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Magnetized Inhomogeneous Universe with Variable Magnetic Permeability and Cosmological Term Λ

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***Abstract*—Magnetized inhomogeneous cosmological models of the universe for perfect fluid distribution of matter with variable magnetic permeability and cosmological term Λ are investigated. The source of the magnetic field is due to an electric current produced along the z-axis. F_{12} is the non-vanishing component of electromagnetic field tensor. To get a deterministic solution, it has been assumed that the expansion θ in the model is proportional to the shear σ. We have considered two cases for solutions (i) when Λ is constant and (ii) when Λ is time dependent. It has been observed that the cosmological term Λ in the solution is found to be consistent with the recent observations. Some physical and geometric properties of the models are also discussed in presence and absence of magnetic field.**

***Index Terms*—Inhomogeneous universe, Variable magnetic permeability, Variable-Λ**
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I. INTRODUCTION

Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have been done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3], [4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7], Szafron and Collins [8]. Recently, Senovilla [9] obtained a new class of exact solutions of Einstein's equations without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruiz and Senovilla [10] have examined a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable function of r and t as metric coefficients. Dadhich et al. [11] have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former. Recently, Patel et al. [12] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background space-time admits two space-like commuting Killing vectors and has separable metric coefficients. Singh, Mehta and Gupta [13] obtained inhomogeneous cosmological models of perfect fluid distribution with electromagnetic field. Recently, Pradhan et al. [14]– [19] have investigated plane-symmetric inhomogeneous cosmological models in various contexts. Cylindrically symmetric space-time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Roy and Singh [20], Bali and Tyagi [21], [22], Chakrabarty et al. [23] and Pradhan et al. [24] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field.

The occurrence of magnetic field on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [25]. Also Harrison [26] has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [27]. The presence of primordial magnetic field in the early stages of the evolution of the universe has been discussed by several authors [28]– [37]. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than the case when the pressure was isotropic [38], [39]. Such fields can be generated at the end of an inflationary epoch [40]– [44]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [45] obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Chakrabarty et al. [23] and Pradhan et al. [46]– [51] have investigated magnetized viscous fluid cosmological models in various contexts.

Maxwell considered the magnetic permeability ($\bar{\mu}$) to be a constant for a given material. Maxwell considered the spatial gradient of the magnetic field intensity in the steady state to be exclusively determined by a variation in the velocity of the molecular vortices within the magnetic lines of force. To this day, it is assumed that the magnetic permeability is a constant for a given material. But from 'The Double Helix Theory of the Magnetic Field' [52], we must look to a variable magnetic permeability in order to account for variations in magnetic flux density in the steady state, and if we look at the solenoidal magnetic field pattern around a bar magnet, this is not very difficult to visualize. The magnetic field lines are clearly more concentrated at the poles of the magnet than elsewhere. It should be quite obvious that the density of the vortex sea, as denoted by the quantity $\bar{\mu}$, is a variable quantity and that this density visibly varies according to how tightly the magnetic lines of force are packed together [53]. Bali [54], [55] obtained Bianchi type-V magnetized string dust cosmological models in perfect and bulk viscous fluid respectively with variable magnetic permeability. Tyagi and Sharma [56] investigated Bianchi type-V magnetized string cosmological model with variable magnetic permeability in presence of bulk viscosity. Recently, Pradhan and Ram [57] and Ali and Rahaman [58] have discussed respectively a plane-symmetric and Bianchi type-I inhomogeneous cosmological models of perfect fluid with variable magnetic permeability.

Motivated by the situation discussed above, in this paper, we have obtained a new magnetized inhomogeneous cosmological model for perfect fluid distribution in presence of electromagnetic field. This paper is organized as follows. The introduction and motivation are laid down in Sec. 1. The metric and the field equations are given in Sec. 2. In Sec. 3, solution representing inhomogeneous cosmological model with perfect fluid are obtained imposing the condition when the expansion θ in the model is proportional to the shear σ. Some physical and geometric behaviour of the model in presence of variable magnetic permeability are discussed in Sec. 4. Section 5 describes the solution of field equations when Λ is time dependent. Concluding remarks are given in the last Sec. 6.

II. THE METRIC AND FIELD EQUATIONS

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \tag{1}$$

where A is the function of t alone and B and C are functions of x and t. The energy momentum tensor is taken as has the form

$$T_i^j = (\rho + p)u_iu^j + pg_i^j + E_i^j, \tag{2}$$

where ρ and p are, respectively, the energy density and pressure of the cosmic fluid, and u_i is the fluid four-velocity vector satisfying the condition

$$u^iu_i = -1, \quad u^ix_i = 0. \tag{3}$$

In Eq. (2), E_i^j is the electromagnetic field given by Lichnerowicz [59]

$$E_i^j = \bar{\mu}\left[h_lh^l\left(u_iu^j + \frac{1}{2}g_i^j\right) - h_ih^j\right], \tag{4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}}\,{}^*F_{ji}u^j, \tag{5}$$

where the dual electromagnetic field tensor ${}^*F_{ij}$ is defined by Synge [60]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl}. \tag{6}$$

Here F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density.

The co-ordinates are considered to be comoving so that $u^1 = 0 = u^2 = u^3$ and $u^4 = \frac{1}{A}$. If we consider that the current flows along the z-axis, then F_{12} is the only non-vanishing component of F_{ij}. The Maxwell's equations

$$F_{[ij;k]} = 0, \tag{7}$$

$$\left[\frac{1}{\bar{\mu}}F^{ij}\right]_{;j} = 0, \tag{8}$$

require that F_{12} is the function of x-alone. We assume that the magnetic permeability is the functions of x and t both. Here the semicolon represents a covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{9}$$

for the line-element (1) lead to the following system of equations:

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_4C_4}{BC} + \frac{B_1C_1}{BC}\right] - \Lambda = 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{10}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{C_{11}}{C}\right) - \Lambda = 8\pi\left(p + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{11}$$

$$\frac{1}{A^2}\left(\frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{B_{11}}{B}\right) - \Lambda = 8\pi\left(p - \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{12}$$

$$\frac{1}{A^2}\left[-\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC}\right] + \Lambda = 8\pi\left(\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{13}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) = 0, \tag{14}$$

where the sub indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to x and t respectively.

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III. SOLUTION OF FIELD EQUATIONS WHEN Λ IS CONSTANT

Equations (10)−(14) are five independent equations in six unknowns A, B, C, ρ, p and F_{12}. For the complete determinacy of the system, we need one extra condition. The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations.

To get determinate solution we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$A = \left(\frac{B}{C}\right)^n, \tag{15}$$

where n is a constant. From Eqs. (10)−(12), we have

$$\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{B_{44}}{B} - \frac{B_4C_4}{BC} = \frac{C_{11}}{C} - \frac{B_1C_1}{BC} = \text{K (constant)} \tag{16}$$

and

$$\frac{8\pi F_{12}^2}{\bar{\mu}B^2} = -\frac{C_{44}}{C} + \frac{C_{11}}{C} + \frac{B_{44}}{B} - \frac{B_{11}}{B}. \tag{17}$$

We also assume that

$$B = f(x)g(t)$$

$$C = f(x)k(t). \tag{18}$$

Using Eqs. (15) and (18) in (14) and (16) lead to

$$\frac{k_4}{k} = \frac{(2n-1)}{(2n+1)}\frac{g_4}{g}, \tag{19}$$

$$(n-1)\frac{g_{44}}{g} - n\frac{k_{44}}{k} - \frac{g_4}{g}\frac{k_4}{k} = K, \tag{20}$$

$$ff_{11} - f_1^2 = Kf^2. \tag{21}$$

Equation (19) leads to

$$k = cg^{\alpha}, \tag{22}$$

where $\alpha = \frac{2n-1}{2n+1}$ and c is the constant of integration. From Eqs. (20) and (22), we have

$$\frac{g_{44}}{g} + \beta\frac{g_4^2}{g^2} = N, \tag{23}$$

where

$$\beta = \frac{n\alpha(\alpha-1)+\alpha}{n(\alpha-1)+1},\ N = \frac{K}{n(1-\alpha)-1}.$$

Equation (21) leads to

$$f = \exp\left(\frac{1}{2}K(x+x_0)^2\right), \tag{24}$$

where x_0 is an integrating constant. Equation (23) leads to

$$g = \left(c_1e^{bt} + c_2e^{-bt}\right)^{\frac{1}{(\beta+1)}}, \tag{25}$$

where $b = \sqrt{(\beta+1)N}$. Hence from (22) and (25), we have

$$k = c\left(c_1e^{bt} + c_2e^{-bt}\right)^{\frac{\alpha}{(\beta+1)}}. \tag{26}$$

Therefore we obtain

$$B = \exp\left(\frac{1}{2}K(x+x_0)^2\right)\left(c_1e^{bt} + c_2e^{-bt}\right)^{\frac{1}{(\beta+1)}}, \tag{27}$$

$$C = \exp\left(\frac{1}{2}K(x+x_0)^2\right)c\left(c_1e^{bt} + c_2e^{-bt}\right)^{\frac{\alpha}{(\beta+1)}}, \tag{28}$$

$$A = a\left(c_1e^{bt} + c_2e^{-bt}\right)^{\frac{n(1-\alpha)}{(\beta+1)}}, \tag{29}$$

where $a = \frac{c_3}{c}$, c_3 being a constant of integration.

After using suitable transformation of the co-ordinates, the model (1) reduces to the form

$$ds^2 = a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}}(dX^2 - dT^2) + e^{KX^2}(c_1e^{bT} + c_2e^{-bT})^{\frac{2}{(\beta+1)}}dY^2$$

$$+e^{KX^2}(c_1e^{bT} + c_2e^{-bT})^{\frac{2\alpha}{(\beta+1)}}dZ^2, \tag{30}$$

where $x + x_0 = X$, $t = T$, $y = Y$, $cz = Z$.

IV. SOME PHYSICAL AND GEOMETRIC FEATURES OF THE SOLUTIONS

The expressions for pressure p and density ρ for the model (30) are given by

$$8\pi p = \frac{1}{a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}}}\left[\frac{b^2\{2n(1-\alpha^2) + 2\beta + 2\alpha(\beta-\alpha)(1-\alpha)\}}{2(\beta+1)^2}\times\right.$$

$$\left.\frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2} - \frac{b^2(3\alpha+1)}{2(\beta+1)} + K^2X^2\right] - \Lambda, \tag{31}$$

$$8\pi\rho = \frac{1}{a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}}}\left[\frac{b^2\{2n(1-\alpha^2) + 2\alpha + (\beta-\alpha)(1-\alpha)\}}{2(\beta+1)^2}\times\right.$$

$$\left.\frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2} - \frac{b^2(1-\alpha)}{2(\beta+1)} - K(2 + 3KX^2)\right] + \Lambda. \tag{32}$$

Since the magnetic permeability is a variable quantity, we have assumed it as

$$\bar{\mu} = \frac{(c_1e^{bT} + c_2e^{-bT})^{-\frac{2}{(\beta+1)}}}{\left[1 - \frac{(\beta-\alpha)}{(\beta+1)}\frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2}\right]} \tag{33}$$

Thus $\bar{\mu} \to 0$ as $T \to \infty$ and $\bar{\mu} = 1$ when $T \to 0$. Zel'dovich [61] has explained that $\rho_s/\rho_c \sim 2.5 \times 10^{-3}$, where ρ_s is the mass density and ρ_c the critical density then the bodies frozen in plasma would change their density like a^{-2} i.e. like t^{-1} in the radiation dominated universe where a is the radius of the universe. The non-vanishing component F_{12} of electromagnetic field tensor is obtained as

$$F_{12}^2 = \frac{b^2(1-\alpha)}{8\pi(\beta+1)}e^{KX^2}, \tag{34}$$

which is the function of X alone. So it is consistent as the Maxwell's equations (7) and (8) require F_{12} to be function of x alone.

The expressions for the expansion θ, shear scalar σ^2, acceleration vector $\dot{u}_i$ and proper volume V^3 are given by

$$\theta = \frac{b\{n(1-\alpha) + (1+\alpha)\}}{(\beta+1)a(c_1e^{bT} + c_2e^{-bT})^{\frac{n(1-\alpha)}{(\beta+1)}}}\frac{(c_1e^{bT} - c_2e^{-bT})}{(c_1e^{bT} + c_2e^{-bT})}, \tag{35}$$

$$\sigma^2 = \frac{b^2\left[\{n(1-\alpha) + (1+\alpha)\}^2 - 3n(1-\alpha)(1+\alpha) - 3\alpha\right]}{3(\beta+1)^2a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}}}\frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2}, \tag{36}$$

$$\dot{u}_i = (0, 0, 0, 0), \tag{37}$$

$$V^3 = \sqrt{-g} = a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(1-\alpha)+(1+\alpha)}{(\beta+1)}}, \tag{38}$$

From Eqs. (35) and (36), we have

$$\frac{\sigma^2}{\theta^2} = \frac{\left[\{n(1-\alpha) + (1+\alpha)\}^2 - 3n(1-\alpha^2) - 3\alpha\right]}{3\{n(1-\alpha) + (1+\alpha)\}^2} = \text{constant}. \tag{39}$$

The rotation ω is identically zero.

The dominant energy conditions (Hawking and Ellis [62])

$$(i)\ \ \rho - p \geq 0 \quad (ii)\ \ \rho + p \geq 0$$

lead to

$$\frac{b^2(\alpha-\beta)\{2-(1-\alpha)(1-2\alpha)\}}{2(\beta+1)^2}\frac{(c_1e^{bT}-c_2e^{-bT})^2}{(c_1e^{bT}+c_2e^{-bT})^2}+\frac{2\alpha\beta}{(\beta+1)}-$$

$$2K(1+2KX^2)+2\Lambda a^2(c_1e^{bT}+c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}} \geq 0, \tag{40}$$

and

$$\frac{b^2}{2(\beta+1)^2}\left[(1-\alpha)\{4n(1+\alpha)+(\beta-\alpha)(1+2\alpha)\}+2(\alpha+\beta)\right]\times$$

$$\frac{(c_1e^{bT}-c_2e^{-bT})^2}{(c_1e^{bT}+c_2e^{-bT})^2}-\frac{b^2(1+\alpha)}{(\beta+1)}-2K(1+KX^2) \geq 0. \tag{41}$$

The reality conditions (Ellis [63])

$$(i)\ \ \rho + p > 0, \quad (ii)\ \ \rho + 3p > 0,$$

lead to

$$\frac{b^2}{2(\beta+1)^2}\left[(1-\alpha)\{4n(1+\alpha)+(\beta-\alpha)(1+2\alpha)\}+2(\alpha+\beta)\right]\times$$

$$\frac{(c_1e^{bT}-c_2e^{-bT})^2}{(c_1e^{bT}+c_2e^{-bT})^2}-\frac{b^2(1+\alpha)}{(\beta+1)}-2K(1+KX^2) > 0, \tag{42}$$

and

$$\frac{b^2}{2(\beta+1)^2}\left[(1-\alpha)\{8n(1+\alpha)+(\beta-\alpha)(1+6\alpha)\}+2(\alpha+3\beta)\right]\times$$

$$\frac{(c_1e^{bT}-c_2e^{-bT})^2}{(c_1e^{bT}+c_2e^{-bT})^2}-\frac{b^2(2\alpha+1)}{(\beta+1)}-2K-2\Lambda a^2(c_1e^{bT}+c_2e^{-bT})^{\frac{2n(1-\alpha)}{(\beta+1)}} > 0. \tag{43}$$

The condition (40) imposes a restriction on Λ. The model starts expanding at $T > 0$ and goes on expanding indefinitely when $\frac{n(1-\alpha)}{(\beta+1)} < 0$. The model (30) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy. As T increases the proper volume also increases. The model is non-accelerating. The physical quantities p and ρ decrease as F_{12} increases. However, if $\frac{n(1-\alpha)}{(\beta+1)} > 0$, the process of contraction starts at $T > 0$ and at $T = \infty$ the expansion stops. The electromagnetic field tensor does not vanish when $b \neq 0$, and $\alpha \neq 1$.

V. SOLUTION OF FIELD EQUATIONS WHEN Λ IS TIME DEPENDENT

There are significant observational evidence for the detection of Einstein's cosmological constant, Λ or a component of material content of the universe that varies slowly with time and space to act like Λ. For recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant, see the references [64−68]. There is a plethora of astrophysical evidence today, from supernovae measurements (Perlmutter et al. [69], Riess et al. [70]–[72]), the spectrum of fluctuations in the Cosmic Microwave Background (CMB) [73], baryon oscillations [74] and other astrophysical data, indicating that the expansion of the universe is currently accelerating. These recent observations strongly favour a significant and a positive value of Λ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$.

In absence of matter described by the stress energy tensor T_{ij}, Λ must be constant, since the Bianchi identities guarantee vanishing covariant divergence of the Einstein tensor, $G^{ij}_{;j} = 0$, while $g^{ij}_{;j} = 0$ by definition. If Hubble parameter and age of the universe as measured from high red-shift would be found to satisfy the bound $H_0t_0 > 1$ (index zero labels values today), it would require a term in the expansion rate equation that acts as a cosmological constant. Therefore the definitive measurement of $H_0t_0 > 1$ and wide rage of observations would necessitate a non-zero cosmological constant today or the abandonment of the standard big bang cosmology [75]. However, a constant Λ, as it was originally introduced by Einstein in 1917, cannot explain why the calculated value of vacuum energy density at Plank epoch following quantum field theory is 123 orders of magnitude larger than its value as observed or as predicted by standard cosmology at the present epoch [76]. In attempt to solve this problem, variable Λ was introduced such that Λ was larger in the early universe and then decayed with the evolution [77]. The idea that Λ might be variable has been studied for more than two decades (see [78], [79] and references therein). Linde [80] has suggested that Λ is a function of temperature and is related to the process of broken symmetries. Therefore, it could be a function of time in a spatially homogeneous, expanding universe [74]. In a paper on Λ-variability, Overduin and Cooperstock [81] suggested that Λg_{ij} is shifted onto the right-hand side of the Einstein field equation and treated as part of the matter content. In general relativity, Λ can be regarded as a measure of the energy density of the vacuum and can in principle lead to the avoidance of the big bang singularity that is characterized of other FRW models. However, the rather simplistic properties of the vacuum that follows from the usual form of Einstein equations can be made more realistic if that theory is extended, which in general leads to a variable Λ. Recently, Overduin [82], [83] has given an account of variable Λ-models that have a non-singular origin. Liu and Wesson [84] have studied universe models with variable cosmological constant. Podariu and Ratra [85] have examined the consequences of also incorporating constraints from recent measurements of the Hubble parameter and the age of the universe in the constant and time-variable cosmological constant models. Recently, Pradhan et al. [86−88] have studied Bianchi type-I cosmological models with time dependent cosmological term-Λ in different context.

For the specification of $\Lambda(t)$, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{44}$$

where γ $(0 \leq \gamma \leq 1)$ is a constant.

Using (44) in (31) and (32), we obtain

$$\rho = \frac{a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(\alpha-1)}{(\beta+1)}}}{8\pi(1+\gamma}\left[\frac{b^2\{4n(1-\alpha^2) + (1-\alpha)(\beta-\alpha)\}}{2(\beta+1)^2}\times \frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2} - \frac{b^2(\alpha+1)}{(\beta+1)} - 2K(2KX^2+1)\right], \tag{45}$$

and

$$\Lambda = \frac{a^2(c_1e^{bT} + c_2e^{-bT})^{\frac{2n(\alpha-1)}{(\beta+1)}}}{(1+\gamma}\left[(K_1 - \gamma K_2)\times\frac{(c_1e^{bT} - c_2e^{-bT})^2}{(c_1e^{bT} + c_2e^{-bT})^2} - \frac{b^2\{(3\alpha+1) - (1-\alpha)\gamma\}}{2(\beta+1)} - K\{KX^2 - \gamma(2+3KX^2)\}\right], \tag{46}$$

where

$$K_1 = \frac{b^2\{2n(1-\alpha^2) + 2\beta + 2\alpha(\beta-\alpha)(1-\alpha)\}}{2(\beta+1)^2},$$

$$K_2 = \frac{b^2\{2n(1-\alpha^2) + 2\alpha + (\beta-\alpha)(1-\alpha)\}}{2(\beta+1)^2}.$$

From Eq. (45), it is observed that the rest energy density ρ is a decreasing function of time and $\rho > 0$ always. The behaviour of the universe in this model will be determined by the cosmological term Λ, this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda$ which is constant in time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ the expansion will tend to accelerate whereas in the universe with negative value of Λ the expansion will slow down, stop and reverse. In a universe with both matter and vacuum energy, there is a competition between the tendency of Λ to cause acceleration and the tendency of matter to cause deceleration with the ultimate fate of the universe depending on the precise amounts of each component. This continues to be true in the presence of spatial curvature, and with a nonzero cosmological constant it is no longer true that the negatively curved ("open") universes expand indefinitely while positively curved ("closed") universes will necessarily re-collapse - each of the four combinations of negative or positive curvature and eternal expansion or eventual re-collapse become possible for appropriate values of the parameters. There may even be a delicate balance, in which the competition between matter and vacuum energy is needed drawn and the universe is static (non expanding). The search for such a solution was Einstein's original motivation for introducing the cosmological constant. From Eq. (46), it is observed that the cosmological constant Λ is a decreasing function of time and it approaches a small positive value at late time.

VI. CONCLUSION

We have obtained a new inhomogeneous cosmological model of electro-magnetic perfect fluid as the source of matter with variable magnetic permeability. We have solved the field equations for two cases: (i) when Λ is constant and (ii) when Λ is time dependent. Generally the models (30) represent expanding, shearing and non-rotating universe in which the flow vector is geodetic. In the derived model $\frac{\sigma}{\theta} =$ constant and hence it does not approach isotropy. The model is non-accelerating. The observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. But this does not rule out the decelerating ones which are also consistent with these observations (Vishwakarma [89]). Thus the nature of Λ in our derived models are supported by recent observations. In this solution all physical quantities depend on at most one space co-ordinate and time. It is important to note here that the model (30) in presence of magnetic field reduces to homogeneous universe when $K = 0$. This shows that for $K = 0$, inhomogeneity dies out.

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