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Comments on the Papers by Selleri and by Klauber Published in Foundations of Physics

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ABSTRACT

The explanation of the null result in the Michelson-Morley class of experiments appears very early and often in literature. Unfortunately, the explanations resort invariably to inertial frames in linear relative motion while the experiment was actually executed in a rotating frame. In the current paper we clarify several errors and misconceptions introduced by Selleri [10] and later by Klauber [8,9] on the subject of the application of rotating frames to the explanation of the null result of the experiments. We also clear the confusion introduced by Selleri and reprised by Klauber on the alleged anisotropy of light speed in a rotating frame.

Keywords: *length contraction, time dilation, general Lorentz transforms for uniformly rotating frame*

1. LENGTH CONTRACTION IN UNIFORMLY ROTATING FRAMES

Consider that the frame of reference S is rotating with a constant angular speed ω describing a circle of radius r with the origin coincident with the inertial frame S'.

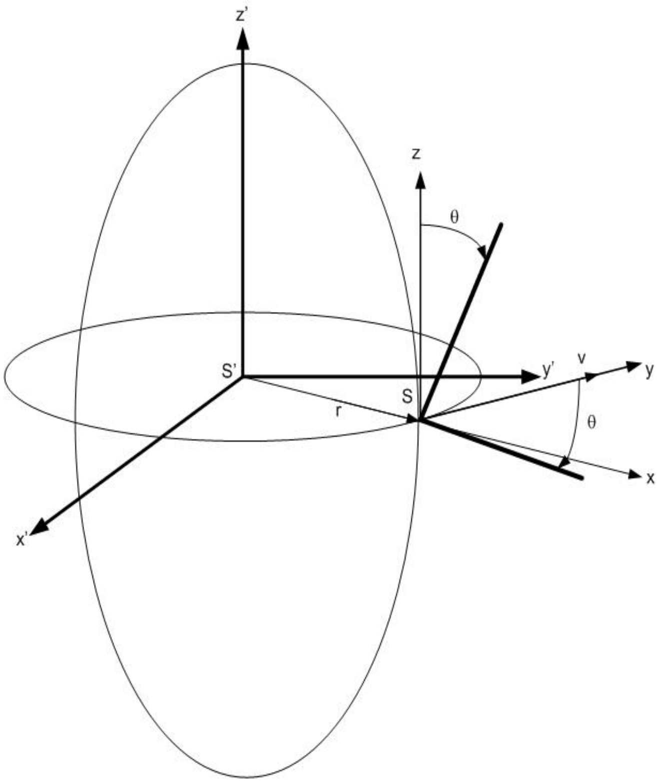

Fig 1: The Michelson-Morley experiment in rotating frames

For an arbitrary orientation of the rod (see **Appendix**) in the rotating frame S, as seen in fig.1:

$$dy' = l_0 \cos\theta \sqrt{1-(\frac{\omega R}{c})^2}$$
$$dz' = l_0 \cos\theta$$

(1.1)

where l_0 is the proper length of the rod in frame S and θ is the angle between the rod and the y-axis.

We must also answer the question of light isotropy in rotating frames. We start with the fact that in the **inertial** frame S', light propagates with speed c along a null geodesic:

$$dx'^2 + dy'^2 + dz'^2 - (cdt')^2 = 0$$

(1.2)

The coordinate transforms [1,2] expressed in terms of infinitesimal quantities are:

$$dx = dx'\cos\gamma\omega t' - \gamma dy'\sin\gamma\omega t' - R\gamma\omega\sin\gamma\omega t'\,dt'$$
$$dy = dx'\sin\gamma\omega t' + \gamma dy'\cos\gamma\omega t' + R\gamma\omega\cos\gamma\omega t'\,dt'$$
$$dz = dz'$$
$$dt = \gamma(dt' + \frac{R\omega}{c^2}dy') \tag{1.3}$$

When making time measurements in a given point in S' $dy' = 0$ so, we recover the time dilation formula for linear uniform motion [2]:

$$dt = \gamma dt' \tag{1.4}$$

Substituting (1.3) into (1.2), we obtain:

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = dx'^2 + dy'^2 + dz'^2 - c^2dt'^2 \tag{1.5}$$

In other words, the light speed measured in the rotating frame S in **a small vicinity** of the origin is also c.

This demonstration, supported by experiment [11-17], contradicts the earlier claims made by Klauber [8,9] and Selleri [10]. Klauber goes so far as to claim [9] that there is "No Lorentz contraction" and the "Lack of invariance of the speed of light", both claim clearly incorrect. On these false conclusions Klauber [9] builds a justification as to why the Michelson-Morley experiment should "yield" a non-null result, similar to the one found experimentally by Dayton Miller.

Now we have all the tools to attack the explanation of the Michelson-Morley experiment as viewed from the rotating frame of the lab.

2. THE EXPERIMENTAL SETUP

Let's derive the equations of the Michelson-Morley in rotating frames. In the Earth centered frame S' the light path along the two arms of the interferometer is calculated by assuming that the arms of the interferometer have the same length, l_0, in the lab frame S.

a. The calculation of the time differential in the interferometer leg making the angle θ with respect to the y-axis uses the fact that while the light travels a path of length $l_2' = dy' = l_0\cos\theta\sqrt{1-(\frac{v}{c})^2}$ the end of the interferometer arm moves at a speed $\pm v$ with respect to the light wave front:

$$\Delta t_2' = \frac{l_2'}{c+v} + \frac{l_2'}{c-v} = \frac{2l_0\gamma(v)\cos\theta}{c} \tag{2.1}$$

b. The calculation of the time differential in the interferometer leg making the angle θ with respect to the z-axis uses the fact that the light path follows a triangular path described by:

$$(ct_1')^2 = l_1'^2 + (vt_1')^2 \tag{2.2}$$

where, according to (1.1), $l_1' = dz' = dz = l_0\cos\theta$, so:

$$\Delta t_1' = 2t_1' = \frac{2l_0\gamma(v)\cos\theta}{c} \tag{2.3}$$

The phase difference between the two arms in the Earth centered frame is:

$$\Delta t' = \Delta t_2' - \Delta t_1' = 0 \tag{2.4}$$

From (2.4) and (1.4) it follows that the phase differential is also null in the lab frame S. Much argument [3-9] has been made in the past about the presence and/or absence of a "signal" in the Michelson-Morley experiment due to the uniform rotation. We can now firmly demonstrate that special relativity predicts no such signal but rather a null result. The "signal" in the Dayton-Miller experiment [3,4] was due to noise being improperly interpreted as signal due to the absence of error bar calculation [6].

In practice the angle between the interferometer arms may be slightly different from 90^0, differing by an angle δ_θ, resulting into a non-null phase differential:

$$\Delta t' = \frac{2l_0\gamma(v)}{c}(\cos\theta - \cos(\theta+\delta_\theta)) = \frac{2l_0\gamma(v)\delta_\theta}{c}\sin\theta \tag{2.5}$$

In the case of the Dayton Miller experiments [3,4], the rotation $\theta = \Omega t$ of the setup results into a sinusoidal variation with time of the phase difference:

$$\Delta t' = \frac{2l_0\gamma(v)\delta_\theta}{c}\sin\Omega t \tag{2.6}$$

The larger the length of the interferometer arms, the more significant the error. Additionally, any inequality

δ_l between the lengths of the two arms also reflects immediately into a phase difference:

$$\Delta t^{'} = \frac{2\gamma(v)\delta_l}{c}\cos\Omega t \quad (2.7)$$

Thus, the worst case departure from the null result due to equipment imperfections is:

$$\Delta t^{'} = \frac{2\gamma(v)}{c}(l_0\delta_\theta \sin\Omega t + \delta_l \cos\Omega t) \quad (2.8)$$

Again, Klauber [9] misinterprets the errors introduced by this type of equipment imperfections in the Brillet-Hall experiment [18] as a signal in the vein of the one present in the Dayton Miller experiments. Given the errors in his derivations, such a conclusion is not surprising. Due to the advancements in equipment technology, neither of these effects has a significant contribution in the modern tests [11-17] using cavity resonators.

3. CONCLUSIONS

Using the predictions of special relativity applied to rotating frames we have derived the theoretical explanation for the null result of the Michelson-Morley experiment for the realistic case of a uniformly rotating lab and we have cleared the misconceptions introduced by the works of Selleri and Klauber.

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APPENDIX

The vector approach allows for generalizing the length contraction equations to the case when the axis of frames S and S' have an arbitrary orientation with respect to each other.

$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(\frac{\gamma - 1}{v^2}\mathbf{r.v} - \gamma t) \quad \text{(A.1)}$$

$$t' = \gamma(t - \frac{\mathbf{r.v}}{c^2}) \quad \text{(A.2)}$$

where $\mathbf{r}'$ is the positional vector of an arbitrary point in S'.

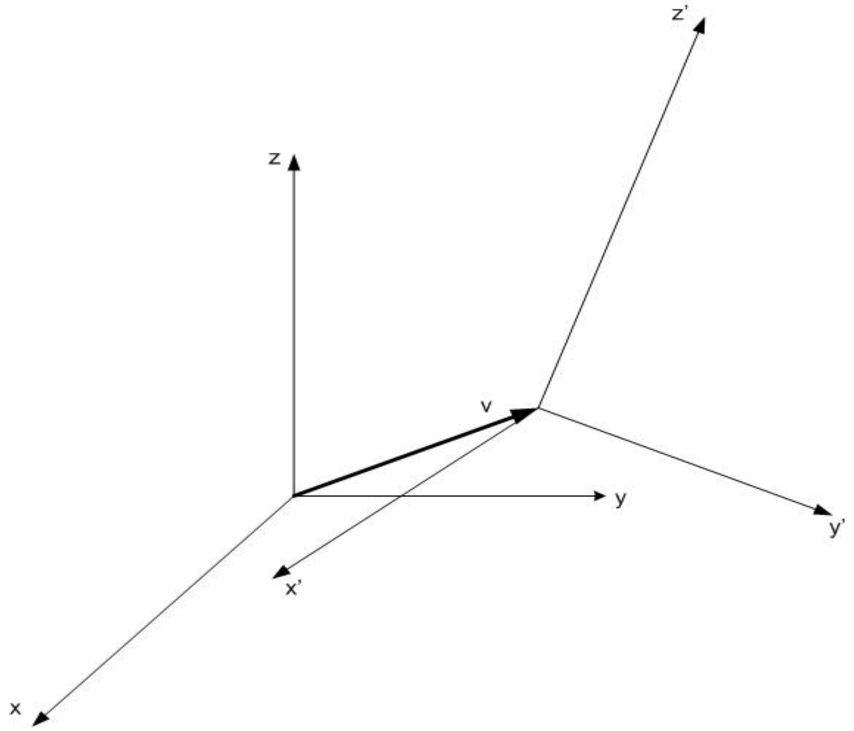

Fig 2: Arbitrary frames with misaligned axes

In frame S' we need to mark both ends of the rod simultaneously in order to perform the length measurement, so $\Delta t' = 0$ .Thus:

$$\Delta t = \frac{\Delta \mathbf{r.v}}{c^2} \quad \text{(A.3)}$$

$$\Delta \mathbf{r}' = \Delta \mathbf{r} + \mathbf{v}(\frac{\gamma - 1}{v^2}\Delta \mathbf{r.v} - \gamma \Delta t) = \Delta \mathbf{r} + \mathbf{v}(\frac{\gamma - 1}{v^2} - \frac{\gamma}{c^2})\Delta \mathbf{r.v} \quad \text{(A.4)}$$

where $\Delta \mathbf{r}'$ is the vector connecting two arbitrary points on S'.

$$\Delta \mathbf{r}' = \Delta \mathbf{r} + \mathbf{v}\frac{\gamma^{-1} - 1}{v^2}\Delta \mathbf{r.v} \quad \text{(A.5)}$$

We can decompose $\Delta \mathbf{r}$ into $\Delta \mathbf{r}_{\parallel} \parallel \mathbf{v}$ and $\Delta \mathbf{r}_{\perp} \perp \mathbf{v}$:

$$\Delta \mathbf{r}' = \Delta \mathbf{r}_{\parallel} + \Delta \mathbf{r}_{\perp} + \mathbf{v}\frac{\gamma^{-1} - 1}{v^2} v \Delta r_{\parallel} =$$

$$= \Delta \mathbf{r}_{\parallel} + \Delta \mathbf{r}_{\perp} + (\gamma^{-1} - 1)\frac{\mathbf{v}}{v}\Delta r_{\parallel} = \Delta \mathbf{r}_{\perp} + \frac{\Delta \mathbf{r}_{\parallel}}{\gamma} \quad \text{(A.6)}$$

Thus:

$$\Delta r'^2 = \Delta r_{\perp}{}^2 + \frac{\Delta r_{\parallel}^2}{\gamma^2} \quad \text{(A.7)}$$

For an arbitrary orientation of a rod of proper length L located in the frame S, $r_{\parallel} = L\cos\alpha, r_{\perp} = L\sin\alpha$, so (A.7) can be rewritten as

$$L' = L\sqrt{1 - (\frac{v\cos\alpha}{c})^2} \quad \text{(A.8)}$$

To conclude, we compute the time dilation as simply:

$$\Delta t' = \gamma(\Delta t - \frac{\Delta \mathbf{r.v}}{c^2}) = \gamma(\Delta t - 0) = \gamma \Delta t \tag{A.9}$$

We can extend the above to rotating frames by considering that the reference frame S is rotating with a constant angular speed describing a circle with the origin coincident with the origin of the inertial frame S'. The coordinate transformations between S and S' are:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\gamma\omega t & -\gamma\sin\gamma\omega t \\ \sin\gamma\omega t & \gamma\cos\gamma\omega t \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + R \begin{bmatrix} \cos\gamma\omega t - 1 \\ \sin\gamma\omega t \end{bmatrix} \tag{A.10}$$

$$z' = z \tag{A.11}$$

$$t' = \gamma(t + \frac{\omega R y}{c^2}) \tag{A.12}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{\omega^2 R^2}{c^2}}}$

Consider a rod of length L situated at $x = y = 0$, in frame S. From the perspective of an observer in the inertial frame S' we need to mark both ends of the rod simultaneously, so $dt' = 0$. For simplicity, the measurement is executed at $t' = 0$. This implies immediately that $t = 0$. Thus:

$$dx' = dx\cos\gamma\omega t - \gamma dy\sin\gamma\omega t - R\gamma\omega\sin\gamma\omega t dt$$
$$dy' = dx\sin\gamma\omega t + \gamma dy\cos\gamma\omega t + R\gamma\omega\cos\gamma\omega t dt$$
$$dt = -\frac{R\omega}{c^2}dy \tag{A.13}$$

For $x = y = t = 0$:

$$dx' = dx$$
$$dy' = \gamma dy + R\gamma\omega dt = \gamma dy(1 - \frac{R^2\omega^2}{c^2}) = \frac{dy}{\gamma} \tag{A.14}$$

The above result coincides with the one obtained by the authors of [19].