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Rough Formal Concept Lattice Based on the Soft Sets

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Abstract—**We define the lower(upper) rough soft formal concept in the rough soft formal context, and using the soft set define the rough soft formal concept lattice, give the properties of the soft rough formal concept lattices.**
Keywords: lower(upper) rough soft formal concept, soft set, soft lattice, rough soft formal concept lattice.

I. Introduction

To slove the problems of uncertainty and complication in economic, engineering, environment, social, and so on, many tools have been studied. These are including probability theory [?], fuzzy sets [?], rough sets [?], [?], vague sets [?], formal context [?] and soft sets [?]. R.Wille [?] proposed a new model to represent the formal concepts associated to a context (U, M, L), named formal concept analysis based on formal context, which is a binary relation between a set of objects and a set of attributes. The theory of rough sets, proposed by Z.Pawlak [?], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. Molodsov [?] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. Because there is no limited condition to description of objects, many researchers discuss the soft set, rough sets and formal concept analysis, and these three theories are progressing rapidly(see [6-23] respectively).

In [24], authors defined the rough formal context, and discussed its properties. In [25], author defined soft lattice, and in [26],author defined the rough soft formal context. In this paper, we defined the rough soft formal concept lattice, and discuss the properties of rough formal concept lattice in soft set.

The rest of the paper is organized as following, in section 2, we review some basic concepts and properties of rough concept formal and soft sets. In section 3, we define the lower and upper rough soft formal concepts, and give some examples. In section 4, we discuss the properties of the rough soft formal concept lattice. Conclusions are given in section 5.

II. Basic knowledge

Definition 2.1 [?] Let (U, M, R) be an information system, where $U = \{a_1, \cdots, a_m\}$ is an object set, $M = \{x_1, \cdots, x_n\}$ is an attribute (property) set, R is an equivalent relation on G, $\forall A \subseteq G$, we can define the upper and the lower approximation of A about R:
$\overline{A}_R = \bigcup\{Y \in U/R \mid Y \cap A \neq \emptyset\} = \{x \in U \mid [x]_R \cap A \neq \emptyset\}$;
$\underline{A}_R = \bigcup\{Y \in U/R \mid Y \subseteq A\} = \{x \in U \mid [x]_R \subseteq A\}$.

$\overline{A}_R$, $\underline{A}_R$ are called $R-$ upper approximation and $R-$ lower approximation of A. $\overline{A}$, $\underline{A}$ are denoted simply respectively. If $\overline{A} = \underline{A}$, we say that A is definable, otherwise, A is rough.

Similarly, we can define the upper and lower approximation of attributes set $B \subseteq M$ about an equivalent relation on M.

Definition 2.2 [?] Let P be a none-empty ordered set.

(i) If $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$, then P is called a **lattice**;

(ii) If $\vee S$ and $\wedge S$ exist for all $S \subseteq P$, then P is called a **complete lattice**.

Definition 2.3 [?] Let U be an initial universe set and E be a set of parameters . Let $\mathcal{P}(U)$ denotes the power set of U and $A \subset E$. Then a pair (F, A) is called a **soft set** over U, where $F : A \to \mathcal{P}(U)$ is a mapping.

Definition 2. 4 [?] Let triplet $M = (f, X, L)$, where L is a complete lattice , $f : X \to L$ is a mapping, X is a universe set, then M is called **the soft lattice** .

Definition 2.5 [?] Let (U, M, R) is a rough formal context, $R \subseteq G \times M$, for a set $B \subseteq M$ of attributes , we define function $^{\downarrow} : 2^M \to 2^U, B^{\downarrow} = \{g \in G \mid (g, m) \in R, \forall m \in B\} = \{R_p(m) \subseteq U \mid m \in B\}$ (the set objects which have all attributes in B).

Correspondingly, for a set $A \subseteq U$ of objects ,we define: $^{\uparrow} : 2^U \to 2^M$, as following:
$A^{\uparrow} = \{m \in M \mid (g, m) \in R, \forall g \in A\} = \{R_s(g) \subseteq M \mid g \in A\}$ (the set of attributes common to the objects of in A).

Definition 2.6 [?] **A rough formal concept** of a rough formal context (U, M, R) is a pair of (A, B) with $A \subseteq U, B \subseteq M, B^{\downarrow} = A, A^{\uparrow} = B$. We call A the extent and B the intent of rough concept (A, B).

Simply denote the singleton $\{x\}^{\downarrow}$ as $x^{\downarrow}$, and $\{x\}^{\uparrow}$ as $x^{\uparrow}$.

Definition 2.7 [?] Let (U, M, R) is a rough formal context, U is objects set which is also the universe, M is attributes set. A pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, furthermore, the lower and upper rough approximations of pair (F, B) are denoted by $\underline{R}(F, B) = (\underline{F}, B), \overline{R}(F, B) = (\overline{F}, B)$, which are soft sets over G with the set-valued mappings given by $\underline{F}(x) = \underline{B}(F(x))$ and $\overline{F}(x) = \overline{B}(F(x))$, where $x \in B$. The operators $\underline{R}(F, B), \overline{R}(F, B)$ are called the lower and upper rough approximation operators on soft set (F, B). If $\overline{R} = \underline{R}$, we say that the soft set (F, B) is definable, otherwise, (F, B) is rough.

we call such quadruple tuple (G, M, R, F) as **rough soft formal context**, and, $\forall B \subseteq M$, soft set (F, B) on the rough soft formal context (G, M, R, F) which is called **rough soft formal set**.

Obviously, $\forall x \in B \subseteq M, F(x) \subseteq U$ is a parameterized family of subsets of U, and $F(x)$ is the set of x- approximate elements in (U, M, R, F).

III. Rough Soft formal Concept

In the following, we consider $\mathcal{P} = (U, M, F, G, R)$ as a rough soft formal context, in which the pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, the pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M.

Definition 3.1 Let $\mathcal{P} = (U, M, F, G, R)$ is a rough soft formal context, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M, the lower and upper rough approximations of B under the relation $R \subseteq U \times M$ are denoted by

$\overline{B} = \cup_{x \in A}\{G(x)|G(x) \cap B \neq \emptyset\}$,

$\underline{B} = \cup_{x \in A}\{G(x)|G(x) \subseteq B\}$.

Correspondingly, the lower and upper rough approximations of A under the relation $R \subseteq U \times M$ are denoted by

$\overline{A} = \cup_{y \in B}\{F(y)|F(y) \cap A \neq \emptyset\}$,

$\underline{A} = \cup_{x \in A}\{F(y)|F(y) \subseteq A\}$.

Definition 3.2 Let $\mathcal{P} = (U, M, F, G, R)$ is a rough soft formal context, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, consider the pair $(F(x), \{x\}), x \in B$, if $x^{\downarrow} = F(x), F(x)^{\uparrow} = \{x\}$. We call the pair $(F(x), \{x\}), x \in B$ is the rough soft formal concept, in which $F(x)$ is the extent and $\{x\}$ the intent of rough soft formal concept $(F(x), \{x\})$.

Example 1 Let (U, M, F, G, R) is a rough soft formal context, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M, where $U = \{h_1, h_2, h_3, h_4, h_5\}$ which is the set of houses, $M = \{e_1, \cdots, e_9\}$, in which e_1 stands for "$expensive$", e_2 : "$beautiful$", e_3 : "$wooden$", e_4 : "$cheap$", e_5 : "$in\ the\ green\ surrounding$", e_6 : "$good\ communication$", e_7 : "$having\ elementary\ school$", e_8 : "$having\ kindergarten$", e_9 : "$having\ the\ supermarket$". and suppose that:

$F(e_1) = \{h_1, h_2, h_3\}, F(e_2) = \{h_4, h_5\}, F(e_3) = \{h_1, h_2, h_4\}, F(e_4) = \{h_3\}, F(e_5) = \{h_5\}, F(e_6) = \{h_1, h_4\}, F(e_7) = \{h_2, h_3, h_5\}, F(e_8) = \{h_1, h_4\}, F(e_9) = \{h_2, h_3\}$.

$G(h_1) = \{e_1, e_3, e_6, e_8\}, G(h_2) = \{e_1 e_3, e_7, e_9\}, G(h_3) = \{e_1, e_4, e_7, e_9\}, G(h_4) = \{e_2, e_3, e_6, e_8\}, G(h_5) = \{e_2, e_5, e_7\}$.

Using 1 represents the object x have the property y under the arbitrary relation $R \subseteq U \times M$, and 0 means does not have this relation, we can represent this rough soft formal context as:

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
h_1	1	0	1	0	0	1	0	1	0
h_2	1	0	1	0	0	0	1	0	1
h_3	1	0	0	1	0	0	1	0	1
h_4	0	1	1	0	0	1	0	1	0
h_5	0	1	0	0	1	0	1	0	0

Tab1. rough soft formal context

Consider $e_1 \in B, F(e_1)^{\uparrow} = \{h_1, h_2, h_3\}^{\uparrow} = \{a\}, \{a\}^{\downarrow} = \{h_1, h_2, h_3\}$, so the pair $(F(e_1), e_1)$ is a rough soft formal concept; $e_9 \in B, F(e_9)^{\uparrow} = \{h_2, h_3\}^{\uparrow} = \{a, g, i\} \neq \{i\}, \{i\}^{\downarrow} = \{h_2, h_3\} = F(i)$, so the pair $(F(e_9), e_9)$ is not a rough soft formal concept.

More general, we have:

Definition 3.3 Let $\mathcal{P} = (U, M, F, G, R)$ is a rough soft formal context, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M, consider the pair (A, B), if $B^{\downarrow} = A, A^{\uparrow} = B$. We call the pair (A, B) is the rough soft formal concept, in which A is the extent and B the intent of rough soft formal concept (A, B) over $\mathcal{P}$.

Remark 1 Specially, if we define $A^{\uparrow} = \{G(g) \mid g \in A\} \subseteq M$, and $B^{\downarrow} = \{F(m) \mid m \in B\} \subseteq U$, then $A^{\uparrow} = B$, and $B^{\downarrow} = A \subseteq U$, that is, in this case the pair (A, B) must be a formal concept of rough soft formal context, that is, the pair (A, B) is a rough soft formal concept over $\mathcal{P}$.

Example 2 Consider the example 1, let $A = \{h_1, h_2\} \subseteq U, B = \{e_1, e_3\} \subseteq M$, then $A^{\uparrow} = \{h_1, h_3\}^{\uparrow} = \{e_1, e_3\} = B$, and $B^{\downarrow} = \{e_1, e_3\}^{\downarrow} = \{h_1, h_3\} = A$, therefore, (A, B) is rough soft formal concept over $\mathcal{P}$.

Definition 3.4 Let $\mathcal{P} = (U, M, F, G, R)$ is a rough soft formal context, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M, if $B^{\downarrow} = \overline{A}, \overline{A}^{\uparrow} = B$. We call the pair $(\overline{A}, B)$ is the upper rough soft formal concept, and if $B^{\downarrow} = \underline{A}, \underline{A}^{\uparrow} = B$. We call the pair $(\underline{A}, B)$ is the lower rough soft formal concept. For the pair (A, B) , if it is both and a lower rough soft formal concept and a upper rough soft formal concept, then it is a rough soft formal concept over the rough soft formal context $\mathcal{P}$.

Example 3 Let $\mathcal{P} = (U, M, F, G, R)$ is a rough soft formal context, where $U = \{h_1, h_2, h_3, h_4, h_5\}$ which is the houses set, $M = \{e_1, \cdots, e_5\}$, in which e_1 stands for "$expensive$", e_2 : "$beautiful$", e_3 : "$wooden$", e_4 : "$cheap$", e_5 : "$in\ the\ green\ surrounding$", let $F : M \to \mathcal{P}(U)$, and suppose that: $F(e_1) = \{h_2, h_4\}, F(e_2) = \{h_1, h_3\}, F(e_3) = \emptyset, F(e_4) = \{h_1, h_3, h_5\}, F(e_5) = \{h_1\}$. We can represent using table as following:

	e_1	e_2	e_3	e_4	e_5
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	0	1	0
h_4	1	0	0	0	0
h_5	0	0	0	1	0

Let $A = \{h_1, h_3\} \subseteq U, B = \{e_2, e_4\} \subseteq M$, because $F(e_2) = \{h_1, h_3\} \subseteq A, F(e_4) = \{h_1, h_3, h_5\} \subseteq A$, and $F(e_2) = \{h_1, h_3\} \cap A \neq \emptyset, F(e_4) = \{h_1, h_3, h_5\} \cap A \neq \emptyset$,

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we can get $\underline{A}=\{h_1,h_3\},\overline{A}=\{h_1,h_3,h_5\}$, and $\underline{A}^{\uparrow}=\{h_1,h_3\}^{\uparrow}=\{e_2,e_4\}$, $B^{\downarrow}=\{e_2,e_4\}^{\downarrow}=\{h_1,h_3\}=\underline{A}$, $\overline{A}^{\uparrow}=\{h_1,h_3,h_5\}^{\uparrow}=\{e_4\}\neq B$, therefore, the pair $(\overline{A},B)$ is not the upper rough soft formal concept, and the pair $(\underline{A},B)$ is the lower rough soft formal concept over the rough soft formal context $\mathcal{P}$.

If let $A=\{h_2,h_4\}\subseteq U, B=\{e_1\}\subseteq M$, because $F(e_1)=\{h_2,h_4\}\subseteq A$, we can get $\underline{A}=\{h_2,h_4\}=\overline{A}$, and $\underline{A}^{\uparrow}=\{h_2,h_4\}^{\uparrow}=\{e_1\}$, $B^{\downarrow}=\{e_1\}^{\downarrow}=\{h_2,h_4\}=\underline{A}=\overline{A}$, therefore, the pair $(\overline{A},B)$ is not the upper rough soft formal concept, and the pair $(\underline{A},B)$ is the lower rough soft formal concept, so the pair (A,B) is a rough soft formal concept over the rough soft formal context $\mathcal{P}$.

Remark 2 In this example, there is not any object (house) having the attribute e_3, so we can not consider this attribute, that is, attribute e_3 can reduct.

IV. The property of Rough Soft Formal concept lattice

Let $\mathcal{P}=(U,M,F,G,R)$ be a rough soft formal context, $\mathcal{B}(U,M,F,G,R)\subseteq 2^U\times 2^M$ denotes the set of all rough concepts over $\mathcal{P}$, which is a complete lattice with order relation $(A_1,B_1)\leq(A_2,B_2)\Longleftrightarrow A_1\subseteq A_2\Longleftrightarrow B_2\subseteq B_1$, the pair $(A_i,B_i), i=1,2$ is a rough soft formal concept set over $\mathcal{P}$, where $B_i\subseteq M$, and $F:B_i\rightarrow\mathcal{P}(U)$ is a set-value mapping over U, pair $(G,A_i), i=1,2$ is a soft set over M, where $A_i\subseteq U$, and $G:A_i\rightarrow\mathcal{P}(M)$ is a set-value mapping over M.

Let $\mathcal{P}=(U,M,F,G,R)$ is a rough soft formal context, pair (F,B) is a soft set over U, where $B\subseteq M$, and $F:B\rightarrow\mathcal{P}(U)$ is a set-value mapping over U, pair (G,A) is a soft set over M, where $A\subseteq U$, and $G:A\rightarrow\mathcal{P}(M)$ is a set-value mapping over M, and the pair $(\overline{A},B)$ is the upper rough soft formal concept, $(\underline{A},B)$ is the lower rough soft formal concept. Then all the lower rough soft formal concepts and upper rough soft formal concepts can form **lower rough soft formal concept lattice** and **upper rough soft formal concept lattice** over the rough soft formal context $\mathcal{P}$, respectively. And all rough soft formal concepts form the rough soft formal concept lattice, they are all complete lattices.

The rough soft formal contexts in example 1 and example 3 having the lattice structure (Fig1).

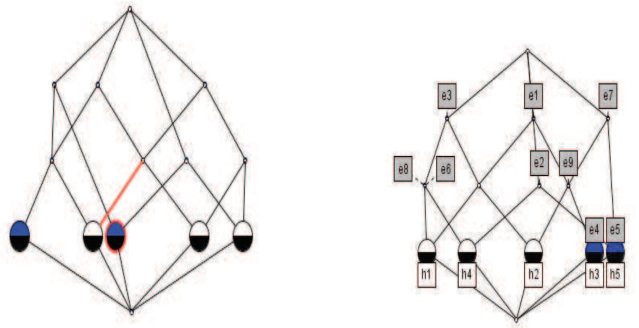
Proposition 4.1 If $\mathcal{P}=(U,M,F,G,R)$ is a rough soft formal context, the pair $(A_i,B_i), i=1,2$ is a rough soft formal concept set over $\mathcal{P}$, where $B_i\subseteq M$, and $F_i:B_i\rightarrow\mathcal{P}(U)$ is a set-value mapping over U, pair $(G_i,A_i), i=1,2$ is a soft set over M, where $A_i\subseteq U$, and $G_i:A_i\rightarrow\mathcal{P}(M)$ is a set-value mapping over M, then

(1) $A_1\subseteq A_2\Longrightarrow A_2^{\uparrow}\subseteq A_1^{\uparrow}, B_1\subseteq B_2\Longrightarrow B_2^{\downarrow}\subseteq B_1^{\downarrow}$;
(2) $A\subseteq A^{\uparrow\downarrow}, B\subseteq B^{\downarrow\uparrow}$;
(3) $A=A^{\uparrow\downarrow\uparrow}, B=B^{\downarrow\uparrow\downarrow}$;
(4) $A\subseteq B\Longleftrightarrow B\subseteq A^{\uparrow}\Longleftrightarrow A\times B\subseteq R$;

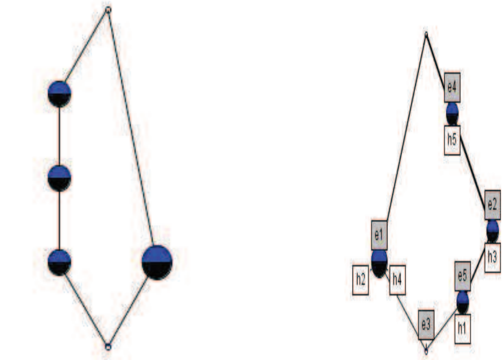
Obviously, $^{\downarrow}: 2^M\rightarrow 2^U, ^{\uparrow}: 2^U\rightarrow 2^M$, that is $(^{\uparrow}, ^{\downarrow})$ forms Galois connection, and calls the **rough soft Galois connection**.

proof Using the definition of $^{\uparrow}, ^{\downarrow}$, we can show these easily.

Proposition 4.2 A pair $(^{\uparrow}, ^{\downarrow})$ is a rough soft Galois connection if and only if $A\subseteq B^{\downarrow}\Longleftrightarrow B\subseteq A^{\uparrow}$, where $A\subseteq G, B\subseteq M$.



Lattice structure of Example 1



Lattice structure of Example 3
Fig 1

Proof By proposition 2.1, $A\subseteq B^{\downarrow}\Longrightarrow B^{\downarrow\uparrow}\subseteq A^{\uparrow}$, and $B\subseteq B^{\downarrow\uparrow}$, hence, $B\subseteq A^{\uparrow}$, i.e. one direction holds, the other follows symmetrically.

Definition 4.1 If $\mathcal{P}=(U,M,F,G,R)$ is a rough soft formal context, $B_i\subseteq M$, $A_i\subseteq U$, pair (F_i,B_1) is a soft set over U, $i=1,2$ $F_i:B_i\rightarrow\mathcal{P}(U)$ is a set-value mapping over U, then

(1) $(F_1,B_1)\wedge(F_2,B_2)$ is a soft set defined by $(F_1,B_1)\wedge(F_2,B_2)=(H,A\times B$, where $H(x,y)=F_1(x)\cap F_2(y),\forall(x,y)\in B_1\times B_2$, and the operator $\cap$ is the intersection operation of sets.

(2) $(F_1,B_1)\vee(F_2,B_2)$ is a soft set defined by $(F_1,B_1)\vee(F_2,B_2)=(H,A\times B$, where $H(x,y)=F_1(x)\cup F_2(y),\forall(x,y)\in B_1\times B_2$, and the operator $\cup$ is the union operation of sets.

Theorem 4.1 Let $\mathcal{P}=(U,M,F,G,R)$ is a rough soft formal context, pair (F_t,B_t) is a soft set over U, where $B_t\subseteq M$, and $F_t:B_t\rightarrow\mathcal{P}(U)$ is a set-value mapping over U, T is an index set, for all $t\in T$, the pair (G_t,A_t) is a soft set over M, where $A_t\subseteq U$, and $G_t:A_t\rightarrow\mathcal{P}(M)$ is a set-value mapping over M, in which $B_t^{\downarrow}=\overline{A_t},\overline{A_t}^{\uparrow}=B_t, t\in T$. that is the pair $(\overline{A_t},B_t)$ is the upper rough soft formal concept, and $B_t^{\downarrow}=\underline{A_t},\underline{A_t}^{\uparrow}=B_t$. that is, the pair $(\underline{A_t},B_t)$ is the lower rough soft formal concept over the rough soft formal context $\mathcal{P}$, the upper rough soft concept lattice $(\mathcal{P},\vee,\wedge)$ is a complete in which the infimum and supremum are given by

$$\bigwedge_{t\in T}(\overline{A_t}, B_t) = (\bigcap_{t\in T}\overline{A_t}, (\bigcup_{t\in T}B_t)^{\downarrow\uparrow})$$

$$\bigvee_{t\in T}(\overline{A_t}, B_t) = ((\bigcup_{t\in T}\overline{A_t})^{\uparrow\downarrow}, \bigcap_{t\in T}B_t)$$

.

For the lower rough soft concept lattice, we have similar results, that is :

$$\bigwedge_{t\in T}(\underline{A_t}, B_t) = (\bigcap_{t\in T}\underline{A_t}, (\bigcup_{t\in T}B_t)^{\downarrow\uparrow})$$

$$\bigvee_{t\in T}(\underline{A_t}, B_t) = ((\bigcup_{t\in T}\underline{A_t})^{\uparrow\downarrow}, \bigcap_{t\in T}B_t)$$

.

Theorem 4.2 Let $\mathcal{P} = \mathcal{B}(U, M, F, G, R)$ be the rough soft formal context, the rough soft concept lattice $(\mathcal{P}, \vee, \wedge)$ is a complete in which the infimum and supremum are given by

$$\bigwedge_{t\in T}(A_t, B_t) = (\bigcap_{t\in T}A_t, (\bigcup_{t\in T}B_t)^{\downarrow\uparrow})$$

$$\bigvee_{t\in T}(A_t, B_t) = ((\bigcup_{t\in T}A_t)^{\uparrow\downarrow}, \bigcap_{t\in T}B_t)$$

where T is a index set and for every $t \in T$, $A_t \subseteq U, B_t \subseteq M$, $(A_t, B_t)$is the rough soft formal concept.

Furthermore, a complete lattice $\mathcal{V} = (V, \leq)$ is isomorphic to the rough formal concept lattice $(\mathcal{P}, \vee, \wedge)$ if and only if there are mappings $\gamma : U \longrightarrow V$ and $\mu : M \longrightarrow V$ such that $\gamma(U)$ is supremum-dense in V, $\mu(M)$ is infimum-dense in V, and gRm is equivalent to $\gamma \leq \mu$, for all $g \in U$ and all $m \in M$.

Proof We will explain the formula for the infimum . Since $A_t = B_t^{\downarrow}$, for each $t \in T$, by proposition 4.1, having $(\bigcap_{t\in T}A_t, (\bigcup_{t\in T}B_t)^{\downarrow\uparrow}) = ((\bigcap_{t\in T}B_t)^{\downarrow}, (\bigcup_{t\in T}B_t)^{\downarrow\uparrow})$, and is therefore a rough concept, that this can only be the infimum. The formula for the supremum is substantiated correspondingly. Thus, we have proven that $\mathcal{B}(U, M, F, G, R)$ is a complete lattice.

The proof of $\mathcal{V} \cong (\mathcal{P}, \vee, \wedge)$, we can use the ideas in [6], in here,we omit its.

Remark 3 We should note that the rough soft formal concept is a soft formal concept, and the rough soft formal concept lattice is a soft formal concept lattice, however, the converse results do not hold.

Example 3 Consider the rough soft formal context (U, M, F, G, R) formed by the relation between the living being and water, pair (F, B) is a soft set over U, where $B \subseteq M$, and $F : B \to \mathcal{P}(U)$ is a set-value mapping over U, pair (G, A) is a soft set over M, where $A \subseteq U$, and $G : A \to \mathcal{P}(M)$ is a set-value mapping over M, where $U = \{1, \cdots, 8\}$ which is the set of living being, $M = \{e_1, \cdots, e_9\}$, in which e_1 stands for "*lives in water*", e_2 : "*lives on land*", e_3 : "*need chlorophyll to produce food*", e_4 : "*two seed leaves*", e_5 : "*one seed leaf*", e_6 : "*can move around*", e_7 : "*has limbs*", e_8 : "*suckles its offspring*".

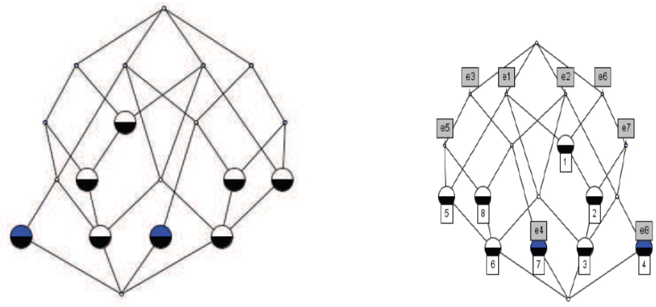


Figure 2

and suppose that:
$F(e_1) = \{1, 2, 3, 5, 6\}, F(e_2) = \{3, 4, 6, 7, 8\}, F(e_3) = \{5, 6, 7, 8\}, F(e_4) = \{7\}, F(e_5) = \{5, 6, 8\}, F(e_76) = \{1, 2, 3, 4\}, F(e_7) = \{2, 3, 4\}, F(e_8) = \{4\}$. $G(1) = \{e_1, e_6\}, G(2) = \{e_1 e_6, e_7\}, G(3) = \{e_1, e_2, e_6, e_7\}, G(4) = \{e_2, e_6, e_7, e_8\}, G(5) = \{e_1, e_3, e_5\}, G(6) = \{e_1, e_2, e_3, e_5\}, G(7) = \{e_2, e_3, e_4\}, G(8) = \{e_2, e_3, , e_5\}$.
We can represent this rough soft formal context as:

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
1	1	0	0	0	0	1	0	0
2	1	0	0	0	0	1	1	0
3	1	1	0	0	0	1	1	0
4	0	1	0	0	0	1	1	1
5	1	0	1	0	1	0	0	0
6	1	1	1	0	1	0	0	0
7	0	1	1	1	0	0	0	0
8	0	1	1	0	1	0	0	0

Its lattice structure as figure 2.

Consider $\{e_2, e_6, e_7\} \subseteq M, \{e_2, e_6, e_7\}^{\downarrow} = \{3, 4\}, \{3, 4\}^{\uparrow} = \{e_2, e_6, e_7\}$, so the pair $(\{3, 4\}, \{e_2, e_6, e_7\})$ is soft formal concept; however, by $F(e_8) = \{4\} \subseteq \{3, 4\}$, and $\{3, 4\} \cap F(e_1), F(e_2), F(e_6), F(e_7), F(e_8) \neq \emptyset$, so, $\underline{\{3, 4\}}^{\uparrow} = \{4\}^{\uparrow} = \{e_2, e_6, e_7, e_8\}, \overline{\{3, 4\}}^{\uparrow} = U^{\uparrow} = \emptyset$, so the pair $(\underline{\{3, 4\}}, \{e_2, e_6, e_7\})$ is not lower rough soft formal concept, and the pair $(\overline{\{3, 4\}}, \{e_2, e_6, e_7\})$ is not upper rough soft formal concept.

Of course, $(\underline{\{3, 4\}}, \{e_2, e_6, e_7, e_8\})$ is the lower rough soft formal concept, and the pair $(\overline{\{3, 4\}}, U)$ is the upper rough soft formal concept.

Remark 4 In fact, we can replace the equivalent relation R between U and M by any arbitrary binary relation.

V. Conclusion

In this paper, we define the lower(upper) rough soft formal concept in the rough soft formal context, and give some example to explain it. We define the rough soft formal concept lattice using the soft set , give the properties of the soft rough formal concept lattices. It offers a new method and tool in data analysis. Therefore, we can deal with lots of data more easily and do more efficient decision in data mining, information system, human reasoning and so on.

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