

Beyond Gravitoelectromagnetism: Critical Speed in Gravitational Motion

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ABSTRACT

In the present review we will present a rebuttal of a recent paper by Mashhoon published in ", IJMP D, 14,12,(2005) pp. 2025-2037. We will rebut the idea put forward by the author that there is such a thing as a "critical speed" v_c during the fall of a test particle where the "the gravitational attraction turns to repulsion". The conclusion drawn by Mashhoon is unphysical, there is no such thing as a gravitational repulsion. The root of the error can be found in the author basing his derivation on coordinate acceleration. The correct analysis should have used proper acceleration. We demonstrate that, contrary to the author's claims, there is no such thing as gravitational "repulsion".

Keywords: *General relativity, Schwarzschild metric, Euler-Lagrange formalism*

1. ANALYSIS AND DISPROOF OF MASHHOON'S CONCLUSIONS VIA THE EULER-LAGRANGE FORMALISM

Exactly as in², in order to find the equations of motion of a test particle moving radially in a gravitational field we start with the Schwarzschild metric for the particular case of absence of rotation ($d\theta = d\phi = 0$). Throughout this note we will use the formalism and the results developed in². We start with the simplified metric³:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{1}{1 - \frac{r_s}{r}} dr^2 \quad (1)$$

where r_s is the Schwarzschild radius. From the metric we obtained² the proper acceleration:

$$\frac{d^2 r}{ds^2} = -\frac{r_s}{2r^2} \quad (2)$$

From (2) we can see that the acceleration increases monotonically as the radial coordinate decreases. The proper speed for a test particle dropped from infinity is derived² by integrating (3):

$$\frac{dr}{ds} = \sqrt{\frac{r_s}{r}} \quad (3)$$

From (3) we can see that the proper speed increases monotonically as well as the radial coordinate decreases. This is in line with our knowledge derived from Newtonian mechanics. For the test particle dropped from infinity the coordinate acceleration is²:

$$\frac{d^2 r}{dt^2} = -\frac{r_s}{2r^2} \left(1 - \frac{r_s}{r}\right) \left(1 - \frac{3r_s}{r}\right) \quad (4)$$

while the corresponding coordinate speed is:

$$\frac{dr}{dt} = \left(1 - \frac{r_s}{r}\right) \sqrt{\frac{r_s}{r}} \quad (5)$$

At $r = 3r_s$ the coordinate speed reaches a maximum:

$$\left. \frac{dr}{dt} \right|_{r=3r_s} = \frac{2}{3\sqrt{3}} \quad (6)$$

The corresponding proper speed for $r = 3r_s$ is:

$$\left. \frac{dr}{ds} \right|_{r=3r_s} = \frac{1}{\sqrt{3}} = v_c \quad (7)$$

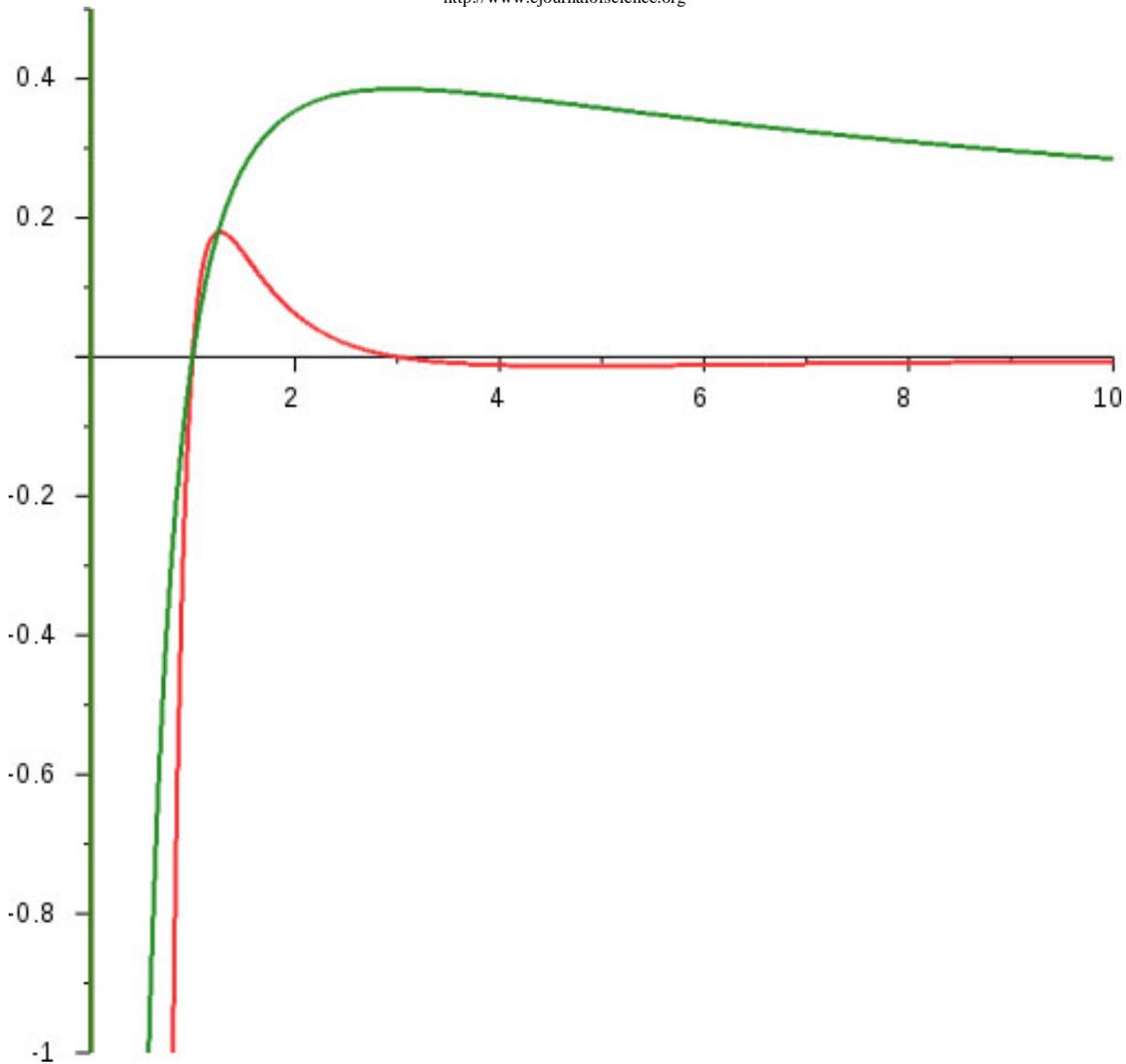


Fig 1: Coordinate acceleration (in red) and coordinate speed (in green)

From (4) we see that the coordinate acceleration changes sign, from positive to negative, at $r = 3r_s$. The radial coordinate $r = 3r_s$ coincides with the point where the proper speed equals what Mashhoon calls “critical speed” $v_c = 1/\sqrt{3}$. The problem with Mashhoon’s paper is that the author is drawing his conclusions based on the change of sign of **coordinate** acceleration instead of analyzing the behavior of **proper** acceleration. The coordinate acceleration and coordinate speed are not meaningful from a physical point of view, only the proper acceleration and proper speed are. Therefore, contrary to Mashhoon’s conclusions¹, there is no such thing as “...for motion with $v < v_c$, we have the standard attractive force of gravity familiar from Newtonian physics, while for $v = v_c$, the particle experiences no force and for $v > v_c$ the

gravitational attraction turns to repulsion” since there is no change of sign in the proper acceleration whatsoever.

2. CONCLUSION

We have shown that the conclusions drawn by Mashhoon are based in fact on the behavior of coordinate acceleration. Since coordinate acceleration has no bearing on the underlying physics and since the proper acceleration never changes sign, it follows that the conclusion drawn by Mashhoon relative to the gravitational force “turning to repulsion” at $v = v_c$ is false.

REFERENCES

- [1] Mashhoon, B. “Beyond Gravitoelectromagnetism: Critical Speed in Gravitational Motion”, *IJMP D*, **14**,12,(2005) pp. 2025-2037

<http://www.ejournalofscience.org>

- [2] Sfarti, A. "Euler-Lagrange Solution for Calculating Particle Orbits in Gravitational Fields", *Fizika A*, **19**, 4, (2010)
- [3] Sfarti, A., "Relativistic electrodynamics Lagrangian and Hamiltonian for particle accelerators" , *IJNEST*, **5**, 3, (2010) , pp. 189-194