	408.59	401.48	208.58	200.43
	2	309.59	227.94	172.98	299.12	169.72	157.21	97.31	96.76
	3	299.26	251.46	233.60	567.09	460.95	462.17	152.42	197.90
	4	51.75	101.58	242.42	525.55	481.11	501.37	55.96	49.78
	5	60.71	56.55	46.46	145.32	132.02	132.04	27.83	29.77
	6	1.86	1.78	1.82	1.99	1.95	1.95	1.58	1.78
	7	288.36	246.99	230.36	398.11	458.13	458.92	167.61	190.89
	8	74.39	60.22	53.49	74.98	62.78	64.00	28.30	39.08
	9	0.013	0.013	0.013	0.045	0.036	0.036	0.012	0.019
	10	0.425	0.505	0.420	0.772	0.738	0.763	0.378	0.344
	11	0.064	0.069	0.063	0.321	0.288	0.290	0.086	0.061
	12	0.101	0.111	0.125	0.241	0.239	0.237	0.103	0.109
	13	0.095	0.096	0.094	0.211	0.201	0.206	0.065	0.076
	14	348.99	407.16	334.31	435.23	253.63	247.56	162.57	187.32
	15	7.32	6.98	6.09	7.98	7.65	7.66	2.32	2.87
	16	0.108	0.106	0.105	0.134	0.112	0.101	0.079	0.098
	17	0.069	0.068	0.067	0.191	0.186	0.187	0.048	0.069
	18	2.149	2.067	2.063	4.342	4.224	4.341	1.810	2.003
	19	1.713	1.618	1.579	2.007	1.957	2.017	0.828	0.859
	20	1761.61	1791.77	1632.03	5845.87	5757.30	5708.47	1699.87	1625.12
	21	7851.05	6933.16	7789.31	8995.03	8993.01	8475.11	4918.95	5067.76
	22	0.002	0.002	0.002	0.004	0.003	0.003	0.001	0.001
10	1	19.77	21.05	13.49	22.36	18.27	18.09	8.75	9.45
	2	166.62	122.14	95.62	169.17	109.89	96.46	59.36	90.62
	3	169.69	129.62	102.10	154.12	116.56	131.70	70.49	82.89
	4	213.77	228.62	215.00	226.21	164.23	175.32	139.17	127.85
	5	2.59	2.53	2.09	7.43	6.60	6.60	1.57	2.09
	6	0.86	0.81	0.79	0.89	0.55	0.56	0.22	0.27
	7	129.23	99.99	81.81	97.09	155.91	164.85	76.46	69.33
	8	34.85	26.12	21.18	34.78	14.08	19.85	13.67	14.12
	9	0.004	0.004	0.005	0.009	0.005	0.005	0.001	0.001
	10	0.043	0.044	0.046	0.062	0.030	0.030	0.017	0.035
	11	0.028	0.029	0.022	0.100	0.071	0.074	0.022	0.019
	12	0.034	0.038	0.043	0.041	0.031	0.030	0.034	0.032
	13	0.032	0.032	0.032	0.089	0.061	0.061	0.012	0.024
	14	113.63	138.32	112.35	140.44	112.24	102.62	74.51	80.54
	15	3.41	3.22	3.10	3.98	2.16	2.17	0.086	0.096
	16	0.031	0.031	0.031	0.035	0.034	0.032	0.021	0.036
	17	0.027	0.028	0.020	0.060	0.062	0.059	0.020	0.023
	18	0.56	0.55	0.53	0.34	0.49	0.48	0.33	0.33
	19	0.54	0.54	0.56	0.56	0.51	0.57	0.26	0.31
	20	567.37	594.39	504.54	1715.00	1746.90	1636.72	424.17	407.71
	21	2864.41	2828.06	2747.85	3289.23	3236.53	3129.78	2565.99	2589.54
	22	0.0006	0.0006	0.0005	0.0005	0.0005	0.0006	0.0003	0.0005

Table 5.3: Coverage Rate of the Estimators

n	Pop. No.	$\hat{\mathcal{V}}_0$	$\hat{\mathcal{V}}_1$	$\hat{\mathcal{V}}_2$	$\hat{\mathcal{V}}_3$	$\hat{\mathcal{V}}_4$	$\hat{\mathcal{V}}_5$	$\hat{\mathcal{V}}_6$	$\hat{\mathcal{V}}_7$
6	1	53	55	56	60	76	74	78	75
	2	85	86	87	65	86	86	89	88
	3	68	69	70	55	75	73	75	74
	4	60	70	73	54	73	72	74	76
	5	64	65	65	48	77	77	78	77
	6	52	53	53	38	54	53	58	57
	7	71	71	72	60	75	73	76	74
	8	69	70	72	50	73	70	74	72
	9	72	72	72	52	77	75	78	78
	10	55	56	65	42	69	65	70	70
	11	85	87	88	69	95	87	95	88
	12	90	91	91	83	90	89	96	95
	13	91	90	92	66	93	91	94	92
	14	62	66	71	47	89	84	87	88
	15	54	54	55	38	56	55	59	56
	16	81	86	86	76	87	85	84	87
	17	76	77	79	68	88	88	89	90
	18	88	88	89	92	98	92	93	96
	19	75	77	79	64	80	79	79	78
	20	75	79	84	70	94	91	92	92
	21	66	70	77	59	88	84	86	85
	22	88	88	88	73	88	87	89	87
10	1	96	96	97	69	98	97	99	97
	2	96	96	97	79	99	98	100	99
	3	76	81	78	67	83	81	84	82
	4	56	75	80	59	76	75	87	88
	5	16	15	17	20	31	28	31	29
	6	26	26	27	11	42	39	47	40
	7	98	97	99	86	99	98	100	99
	8	97	97	98	96	99	99	100	100
	9	57	58	59	30	74	73	75	74
	10	31	31	32	14	39	31	39	32
	11	85	86	88	54	95	92	94	95
	12	98	98	98	95	99	96	98	98
	13	95	95	96	69	95	94	98	96
	14	59	63	68	49	62	63	87	86
	15	27	28	28	11	32	30	31	32
	16	74	73	79	61	77	76	78	76
	17	75	76	76	53	78	77	78	77
	18	80	80	81	76	88	90	89	91
	19	74	76	77	67	79	76	77	76
	20	77	78	81	73	89	87	88	87
	21	57	60	64	48	73	72	76	73
	22	91	92	93	91	94	91	99	98

Table 5.4: Coefficient of Variation of the Estimators

n	Pop. No.	$\hat{\mathcal{V}}_0$	$\hat{\mathcal{V}}_1$	$\hat{\mathcal{V}}_2$	$\hat{\mathcal{V}}_3$	$\hat{\mathcal{V}}_4$	$\hat{\mathcal{V}}_5$	$\hat{\mathcal{V}}_6$	$\hat{\mathcal{V}}_7$
	1	179.24	96.02	233.61	257.65	489.06	479.34	83.33	81.09
	2	93.54	84.76	89.84	101.12	75.52	74.30	80.23	76.19
	3	59.07	54.79	80.86	82.54	74.47	74.30	53.13	54.76
	4	385.26	186.18	799.85	689.19	891.46	928.48	149.26	150.77
	5	48.30	47.33	190.76	980.76	968.06	968.52	46.74	46.99
	6	72.62	71.58	87.98	88.23	72.06	71.86	70.24	72.12
	7	56.16	51.85	77.94	79.31	75.82	75.79	50.08	52.09

6	8	72.05	63.55	83.15	85.38	58.94	57.90	58.52	59.10
	9	47.24	45.55	73.35	213.67	128.55	128.82	46.65	43.77
	10	123.45	76.12	196.40	176.32	79.61	75.51	76.93	76.12
	11	59.54	58.93	78.08	145.61	148.98	150.20	56.72	58.13
	12	50.85	51.98	70.39	99.46	97.49	45.54	51.67	51.73
	13	75.05	64.13	82.88	92.43	122.32	125.62	64.70	63.75
	14	70.13	76.61	72.81	77.98	58.30	61.63	53.03	61.71
	15	74.62	71.46	88.24	98.52	72.05	71.10	69.82	71.10
	16	59.18	57.25	61.65	57.67	58.21	52.16	55.59	49.22
	17	49.49	48.38	73.49	120.34	118.91	119.28	48.33	49.22
	18	44.26	43.77	67.24	78.43	75.17	77.17	41.52	42.79
	19	72.72	69.04	72.81	78.09	67.63	66.91	66.78	67.32
	20	70.58	69.00	136.54	178.11	224.54	220.60	66.86	66.38
	21	78.44	71.49	72.80	81.74	76.29	69.56	64.36	68.96
	22	85.36	68.02	69.70	90.65	57.63	55.79	49.45	52.04
10	1	60.27	56.38	83.64	52.81	58.32	58.89	53.54	54.80
	2	81.98	72.21	82.07	83.54	64.92	60.94	63.93	64.79
	3	55.29	40.37	62.72	62.95	38.02	37.70	37.17	37.99
	4	345.66	107.59	492.47	497.40	586.56	625.99	82.20	81.54
	5	40.60	40.05	69.31	97.21	93.01	93.12	39.66	41.00
	6	76.66	70.78	86.20	87.55	66.48	68.60	63.72	64.98
	7	46.22	66.98	49.42	70.04	55.41	65.69	29.35	32.13
	8	67.77	46.19	56.01	68.28	34.60	34.90	33.87	34.98
	9	38.33	44.55	74.50	89.00	43.09	43.00	36.59	36.93
	10	55.01	54.25	83.36	91.43	47.09	47.07	55.61	49.00*
	11	42.66	39.22	75.08	85.66	70.83	74.33	35.78	37.88
	12	30.12	31.47	40.89	41.92	24.55	22.85	28.24	28.98
	13	46.85	46.29	73.34	72.53	74.71	74.84	46.22	46.21
	14	59.72	58.08	69.80	73.84	61.23	58.78	55.89	54.17
	15	66.67	64.91	86.91	88.15	65.64	64.73	63.55	64.66
	16	36.54	36.03	37.25	52.39	37.65	35.86	34.06	34.24
	17	28.43	27.99	55.03	83.61	89.02	84.42	27.65	27.86
	18	31.28	29.82	59.53	62.87	29.52	29.00	28.44	29.20
	19	55.28	53.51	64.67	67.28	58.89	52.36	52.78	51.53
	20	58.17	57.95	71.81	67.93	57.57	57.51	56.93	56.12
	21	65.47	65.13	87.28	85.35	74.50	64.26	64.01	63.13
	22	33.83	34.54	50.00	61.75	39.20	32.30	29.44	31.37

5.2 Results Based on the Coverage Rate

The coverage rates of nominal 95% confidence intervals for $\bar{Y}$ of the comparable estimators are shown in Table 5.3. From the computed values, it is clear that the CR of the estimators (except some few cases) usually bears no resemblance to the nominal rates aimed at. On the basis of the achieved coverage rates, $\hat{\mathcal{V}}_3$ turns out as the worst performer whereas $\hat{\mathcal{V}}_2$ turns out as better performers than $\hat{\mathcal{V}}_0$ and $\hat{\mathcal{V}}_1$. The estimator $\hat{\mathcal{V}}_6$ emerged out as the best performer as its CR is the highest in 14 populations for $n = 6$ and 10, and ranked as second in 5 and 7 populations for $n = 6$ and $n = 10$ respectively. However, on the ground of the achieved CR we may consider $\hat{\mathcal{V}}_4$ and $\hat{\mathcal{V}}_7$ as the second best and the third best performers respectively.

5.3 Results Based on the Coefficient of Variation

From the displayed numerical values on the CV of different estimators available in Table 5.4, we note that the stability of $\hat{\mathcal{V}}_3$, on the average, is poor compared to other estimators as in the cases of other performance measures. The estimator $\hat{\mathcal{V}}_1$ appears to be more stable than $\hat{\mathcal{V}}_0$ and of $\hat{\mathcal{V}}_2$. $\hat{\mathcal{V}}_6$ is regarded as the most stable estimator as, on the basis of the computed values on the CV, it secures respectively the first, second and third positions in 13, 4 and 3 populations for $n = 6$, and 13, 7 and 1 populations for $n = 10$. On the same consideration, we rank $\hat{\mathcal{V}}_7$ as the second best performer and $\hat{\mathcal{V}}_5$ as the third best performer.

6. CONCLUSIONS

Our numerical comparison of different variance estimators of t_R through the simulation study shows that no estimator is uniformly better than others in respect of all performance criteria. But, from this study we draw only two consistent conclusions *i.e.*, selection of $\hat{\mathcal{V}}_6$ as the most efficient estimator, and selection of $\hat{\mathcal{V}}_3$ as the poorest estimator on the grounds of SE, CR and SE. On the other hand, we also conclude that the estimator $\hat{\mathcal{V}}_7$ may be preferred as the second best choice in respect of both SE and CV whereas $\hat{\mathcal{V}}_4$ as the second best choice in respect of CR.

Of course, these results are only indicative and cannot be able to reveal essential features of the comparable estimators in a straightforward manner. But we hope that this study will eventually lead to better understanding of the unsettled problem of choosing good variance estimators of t_R. However, we stress on further investigation in this direction for arriving at better conclusions.

REFERENCES

[1] Bayless, D. L. and Rao, J. N. K. (1970). An empirical study of stabilities of estimators and variance estimators in unequal probability sampling. Journal of the American Statistical Association, 65, 1645-1667.

[2] Biradar, R. S. and Singh, H. P. (1998). Predictive estimation of finite population variance. Calcutta Statistical Association Bulletin, 48, 229-235.

[3] Bolfarine, H. and Zacks, S. (1992). Prediction Theory for Finite Populations. Springer- Verlag.

[4] Cochran , W. G. (1977). Sampling Techniques (3rd Edition). Wiley Eastern Limited.

[5] Das, A. and Tripathi, T. P. (1978). Use of auxiliary information in estimating the finite population variance. Sankhyā, C40, 139-148.

[6] Dobson, A. J. (1990). An Introduction to Generalized Linear Models. Chapman & Hal, New York.

[7] Grover, L. K. (2007). A wide and efficient class of estimators of population variance under sub-sampling scheme. Model Assisted Statistics and Applications, 2, 41-51.

[8] Horvitz, D. G. and Thompson, D. J. (1952). A generalization of sampling without replacement from a finite universe. Journal of the American Statistical Association, 47, 663-685.

[9] Isaki, C. T. (1983). Variance estimation using auxiliary information. Journal of the American Statistical Association, 78, 117-123.

[10] Kadilar, C. and Cingi, H. (2006). Improvement in variance estimation using auxiliary information. Hacettepe Journal of Mathematics and Statistics, 35, 111-115.

[11] Murthy, M. N. (1967). Sampling Theory and Methods. Statistical Publishing Society, Kolkata.

[12] Nayak, R. K. and Sahoo, L. N. (2012). Some alternative predictive estimators of population variance. Revista Colombiana de Estadistica, 13, 1-13.

[13] Panse, V. G. and Sukhatme, P.V. (1985). Statistical Methods for Agricultural Workers. Indian Council for Agricultural Research, New Delhi.

[14] Rao, J. N. K. (1969). Ratio and regression estimators. New Developments in Survey Sampling. N. L. Johnson and H. Smith Jr. (eds.), Chapel Hill, North Carolina, 213-234.

[15] Rao, J. N. K. and Bayless, D. L. (1969). An empirical study of the stabilities of estimators and variance estimators in unequal probability sampling of two units per stratum. Journal of the American Statistical Association, 64, 540-549.

[16] Sampford, M. R. (1978). Predictive estimation and internal congruency. Contribution to Survey Sampling and Applied Statistics, H.A. David (eds.), Academic Press, New York, 29-39.

[17] Scott, A. J. and Wu, C. F. (1981). On the asymptotic distribution of ratio and regression estimators. Journal of the American Statistical Association, 76, 98-102.

[18] Shukla, G. K. (1966). An alternative multivariate ratio estimate for finite population. Calcutta Statistical Association Bulletin, 15, 127-134.

[19] Singh, D. and Chaudhaury, F.S. (1986). Theory and Analysis of Sample Survey Designs. Willey Eastern Limited, New Delhi.

[20] Steel, Robert G. D. and Torrie, James H. (1960). Principles and Procedures of Statistics. Mc. Graw Hill Book Company.

[21] Sukhatme, P. V. and Sukhatme, B. V. (1977). Sampling Theory of Surveys with Applications. Asia Publishing House, New Delhi.

[22] Wetherill, G. B. (1981). Intermediate Statistical Methods. Chapman & Hall, London.

[23] Wu, C. F. (1982). Estimation of variance of the ratio estimators. Journal of the American Statistical Association, 69, 183-89.

[24] Wu, C. F. J. and Deng, L. Y. (1983). Estimation of variance of the ratio estimator: An empirical study. Scientific Inference, Data Analysis and Robustness. G.E.P. Box et al. (eds.), New York: Academic Press, 245-277.

[25] Yadav, S. K. (2011). Efficient estimators for population variance using auxiliary information. Global Journal of Mathematical Sciences: Theory and Practical, 3, 369-376.

[26] Yates, F. (1960). Sampling Methods for Censuses and Surveys. Charls & Griffin, London.