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Generation of Bulk Viscous Fluid Massive String Cosmological Models with Electromagnetic Field in Bianchi Type VI_0 Space-Time

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***Abstract*—The present study deals with spatially homogeneous and anisotropic Bianchi-VI_0 cosmological models representing a cloud formed by massive strings in presence of electromagnetic field and bulk viscosity. To get a determinate model, we assume that the expansion (θ) in the model is proportional to the shear (σ) and also the fluid obeys the barotropic equation of state. The study reveals that massive strings dominate the early Universe evolving with deceleration and in later phase it disappear from the universe, which is in good agreement with current astronomical observations. The behaviour of the models from physical and geometrical aspects in presence and absence of magnetic field and bulk viscosity is discussed.**

***Index Terms*—Massive string, Bianchi type VI_0, Viscous models, Magnetic field**
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I. Introduction

The explanation of the formation of large scale structure of the universe is one of the basic problem of cosmology even today. In literary production, the widely used mechanisms for this structure formation are the gravitational perturbations generated by topological defects. Certain grand unified theories predict topological defects to have formed in the early universe. According to Big Bang theory, the universe cooled from an initial hot, dense state triggering a series of phase transitions much like what happens in condensed-matter systems. In physical cosmology, a topological defect is an (often) stable configuration of matter predicted by some theories to form at phase transitions in the very early universe. In recent years, there has been considerable interest in string cosmology. Cosmoc strings are topological stable objects, which might be found during phase transition in the early universe [1]. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al. [2]; Kibble [1], [3]; Everett [4]; Vilenkin [5], [6]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies [7]. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies. While matter is accreted onto loops, they oscillate violently and lose their energy by gravitational radiation and therefore they shrink and disappear. These cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [8] who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier [9] first used this idea in obtaining cosmological solutions in Bianchi-I and Kantowski-Sachs space-times. Stachel [10] has also studied massive string. Pradhan et al. [11] and Yadav et al. [12] endured inhomogeneous cosmological models formed by geometric strings and used these models as a source of gravitational fields. In recent past, several authors [13−36] have meditated on cosmic strings in Bianchi type space-times in different context of use.

The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. Most studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. Bulk viscosity is associated with the GUT phase transition and string creation. In past, cosmological models have generally been based on the presumption that the universe undergoes an adiabatic expansion. However, since the discovery of the 2.7^oK radiation background, much attention has been paid to the possible cosmological role of dissipative processes. The role of viscosity has been discussed by Weinberg [37]. A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [38]. The solutions that he found exhibit an interesting feature that the big bang type singularity appears in the infinite past. These motivates to study string cosmological models in presence of bulk viscosity.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged. Several authors (Zeldovich [39], Harrison [40], Misner, Thorne and Wheeler [41], Asseo and Sol [42], Pudritz and Silk [43], Kim, Tribble and Kronberg [44], Perley and Taylor [45], Kronberg et al. [46], Wolfe et al. [47], Kulsrud et al. [48] and Barrow [49]) have pointed out the importance of magnetic field in different context. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The string cosmological models with a magnetic field are also discussed by Benerjee et al. [50], Chakraborty & Chakraborty [51], Tikekar & Patel ([52], and Singh & Singh [53].

Motivated by the above discussions, in this paper, we have obtained some Bianchi type VI_0 string cosmological models in presence and absence of magnetic field and bulk viscosity. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of viscous fluid and magnetic field. In Section 4, we have described some geometric and physical behaviour of the model. Section 5 includes the solution in absence of magnetic field whereas in Section 6, we have given the solution in absence of bulk viscosity. In the last section 7, conclusions are given.

II. The Metric and Field Equations

We consider the Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2, \quad (1)$$

where A, B and C are functions of t alone. The energy-momentum tensor for a cloud of strings in presence of bulk viscosity and magnetic field has the form

$$T_i^k = (\rho+p)u_iu^k + pg_i^k - \lambda x_ix^k - \zeta\theta(g_i^k + u_iu^k) + E_i^k, \quad (2)$$

where E_i^k is the electromagnetic field, given by

$$E_i^k = F_{il}F^{kl} - \frac{1}{4}F_{lm}F^{lm}g_i^k, \quad (3)$$

and v_i and x_i satisfy conditions

$$u^iu_i = -x^ix_i = -1, \;\; u^ix_i = 0. \quad (4)$$

In equations (2), p is isotropic pressure, ρ is rest energy density for a cloud strings, λ is the string tension density, F_{ij} is the components of electromagnetic field tensor, x^i is a unit space-like vector representing the direction of string, and u^i is the four velocity vector satisfying the relation

$$g_{ij}u^iu^j = -1. \quad (5)$$

Here, the co-moving coordinates are taken to be $u^1 = 0 = u^2 = u^3$ and $u^4 = 1$ and $x^i = (\frac{1}{A}, 0, 0, 0)$. The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \quad (6)$$

$$F^{ik}_{;k} = 0, \quad (7)$$

are satisfied by

$$F_{23} = K(\text{say}) = \text{constant}, \quad (8)$$

where a semicolon (;) stands for covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j, \quad (9)$$

for the line-element(1) lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\left[p - \lambda - \zeta\theta - \frac{K^2}{2B^2C^2}\right], \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\left[p - \zeta\theta + \frac{K^2}{2B^2C^2}\right], \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\left[p - \zeta\theta + \frac{K^2}{2B^2C^2}\right], \quad (12)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \left[\rho + \frac{K^2}{2B^2C^2}\right], \quad (13)$$

$$\frac{1}{A}\left[\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0. \quad (14)$$

Here, and also in what follows, a dot indicates ordinary differentiation with respect to t. The velocity field u^i is irrotational. The scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (15)$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right], \quad (16)$$

$$\sigma_{22} = \frac{B^2}{3}\left[\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right], \quad (17)$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right], \quad (18)$$

$$\sigma_{44} = 0. \quad (19)$$

Therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right]. \quad (20)$$

III. Solutions of the Field Equations

The field equations (10)−(14) are a system of five equations with seven unknown parameters A, B, C, ρ, p, λ and ζ. We need two additional conditions to obtain explicit solutions of the system.

Equation (14) leads to

$$C = mB, \quad (21)$$

where m is an integrating constant.

We first assume that the expansion (θ) is proportional to shear (σ). This condition and Eq. (21) lead to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \ell\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right), \quad (22)$$

which yields to

$$\frac{\dot{A}}{A} = n\frac{\dot{B}}{B}, \quad (23)$$

where $n = \frac{(2\ell\sqrt{3}+1)}{(1-\ell\sqrt{3})}$ and ℓ are constants. Eq. (23), after integration, reduces to

$$A = \kappa B^n, \tag{24}$$

where κ is a constant of integration. Eqs. (11) and (13) lead to

$$p = \xi - \frac{K^2}{2B^2C^2} - \left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2}\right), \tag{25}$$

and

$$\rho = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} - \frac{K^2}{2B^2C^2}, \tag{26}$$

respectively, where $\zeta\theta = \xi(\text{say}) = \text{constant}$. Now let us consider that the fluid obeys the barotropic equation of state

$$p = \gamma\rho, \tag{27}$$

where $\gamma(0 < \gamma < 1)$ is a constant. Eqs. (25)−(27) lead to,

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + (1+\gamma)\frac{\dot{A}\dot{C}}{AC} + \gamma\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}\right) - (1+\gamma)\frac{1}{A^2} +$$

$$(1-\gamma)\frac{K^2}{2B^2C^2} - \xi = 0. \tag{28}$$

Eq. (28) with the help of (21) and (24) reduces to

$$2\ddot{B} + \frac{2(n^2+2\gamma n+\gamma)}{(n+1)}\frac{\dot{B}^2}{B^2} = \frac{2(1+\gamma)}{\kappa^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2B^3} - 2\xi B. \tag{29}$$

Let us consider $\dot{B} = f(B)$ and $f' = \frac{df}{dB}$. Hence Eq. (29) reduces to the form

$$\frac{d}{df}(f^2) + \frac{2\alpha}{B}f^2 = \frac{2(1+\gamma)}{\kappa^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2B^3} - 2\xi B, \tag{30}$$

where $\alpha = \frac{(n^2+2n\gamma+\gamma)}{(n+1)}$. Eq. (30) after integrating reduces to

$$f^2 = \frac{2(1+\gamma)B^{-2n+2}}{\kappa^2(2\alpha-2n+2)} + \frac{(1-\gamma)K^2}{2m^2(\alpha-1)} - \frac{\xi B^2}{(\alpha+1)} + MB^{-2\alpha}, \quad \gamma \neq 1. \tag{31}$$

To get deterministic solution in terms of cosmic string t, we suppose $M = 0$. In this case Eq. (31) takes the form

$$f^2 = aB^{-2(n-1)} + bB^{-2} + NB^2, \tag{32}$$

where

$$a = \frac{2(1+\gamma)}{\kappa^2(2\alpha-2n+2)}, \ b = \frac{(1-\gamma)K^2}{2m^2(\alpha-1)}, \ N = -\frac{\xi}{(\alpha+1)}.$$

Therefore, we have

$$\frac{dB}{\sqrt{aB^{-2(n-1)} + bB^{-2} + NB^2}} = dt. \tag{33}$$

To get deterministic solution, we assume $n = 2$. In this case integrating Eq. (33), we obtain

$$B^2 = \sqrt{(a+b)}\ \frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}. \tag{34}$$

Hence, we have

$$C^2 = m^2\sqrt{(a+b)}\ \frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}, \tag{35}$$

$$A^2 = \kappa^2(a+b)\ \frac{\sinh^2(2\sqrt{N}t)}{N}, \tag{36}$$

where $N > 0$ without any loss of generality.

Therefore, the metric (1) in presence of magnetic field and bulk viscosity, reduces to the form

$$ds^2 = -dt^2 + \kappa^2(a+b)\ \frac{\sinh^2(2\sqrt{N}t)}{N}\ dx^2 +$$

$$\sqrt{(a+b)}\ \frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}\ e^{2x}\ dy^2 + m^2\sqrt{(a+b)}\ \frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}\ e^{-2x}\ dz^2. \tag{37}$$

IV. The Geometric and Physical Significance of Model

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (37) are given by

$$p = \frac{N}{(a+b)}\left(\frac{1}{\kappa^2} - \frac{K^2}{2m^2}\right)\coth^2(2\sqrt{N}t) +$$

$$\frac{N}{(a+b)}\left[\frac{K^2}{2m} - \frac{1}{\kappa^2} - 8(a+b)\right] + \xi, \tag{38}$$

$$\rho = \frac{N}{(a+b)}\left[5(a+b) - \frac{K^2}{2m^2} - \frac{1}{\kappa^2}\right]\coth^2(2\sqrt{N}t) +$$

$$\frac{N}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\kappa^2}\right), \tag{39}$$

where $p = \gamma\rho$ is satisfied by (28).

$$\lambda = \frac{N}{(a+b)}\left[\frac{2}{\kappa^2} - \frac{K^2}{m^2} - (a+b)\right]\coth^2(2\sqrt{N}t) +$$

$$\frac{N}{(a+b)}\left\{\frac{K^2}{m^2} - \frac{2}{\kappa^2} - 4(a+b)\right\}, \tag{40}$$

$$\rho_p = \rho - \lambda = \frac{N}{(a+b)}\left[\frac{K^2}{2m^2} - \frac{3}{\kappa^2} + 6(a+b)\right]\coth^2(2\sqrt{N}t) +$$

$$\frac{N}{(a+b)}\left\{\frac{3}{\kappa^2} - \frac{K^2}{2m^2} + 4(a+b)\right\}, \tag{41}$$

$$\theta = 4\sqrt{N}\coth(2\sqrt{N}t), \tag{42}$$

$$\sigma = \sqrt{\frac{N}{3}}\coth(2\sqrt{N}t), \tag{43}$$

$$V^3 = \frac{\kappa m(a+b)}{N}\sinh^2(2\sqrt{N}t). \tag{44}$$

From Eqs. (31) and (32), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{45}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -\left[\frac{\frac{8N}{3} - \frac{8N}{9}\coth^2(2\sqrt{N}t)}{\frac{16N}{9}\coth^2(2\sqrt{N}t)}\right]. \tag{46}$$

From (46), we observe that

$$q < 0 \quad if \quad \coth^2(2\sqrt{N}t) < 3, \tag{47}$$

and

$$q > 0 \quad if \quad \coth^2(2\sqrt{N}t) > 3. \tag{48}$$

The sign of q indicates whether the model inflates or not. The positive sign of q correspond to "standard" decelerating model whereas the negative sign of q indicates inflation. It is remarkable to mention here that though the current observations of SNe Ia and CMBR favour accelerating models, but both do not altogether rule out the decelerating ones which are also consistent with these observations (see, Vishwakarma [54]).

From (39), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{N}{(a+b)}\left(\frac{K^2}{2m^2}+\frac{1}{\kappa^2}\right)}{\frac{N}{(a+b)}\left\{\frac{K^2}{2m^2}+\frac{1}{\kappa^2}-5(a+b)\right\}}\right]. \quad (49)$$

Also from (41), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{N}{(a+b)}\left\{\frac{3}{\kappa^2}-\frac{K^2}{2m^2}+4(a+b)\right\}}{\frac{N}{(a+b)}\left\{\frac{3}{\kappa^2}-\frac{K^2}{2m^2}-6(a+b)\right\}}\right]. \quad (50)$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (49) and (50).

The model (37) starts with a big bang at $t = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta}$ = constant, hence the model does not approach isotropy. There is a Point Type singularity (MacCallum [55]) in the model at $t = 0$. For the condition $\coth^2(2\sqrt{N}t) < 3$, the solution gives accelerating model of the universe. It can be easily seen that when $\coth^2(2\sqrt{N}t) > 3$, our solution represents decelerating model of the universe.

V. Solutions in Absence of Magnetic Field

In absence of magnetic field, i.e. when $b \to 0$ i.e. $K \to 0$, we obtain

$$B^2 = 2\sqrt{2}\,\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}}, \quad (51)$$

$$C^2 = 2m^2\sqrt{a}\,\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}}, \quad (52)$$

$$A^2 = 4a\kappa^2\,\frac{\sinh^2(2\sqrt{N}t)}{4N}. \quad (53)$$

Hence, in this case, the geometry of the universe (37) reduces to

$$ds^2 = -dt^2 + 4\kappa^2 a\,\frac{\sinh^2(2\sqrt{N}t)}{4N}\,dx^2 + 2\sqrt{2}\,\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}}\,e^{2x}\,dy^2 + 2m^2\sqrt{a}\,\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}}\,e^{-2x}\,dz^2. \quad (54)$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (54) are given by

$$p = \frac{N}{a\kappa^2}\coth^2(2\sqrt{N}t) + \xi - \left(\frac{1}{a\kappa^2}+8\right)N, \quad (55)$$

$$\rho = \left(5N - \frac{N}{a\kappa^2}\right)\coth^2(2\sqrt{N}t) + \frac{N}{a\kappa^2}, \quad (56)$$

$$\lambda = \left[\frac{2N}{a\kappa^2} - N\right]\coth^2(2\sqrt{N}t) - \left\{\frac{2N}{a\kappa^2}+4N\right\}, \quad (57)$$

$$\rho_p = \rho - \lambda = 3N\left(2 - \frac{1}{a\kappa^2}\right)\coth^2(2\sqrt{N}t) + N\left(\frac{3}{a\kappa^2}+4\right), \quad (58)$$

$$\theta = 4\sqrt{N}\coth(2\sqrt{N}t), \quad (59)$$

$$\sigma = \sqrt{\frac{N}{3}}\coth(2\sqrt{N}t), \quad (60)$$

$$V^3 = \frac{\kappa m a}{N}\sinh^2(2\sqrt{N}t). \quad (61)$$

From Eqs. (59) and (60), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \quad (62)$$

From (56), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{N}{a\kappa^2}}{\frac{N}{a\kappa^2}-5N}\right]. \quad (63)$$

Also from (58), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{3N}{a\kappa^2}+4N}{\frac{3N}{a\kappa^2}-6N}\right]. \quad (64)$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (63) and (64).

The model (54) starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases as time increases. Since $\frac{\sigma}{\theta}$ = constant, hence the anisotropy is maintained throughout. There is a Point Type singularity (MacCallum [55]) in the model at $t = 0$.

VI. Solution in absence of Viscosity

In absence of bulk viscosity i.e. when $N \to 0$, then we obtain

$$B^2 = 2\sqrt{a}t, \quad (65)$$

$$C^2 = 2m^2\sqrt{a}t, \quad (66)$$

$$A^2 = 4\kappa^2 a t^2. \quad (67)$$

Hence, in this case, the geometry of the universe (54 reduces to

$$ds^2 = -dt^2 + 4\kappa^2 a t^2\,dx^2 + 2\sqrt{a}t\,e^{2x}\,dy^2 + 2\sqrt{a}t\,e^{-2x}\,dz^2. \quad (68)$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (68) are given by

$$p = \frac{1}{4a\kappa^2 t^2}, \quad (69)$$

$$\rho = -\frac{1}{4a\kappa^2 t^2}, \quad (70)$$

$$\lambda = \left(\frac{2}{a\kappa^2} - 1\right)\frac{1}{t^2}, \quad (71)$$

$$\rho_p = \rho - \lambda = -\frac{3}{4a\kappa^2 t^2} + \frac{1}{4t^2}, \quad (72)$$

$$\theta = \frac{2}{t}, \quad (73)$$

$$\sigma = \frac{1}{2\sqrt{3}}\frac{1}{t}, \quad (74)$$

$$V^3 = 4a\kappa m t^2. \quad (75)$$

From Eqs. (73) and (74), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{76}$$

It is observed that the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied if $0 < \alpha < 1$.

The model (68) in absence of magnetic field and bulk viscosity, starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases with time. The string tension λ decreases with time . We also observe that $\lambda > 0$ if $\frac{2}{a} > \kappa^2$ and $\lambda < 0$ if $\frac{2}{a} < \kappa^2$. Since $\frac{\sigma}{\theta}$ = constant, hence the anisotropy is maintained throughout. There is a Point Type singularity (MacCallum [55]) in the model at $t = 0$.

VII. Conclusions

In this paper, we have obtained some new Bianchi type VI_0 massive string cosmological models with a bulk viscous fluid as the source of matter in presence and absence of magnetic field. In presence of bulk viscosity it represents an accelerating universe during the span of time mentioned below Eq. (45) as decelerating factor $q < 0$ and it represents decelerating universe as $q > 0$. All the three models obtained in the present study have a Point Type singularity at $t = 0$. The energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under suitable choice of constants.

The models represent an expanding, accelerating, sheering and non-rotating universe. We observe that $\frac{\sigma}{\theta}$ is constant throughout in all three models. Hence the models do not approach isotropy. It is also observed that the rate of expansion of the universe is same in presence and absence of magnetic field. The magnetic field does not affect the behaviour of models but expressions for physical parameters are different whereas kinematics parameters are unchanged. The idea of primordial magnetism is appealing because it can potentially explain all the large-scale fields seen in the universe today, specially those found in remote proto-galaxies. As a result, the literature contains many studies examining the role and the implications of magnetic fields for cosmology.

The strings dominate in the early universe and eventually disappear from the universe for sufficiently large times. At early universe, the possible occupation of cosmic strings is not allowed to exceed over 10% due to constraints of latest CMB data. At late time evolution, the strings become negligible even then still play an important role in astronomical experiments. The models present the dynamics of strings in the accelerating and decelerating modes of evolution of the universe. The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We observe here that Murphyâs conclusion [38] about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid is, in general, not true. The results obtained in Ref. [56] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

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