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Some Bianchi Type-III Bulk Viscous Massive String Cosmological Models with Electromagnetic Field

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***Abstract*—The present study deals with exact solution of Einstein's field equations in a spatially homogeneous and anisotropic Bianchi III space-time representing massive string in presence of bulk viscosity and electromagnetic. The source of the magnetic field is due to an electric current produced along the z-axis. F_{12} is the non-vanishing component of electromagnetic field tensor. To get the deterministic solution of the field equations, we assume (i) the expansion θ in the model is proportional to the eigne value ${\sigma^2}_2$ of the shear tensor ${\sigma^j}_i$ and (ii) the coefficient of bulk viscosity is assumed to be a power function of mass density ($\xi = \xi_0 \rho^\kappa$), where ξ_0 and κ are real constants. It is observed that the particle density and the tension density of the string are comparable at the two ends and they fall off asymptotically at similar rate. But in early stage as well as at the late time of the evolution of the universe we have two types of scenario (i) universe is dominated by massive strings and (ii) universe is dominated by strings depending on the nature of the constants. Some physical and geometric properties of the model are also discussed.**

***Index Terms*—Massive string, Bianchi-III space-time, Magnetic field**
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I. INTRODUCTION

Cosmic strings, in recent years. have drawn considerable attention among researchers for various aspects such that the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1]− [6]. It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies [7]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [8], [9] and Stachel [10]. The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zel'dovich [11]. Also Harrison [12] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [13]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors (Misner et al. [14]; Asseo and Sol [15]; Pudritz and Silk [16]; Kim et al. [17]; Perley and Taylor [18]; Kronberg et al. [19]; Wolfe et al. [20]; Kulsrud et al. [21]; Barrow [22]). Melvin [23], in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic. The string cosmological models with a magnetic field in Bianchi type I, II, III, VI_0, VIII and IX space-times have been discussed by several authors [24]− [36]. Recently, Pradhan et al. [37] studied Bianchi-I massive string cosmological models in general relativity. Recently, Yadav [38] and Yadav et al. [39]− [41] have studied Bianchi type-V, II and cylindrically symmetric inhomogeneous universe with a cloud of strings.

A realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that in an earlier stage of the universe when the radiation in the form of photons as well as neutrinos decoupled from matter, it behaved like a viscous fluid. Misner [42] has studied the effect of viscosity on the evolution of cosmological models. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [43] for a review on cosmological models with bulk viscosity). A number of authors have discussed cosmological solutions with bulk viscosity in various context [44]− [50].

Motivated the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the massive string in Bianchi-III space-time in presence of magnetic field and bulk viscous fluid. Here we have obtained an exact and general solution of Einstein's field equations of Bianchi type-III space-time for a cloud of strings with electromagnetic field. The paper is organized as follows. The metric and the field equations are presented in Section II. In Section III, we deal with an exact solution of the field equations with cloud of strings. In this section we also described three types of models of the universe. In Section IV we describe some physical and geometric properties of the model. Finally, in Section V, concluding remarks are given.

II. THE METRIC AND FIELD EQUATIONS

We consider the general Bianchi type III space-time given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2ax}dy^2 + C^2(t)dz^2, \quad (1)$$

where a is constant. The energy momentum tensor for a cloud of strings with a perfect fluid and electromagnetic field has the form

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p}g_i^j - \lambda x_i x^j + E_i^j, \quad (2)$$

where v_i and x_i satisfy condition

$$v^i v_i = -x^i x_i = -1, \;\; v^i x_i = 0. \quad (3)$$

Here $\bar{p}$ is effective pressure, ρ is the proper energy density for a cloud strings with particles attached to them, λ is the string tension density, v^i is the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string.

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1), \quad x^i = \left(0, 0, \frac{1}{C}, 0\right). \quad (4)$$

In equations (2), E_i^j is the electromagnetic field given by

$$E_i^j = \frac{1}{4\pi}\left[g^{lm}F_{il}F_{jm} - \frac{1}{4}F_{lm}F^{lm}g_{ij}\right], \quad (5)$$

and $\bar{p}$ is given by

$$\bar{p} = p - \xi v^i_{;i}, \quad (6)$$

where ξ is the coefficient of bulk viscosity and p is isotropic pressure.

We assume that magnetic field is in xy plane. Therefore, the current is flowing along z axis. Thus, F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. Subsequently, Maxwell equations

$$F_{ik,l} + F_{kl,i} + F_{li,k} = 0 \;\text{ and }\; [F^{ik}(-g)^{\frac{1}{2}}]_{,k} = 0, \quad (7)$$

lead to

$$F_{12} = Ke^{-ax}, \quad (8)$$

where K is a constant so that magnetic field depends upon that space coordinate x only. From Eqs. (4),(5) and (7), it follows that $F_{14} = 0$. Now the non-vanishing components of E_{ij} corresponding to the line-element (15) are given by

$$E_1^1 = E_2^2 = \frac{1}{8\pi}\frac{K^2}{A^2B^2}, \quad E_3^3 = E_4^4 = -\frac{1}{8\pi}\frac{K^2}{A^2B^2}. \quad (9)$$

If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda. \quad (10)$$

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j + \Lambda g_i^j, \quad (11)$$

where R_i^j is the Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar.

The field equations (11) with (2) subsequently lead to the following system of equations:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = -8\pi(\bar{p} - \lambda) + \frac{K^2}{A^2B^2}, \quad (12)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi\bar{p} - \frac{K^2}{A^2B^2}, \quad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi\bar{p} - \frac{K^2}{A^2B^2}, \quad (14)$$

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} - \frac{a^2}{A^2} = 8\pi\rho + \frac{K^2}{A^2B^2}, \quad (15)$$

$$a\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0. \quad (16)$$

Here and in what follows an over dot denotes ordinary differentiation with respect to t.

The spatial volume for the model (15) is given by

$$V^3 = ABC. \quad (17)$$

We define $V = (ABC)^{\frac{1}{3}}$ as the average scale factor so that the Hubble's parameter is anisotropic models may be defined as

$$H = \frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \quad (18)$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (19)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z respectively.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{V\ddot{V}}{\dot{V}^2}. \quad (20)$$

The velocity field v^i as specified by (4) is irrotational. The scalar expansion θ, components of shear σ_{ij} and the average anisotropy parameter A_m are defined by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (21)$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right], \quad (22)$$

$$\sigma_{22} = \frac{B^2e^{-2ax}}{3}\left[\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right], \quad (23)$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right], \quad (24)$$

$$\sigma_{44} = 0. \quad (25)$$

Therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right]. \quad (26)$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2, \quad (27)$$

where $\triangle H_i = H_i - H(i = 1,2,3)$.

III. SOLUTIONS OF THE FIELD EQUATIONS

The field equations (12)-(16) are a system of five equations with seven unknown parameters A, B, C, ρ, p, λ and ξ. Two additional constraints relating these parameters is required to obtain explicit solution of the system. Firstly, we assume that the expansion (θ) in the model is proportional to the eigne value ${\sigma^2}_2$ of the shear tensor ${\sigma^j}_i$. This condition leads to

$$B = \ell_1(AC)^{m_1}, \tag{28}$$

where ℓ_1 and m_1 are arbitrary constants. Equations (16) leads to

$$A = mB, \tag{29}$$

where m is an integrating constant. Eqs. (13) and (14) reduce to

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = 0. \tag{30}$$

Using (29) in (30) we obtain

$$(1-m)\left(\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC}\right) = 0. \tag{31}$$

As $m \neq 0$, Eq. (31) gives

$$\left(\frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC}\right) = 0, \tag{32}$$

which on integration reduces to

$$\dot{B}C = k_1, \tag{33}$$

where k_1 is an integrating constant.

From Eqs. (28) and (29), we obtain

$$B = \ell_2 C^{\ell}, \tag{34}$$

where $\ell_2 = \ell_1^{\frac{1}{1-m_1}} m^{\ell}$, $\ell = \frac{m_1}{1-m_1}$. Using (34) in (33) we get

$$C^{\ell}\dot{C} = \frac{k_1}{\ell\ell_2}, \tag{35}$$

which on integration gives

$$C = (\ell+1)^{\frac{1}{\ell+1}}\left[\frac{k_1}{\ell\ell_2}t + k_2\right]^{\frac{1}{\ell+1}}, \tag{36}$$

where k_2 is an integrating constant. Using (36) in Eqs. (34) and (29), we obtain

$$B = \ell_2(\ell+1)^{\frac{\ell}{\ell+1}}\left[\frac{k_1}{\ell\ell_2}t + k_2\right]^{\frac{\ell}{\ell+1}}, \tag{37}$$

and

$$A = m\ell_2(\ell+1)^{\frac{\ell}{\ell+1}}\left[\frac{k_1}{\ell\ell_2}t + k_2\right]^{\frac{\ell}{\ell+1}}, \tag{38}$$

respectively.

Hence the metric (15) reduces to the form

$$ds^2 = -dt^2 + \left[m\ell_2(\ell+1)^{\frac{\ell}{\ell+1}}\left(\frac{k_1}{\ell\ell_2}t + k_2\right)^{\frac{\ell}{\ell+1}}\right]^2 dx^2 +$$

$$\left[\ell_2(\ell+1)^{\frac{\ell}{\ell+1}}e^{-ax}\left(\frac{k_1}{\ell\ell_2}t + k_2\right)^{\frac{\ell}{\ell+1}}\right]^2 dy^2 +$$

$$\left[(\ell+1)^{\frac{1}{\ell+1}}\left(\frac{k_1}{\ell\ell_2}t + k_2\right)^{\frac{1}{\ell+1}}\right]^2 dz^2. \tag{39}$$

Using the suitable transformation

$$m\ell_2(\ell+1)^{\frac{\ell}{\ell+1}}x = X,$$
$$\ell_2(\ell+1)^{\frac{\ell}{\ell+1}}y = Y,$$
$$(\ell+1)^{\frac{1}{\ell+1}}z = Z,$$
$$\frac{k_1}{\ell\ell_2}t + k_2 = T, \tag{40}$$

the metric (39) reduces to

$$ds^2 = -\beta^2 dT^2 + T^{2L}dX^2 + T^{2L}e^{-\frac{2a}{N}X}dY^2 + T^{\frac{2L}{\ell}}dZ^2, \tag{41}$$

where

$$\beta = \frac{\ell\ell_2}{k_1},$$
$$M = (\ell+1)^{\frac{1}{\ell+1}},$$
$$N = m\ell_2 M,$$
$$L = \frac{\ell}{\ell+1}. \tag{42}$$

The effective pressure $\bar{p}$, the rest energy density (ρ), the string tension density (λ) and the particle density (ρ_p) for the model (41) are given by

$$8\pi\bar{p} = \frac{\ell}{(\ell+1)^2\beta^2T^2} - \frac{m^2K^2}{N^4T^{4L}}, \tag{43}$$

$$8\pi\rho = \frac{\ell(\ell+2)}{(\ell+1)^2\beta^2T^2} - \frac{a^2}{N^2T^{2L}} - \frac{m^2K^2}{N^4T^{4L}}, \tag{44}$$

$$8\pi\lambda = \frac{(\ell^2+\ell+2)}{(\ell+1)^2\beta^2T^2} - \frac{a^2}{N^2T^{2L}} - 2\frac{m^2K^2}{N^4T^{4L}}, \tag{45}$$

$$8\pi\rho_p = \frac{(\ell-2)}{(\ell+1)^2\beta^2T^2} + \frac{m^2K^2}{N^4T^{4L}}. \tag{46}$$

Thus, given $\xi(t)$, we can solve for the physical parameters. In most investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density [46]– [49]

$$\xi(t) = \xi_0\rho^{\kappa}, \tag{47}$$

where ξ_0 and m are constants. For small density, m may even be equal to unity as used in Murphy's work [51] If $\kappa = 1$, Equation (47) may correspond to a radiative fluid (Weinberg [52]). Near the big bang, $0 \leq m \leq \frac{1}{2}$ is a more appropriate assumption (Belinskii and Khalatnikov [53]) to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following three cases ($\kappa = 0, 1, \frac{1}{2}$).

A. MODEL I: SOLUTION FOR $\xi = \xi_0$

When $\kappa = 0$, Equation (47) reduces to $\xi = \xi_0$ = constant. Hence in this case Equation (43), with the use of (21) leads to

$$8\pi p = \frac{8\pi(2\ell+1)\xi_0}{(\ell+1)\beta T} + \frac{\ell}{(\ell+1)^2\beta^2T^2} - \frac{m^2K^2}{N^4T^{4L}}. \tag{48}$$

B. MODEL II: SOLUTION FOR $\xi = \xi_0 \rho$

When $\kappa = 1$, Equation (47) reduces to $\xi = \xi_0 \rho$. Hence in this case Eq. (43), with the use of (21) and (44) leads to

$$8\pi p = \frac{(2\ell+1)\xi_0}{(\ell+1)\beta T^3}\left[\frac{(\ell+2)L^2}{\ell\beta^2} - \frac{a^2}{N^2T^{2(L-1)}} - \frac{m^2K^2}{N^4T^{2(2L-1)}}\right] + \frac{\ell}{(\ell+1)^2\beta^2T^2} - \frac{m^2K^2}{N^4T^{4L}}. \tag{49}$$

C. MODEL III: SOLUTION FOR $\xi = \xi_0 \rho^{\frac{1}{2}}$

When $\kappa = \frac{1}{2}$, Eq. (47) reduces to $\xi = \xi_0 \rho^{\frac{1}{2}}$. Hence in this case Eq. (43), with the use of (21) and (44) leads to

$$8\pi p = \frac{\sqrt{8\pi}(2\ell+1)\xi_0}{(\ell+1)\beta T^3}\left[\frac{(\ell+2)L^2}{\ell\beta^2} - \frac{a^2}{N^2T^{2(L-1)}} - \frac{m^2K^2}{N^4T^{2(2L-1)}}\right]^{\frac{1}{2}} + \frac{\ell}{(\ell+1)^2\beta^2T^2} - \frac{m^2K^2}{N^4T^{4L}}. \tag{50}$$

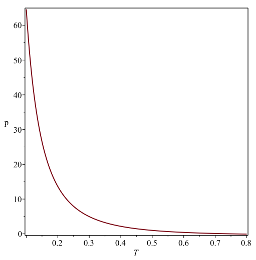

Figure 1. The isotropic pressure p versus T for $L < \frac{1}{2}$

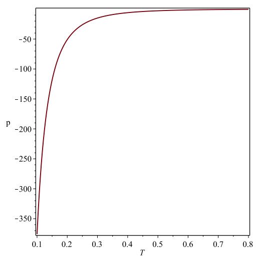

Figure 2. The isotropic pressure p versus T for $L > \frac{1}{2}$

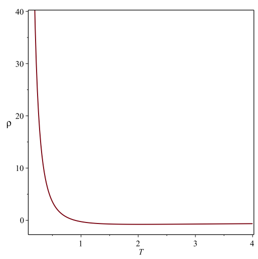

Figure 3. The energy density ρ versus T for $L < \frac{1}{2}$

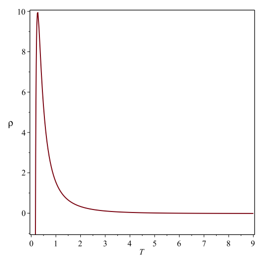

Figure 4. The energy density ρ versus T for $L > \frac{1}{2}$

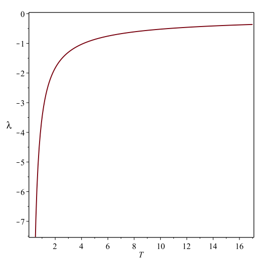

Figure 5. The string tension density λ versus T for $L < \frac{1}{2}$

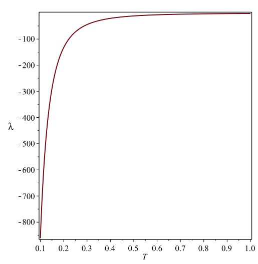

Figure 6. The string tension density λ versus T for $L > \frac{1}{2}$

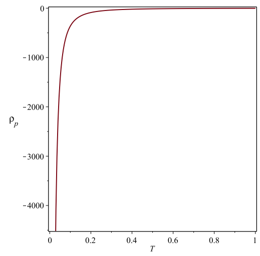

Figure 7. The particle density ρ_p versus T for $L < \frac{1}{2}$

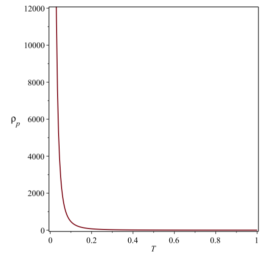

Figure 8. The particle density density ρ_p versus T for $L > \frac{1}{2}$

IV. SOME PHYSICAL AND GEOMETRIC PROPERTIES OF THE MODELS

With regard to the kinematic properties of the velocity vector v^i in the metric (41), a straight forward calculation leads to the following expressions for the scalar of expansion θ, magnitude of shear σ^2, the average anisotropy parameter A_m, deceleration parameter q and proper volume V :

$$\theta = \frac{(2\ell+1)}{(\ell+1)\beta T}, \tag{51}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{(\ell-1)}{(\ell+1)\beta T}\right]^2, \tag{52}$$

$$A_m = 2\left(\frac{\ell-1}{2\ell+1}\right)^2, \tag{53}$$

$$q = -\frac{\ell\beta}{(2\ell+1)}, \tag{54}$$

$$V = \frac{N^2}{m}M^{\frac{1}{\ell}}T^{\frac{L(2\ell+1)}{\ell}}. \tag{55}$$

The rate of expansion H_i in the direction of x, y and z are given by

$$H_1 = H_2 = \frac{\ell}{(\ell+1)\beta T}, \tag{56}$$

$$H_3 = \frac{1}{(\ell+1)\beta T}. \tag{57}$$

Hence the average generalized Hubble's parameter is given by

$$H = \frac{L(2\ell+1)}{3\ell\beta T}. \tag{58}$$

From the above results, it can be seen that the spatial volume is zero at $T = 0$ and it increases with the increase of T. This shows that the universe starts evolving with zero volume at $T = 0$ and expands with cosmic time T. From equations (56)−(58), we observe that all the three directional Hubble parameters are zero at $T \to \infty$. The shear scalar diverges at $T = 0$. As $T \to \infty$, the scale factors $A(t)$, $B(t)$ and $C(t)$ tend to infinity. All the scale factors diverge at $T = 0$, and hence there is a Point Type singularity [54] at $T = 0$. The expansion scalar and shear scalar all tend to zero as $T \to \infty$. The mean anisotropy parameter is uniform throughout whole expansion of the universe when $\ell \neq -\frac{1}{2}$ but for $\ell = -\frac{1}{2}$ it tends to infinity. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. At the initial stage of expansion, when ρ is large, the Hubble parameter is also large and with the expansion of the universe H, θ decrease as does ρ. Since $\frac{\sigma^2}{\theta^2}$ = constant provided $\ell \neq -\frac{1}{2}$, hence the anisotropy is maintained throughout the evolution of the universe. The cosmological evolution of Bianchi type-III space-time is expansionary, with all the three scale factors monotonically increasing function of time. The dynamics of the mean anisotropy parameter depends on the value of ℓ.

From (54) we observe that

$$(i) \qquad \text{for} \quad \ell < -\frac{1}{2}, \quad q > 0$$

i.e., the model is decelerating and

$$(ii) \qquad \text{for} \quad \ell > -\frac{1}{2}, \quad q < 0$$

i.e., the model is accelerating. Thus this case implies an accelerating model of the universe. It follows that our model of the universe is consistent with the recent observations.

From Eq. (44), we observe that $\rho \geq 0$ when

$$T^{2(1-L)}\left(a^2 + \frac{m^2K^2}{N^2T^{2L}}\right) \leq \frac{N^2L^2}{\beta^2}\frac{(\ell+2)}{\ell}. \tag{59}$$

From Eq. (46), we observe that $\rho_p \geq 0$ if $\ell > 2$. Hence the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied under above conditions (59) and $\ell > 2$.

From Eq. (43), it is observed that isotropic pressure p remains negative throughout the evolution as expected which in turn indicates accelerated expansion of the universe. Figures 1 and 2 depict isotropic pressure versus cosmic time for $L < \frac{1}{2}$ and $L > \frac{1}{2}$ respectively. These figures show the negative behaviour of p. The negative pressure makes the expansion tend to speed up. The values of all constants β, m, K, N, and a have been considered equal to 0.1 as a representative case in all figures.

From Eq. (44), it is noted that the rest energy density $\rho(t)$ is a decreasing function of time and it approaches a small positive value at present epoch. This behaviour of energy density is clearly depicted in Figures 3 and 4 for $L < \frac{1}{2}$ and $L > \frac{1}{2}$ respectively. In the first case when $L < \frac{1}{2}$, we note from Fig. 3 that ρ is positive decreasing function of T and it approaches a small constant value near zero. But in the other case when $L > \frac{1}{2}$, we observe from Fig. 4 that ρ is negative in early time and it increases very rapidly in its initial stage in very short time period approaching to maximum positive value at some epoch closer to the early phase of the universe and then decreases with time and follows normal evolution i.e. in the later stage, it decreases from its maximum value with time and approaching to a small positive constant near zero at present epoch.

From Eq. (45), it is found that the tension density λ is negative. Figures 5 and 6 graph string tension density versus T for $L < \frac{1}{2}$ and $L > \frac{1}{2}$ respectively. It is found that in both cases the tension density is negative. It is pointed out by Letelier [9] that λ may be positive or negative. When $\lambda < 0$, the string phase of the universe disappears i.e. we have an anisotropic fluid of particles.

The expression for particle density ρ_p is given in Eq. (46). Figures 7 and 8 describe the variation of string tension density with T for $L < \frac{1}{2}$ and $L > \frac{1}{2}$ respectively. In first case when $L < \frac{1}{2}$, it is observed that ρ_p is negative increasing function of time and it approaches to zero at late time i.e. the string phase of the universe disappears in this case. But in other case when $L > \frac{1}{2}$, we find that ρ_p is always positive and decreasing function of time and it approaches to zero at late time.

When $T \to \infty$), we obtain

$$\frac{\rho_p}{\lambda} = \frac{\ell - 2}{(\ell^2 + \ell + 2)}. \tag{60}$$

We also observe that $L > \frac{1}{2}$ implies $\ell > 1$ whereas $L < \frac{1}{2}$ implies $\ell < 1$. For $\ell = 2$

$$\frac{\rho_p}{\lambda} = 0. \tag{61}$$

According to Kibble [1] and Krori et al. [55], when $\rho_p/ \mid \lambda \mid > 1$, in the process of evolution, the universe is dominated by massive strings, and when $\rho_p/ \mid \lambda \mid < 1$, the universe is dominated by the strings.
Also from Eq. (60), we obtain

$$\frac{\rho_p}{\mid \lambda \mid} < 1,$$

which shows that the universe is dominated by strings in the beginning of evolution of the universe.

Also for $\frac{\ell-2}{\ell^2+\ell+2)} > 1$, we obtain

$$\frac{\rho_p}{\mid \lambda \mid} > 1.$$

which shows that the universe is dominated by the massive strings.

From (61), we note that the strings dominate over the particle at late time when $T \to \infty$).

V. DISCUSSIONS

In this paper we have presented a new exact solution of Einstein's field equations for Bianchi type III space-time with cloud of strings in presence of perfect fluid and decaying vacuum energy density Λ which is different from the other author's solution. In general the model is expanding, shearing and non-rotating. The model starts with a big-bang at $T = 0$ and it goes on expanding until it comes out to rest at $T = \infty$. It is worth mentioned here that $T = 0$ and $T = \infty$ correspond to the proper time $t = 0$ and $t = \infty$ respectively. The initial singularity in the model is the Point Type [54]. Our universe starts evolving with zero volume at $T = 0$ and expand with cosmic time T. We observe that $\frac{\sigma^2}{\theta^2}$ is constant provided $\ell \neq -\frac{1}{2}$, the model does not approach isotropy at any time. Our model is in accelerating phase which is consistent to the recent observations. Thus the model (41) represents a realistic model.

The particle density and the tension density of the string are comparable at the two ends and they fall off asymptotically at similar rate. But in early stage as well as at the late time of the evolution of the universe, we have two types of scenario (i) universe is dominated by massive strings and (ii) universe is dominated by strings depending on the nature of constant L or ℓ. It is observed that $\lambda < 0$ for all time during the evolution of the universe so in our model the string phase of the universe disappears i.e. we have an anisotropic fluid of particles Letelier [9]. In absence of bulk viscosity, our solution represents the solution obtained by Pradhan et al. [30]. Our solutions has a cosmological significance since it is one of theories explaining early universe and it is supposed to be a reasonable representation of the universe at early epoch.

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