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Bianchi-I Massive String Cosmological Models in General Relativity

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***Abstract*—In present paper, we study some massive string cosmological models in a homogeneous and anisotropic Bianchi-I space-time. To get the deterministic solutions of Einstein's field equations, we assume that the expansion scalar in the models is proportional to one of the components of shear tensor and the scale factor $a = (t^k e^t)^{\frac{1}{n}}$ which yields a time-dependent deceleration parameter. It is observed that for $n \leq 1$, the models are not physically feasible whereas for $n \geq 1$, we can generate a class of physically viable models. It is also detected that the models of the universe have a transition from early decelerated phase to the recent accelerating phase at present epoch which is consistent with recent astrophysical observations. The study reveals that massive strings dominate in the early universe. The strings eventually disappear from the universe for sufficiently large times, which is in agreement with current astronomical observations. Some physical and geometric aspects of the models are also discussed.**

***Index Terms*—String, Bianchi type I universe, Variable deceleration parameter, Accelerating universe**

I. Introduction

Cosmic strings play a substantial role in study of the early universe. These strings are topologically stable defects due to the phase transition that occurs as the temperature lowers below some critical temperature of the early stage of the universe [1]. According to Kibble [1] the present day configurations of the universe also support the existence of large scale network of strings in early universe. It is generally assumed that after the big bang explosion, the universe may have undergone a series of phase transitions when the temperature cooled below some critical temperature as predicted by grand unified theories (Zel'dovich et al. [2]; Kibble [1], [3]; Everett [4]; Vilenkin [5], [6]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies [7]. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies. While matter is accreted onto loops, they oscillate violently and lose their energy by gravitational radiation and therefore they shrink and disappear. These cosmic strings have stress-energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [8] who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier [9] first used this idea in obtaining cosmological solutions in Bianchi-I and Kantowski-Sachs space-times. Stachel [10] has also studied massive string. Pradhan et al. [11] and Yadav et al. [12] prevailed inhomogeneous string cosmological models formed by geometric strings and used these models as a source of gravitational fields. In recent past, several authors [13]− [35] have studied cosmic strings in Bianchi type space-times in different context. The simplest of anisotropic models, which, nevertheless, rather completely describe the anisotropic effects, are Bianchi type-I (BI) homogeneous models whose spatial sections are flat but the expansion or contraction rate is directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models. Observations by the Differential Radiometers on NASA's Cosmic Background Explorer recorded anisotropy in various angle scales. It is speculated, that these anisotropies hide in their hearts the entire history of the cosmic evolution down to recombination, and they are conceived to be indicative of the universe geometry and the matter composing the universe. The theoretical argument [36] and the modern experimental data support the existence of an anisotropic phase, which turns into an isotropic one.

The isotropy of the present-day universe makes the BI model a prime candidate for studying the possible effects of an anisotropy in the early universe on modern-day data observations. Recently, studying the accelerating dark energy models in Bianchi type V space-time, Pradhan and Amirhashchi [37] proposed a law of variation of scale factor as increasing function of time which generates a time dependent DP. This law provides explicit form of scale factors governing the Bianchi type-V universe and facilitates to describe accelerating phase of the universe. Recently, in 2012, Yadav [33], [38] generalized the scale factor proposed in [37] in studying Bianchi type-V space-time for string cosmological model and viscous fluid model with time dependent Λ-term respectively. Recently, Chawla et al. [39] studied anisotropic Bianchi-I cosmological models in string cosmology with variable deceleration parameter. Motivated by the above discussions, in this paper, the Einstein's field equations have been solved for massive string with a law of variation of scale factor in Bianchi type-I space-time which also provides a time dependent DP. This scenario facilitates to describe the transition of universe from early decelerated phase to the present accelerating phase. The paper has the following structure. The metric and the field equations are presented in Section 2. In Section 3, we deal with an exact solution of the field equations with cloud of strings. Section 4 describes some physical and geometric properties of the models. Finally, in Section 5, we summarize the results.

II. The Metric and Field Equations

We consider totally anisotropic Bianchi type-I line element, given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \qquad (1)$$

where the metric potentials A, B and C are functions of t alone. This ensures that the model is spatially homogeneous.

We define the following parameters to be used in solving Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-I model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \qquad (2)$$

A volume scale factor V is given by

$$V = a^3 = ABC. \qquad (3)$$

In analogy with FRW universe, we also define the generalized Hubble parameter H and deceleration parameter q as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \qquad (4)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}, \qquad (5)$$

where an over dot denotes derivative with respect to the cosmic time t.
Also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \qquad (6)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively.

The energy-momentum tensor for a cloud of massive string and perfect fluid distribution is taken as

$$T_i^j = (\rho + p)u_iu^j + pg_i^j - \lambda x_ix^j, \qquad (7)$$

where p is the isotropic pressure; ρ is the proper energy density for a cloud string with particles attached to them; λ is the string tension density; $u^i = (0, 0, 0, 1)$ is the four velocity of the particles, and x^i is a unit space-like vector representing the direction of string. The vectors u^i and x^i satisfy the conditions

$$u_iu^i = -x_ix^i = -1, u^ix_i = 0. \qquad (8)$$

Choosing x^i parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0). \qquad (9)$$

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda. \qquad (10)$$

The Einstein's field equations (in gravitational units $8\pi G = c = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j, \qquad (11)$$

in case of the line element (1) and energy distribution (7), lead to the following set of independent differential equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p + \lambda, \qquad (12)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -p, \qquad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p, \qquad (14)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho. \qquad (15)$$

The usual definitions of the dynamical scalars such as the expansion scalar (θ) and the shear scalar (σ) are considered to be

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} \qquad (16)$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \qquad (17)$$

where

$$\sigma_{ij} = u_{i;j} + \frac{1}{2}(u_{i;k}u^ku_j + u_{j;k}u^ku_i) + \frac{1}{3}\theta(g_{ij} + u_iu_j). \qquad (18)$$

The anisotropy parameter (A_m) is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \qquad (19)$$

The energy conservation equation $T^{ij}_{;j} = 0$, leads to the following expression:

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{A}}{A} = 0, \qquad (20)$$

which is a consequence of the field equations (12) − (15).

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III. Solutions of the Field Equations

Equations (12) − (15) are four equations in six unknowns A, B, C, p, ρ and λ. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We first assume that the component σ_1^1 of the shear tensor σ_i^j is proportional to the expansion scalar (θ), i.e., $\sigma_1^1 \propto \theta$. This condition leads to the following relation between the metric potentials:

$$A = (BC)^m, \tag{21}$$

where m is a positive constant. Collins et al. [40] have also pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition that $\frac{\sigma}{\theta}$ is constant.

Secondly, following Yadav [33], [38], we assume the law of variation of scale factor as increasing function of time as given by

$$a = (t^k e^t)^{\frac{1}{n}}, \tag{22}$$

where k and n are positive constants. If we put $n = 2$, Eq. (22) reduces to $a(t) = \sqrt{t^k e^t}$ which is used by Saha et al. [41] and Pradhan & Amirhashchi [37] in studying two-fluid scenario for dark energy models in an FRW universe and accelerating dark energy models in Bianchi type-V space-time respectively. If we put $k = 0$ in (22), we obtain $a(t) = (e^t)^{\frac{1}{n}}$ i.e. an exponential law of variation of scale factor. At this juncture it should be stated that some authors first choose the scale factor in power law, exponential or in other form and then find out other variables with some conditions under these solutions. Hence, the choice of scale factor given by (22) is physically acceptable.

From (5) and (22), we obtain the time varying deceleration parameter as

$$q = \frac{nk}{(t+k)^2} - 1. \tag{23}$$

The motivation to choose such time dependent DP is behind the fact that the universe has accelerated expansion at present as observed in recent observations of Type Ia supernova (Riess et al. [42], [43]; Perlmutter et al. [44]) and CMB anisotropies (Bennett et al. [45]; de Bernardis et al. [46]; Hanany et al. [47]) and decelerated expansion in the past. Also, the transition redshift from decelerated expansion to accelerating expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping (see the Refs. Padmanabhan and Roychowdhury [48]; Amendola [49]; Riess et al. [50]). So, there is no scope for a constant DP at the present epoch. So, in general, the DP is not a constant but time variable. The motivation to choose such scale factor (22) yields a time dependent DP (23).

From Eq. (23), we observe that $q > 0$ for $t < \sqrt{nk} - k$ and $q < 0$ for $t > \sqrt{nk} - k$. The nature of the evolution of the universe depends on two positive constants n and k. It is also observed that $n = 2, k = 2$, our model is in accelerating phase but for $n > 1, k = 1$, our model is evolving from decelerating phase to accelerating phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 \leq q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observations. Figure 1 depicts the variation of the deceleration parameter (q) versus cosmic time (t) which gives the behaviour of q for different value of (n, k). It is also clear from the figure that for $n = 2, k = 2$, the model is evolving only in accelerating phase whereas for $n > 1, k = 1$ the model is evolving from the early decelerated phase to the present accelerating phase.

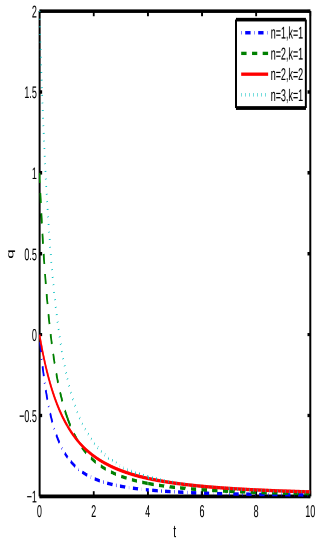

Figure 1. Plots of deceleration parameter q vs time t.

Subtracting (13) from (14), and taking integral of the resulting equation two times, we get

$$\frac{B}{C} = c_1 \exp\left[c_2 \int (ABC)^{-1} dt\right], \tag{24}$$

where c_1 and c_2 are constants of integration.

Solving (2), (21) and (24), we obtain the metric functions as

$$A(t) = a^{\frac{3m}{m+1}}, \tag{25}$$

$$B(t) = \sqrt{c_1} a^{\frac{3}{2(m+1)}} \exp\left[\frac{c_2}{2}\int \frac{1}{a^3} dt\right], \tag{26}$$

$$C(t) = \frac{1}{\sqrt{c_1}} a^{\frac{3}{2(m+1)}} \exp\left[-\frac{c_2}{2}\int \frac{1}{a^3} dt\right]. \tag{27}$$

Using (22) in Eqs. (25)-(27), we obtain the following expressions for scale factors:

$$A(t) = (t^k e^t)^{\frac{3m}{n(m+1)}}, \tag{28}$$

$$B(t) = \sqrt{c_1} (t^k e^t)^{\frac{3}{2n(m+1)}} \exp\left[\frac{c_2}{2} F(t)\right], \tag{29}$$

$$C(t) = \frac{1}{\sqrt{c_1}} (t^k e^t)^{\frac{3}{2n(m+1)}} \exp\left[-\frac{c_2}{2} F(t)\right]. \tag{30}$$

where

$$F(t) = \sum_{i=1}^{\infty} \frac{(-3)^{i-1}}{n^{i-2}(ni - 3k)(i-1)!} t^{i - \frac{3k}{n}}.$$

Hence the geometry of the universe (1) is reduced to

$$ds^2 = -dt^2 + (t^k e^t)^{\frac{6m}{n(m+1)}} dx^2 + c_1 (t^k e^t)^{\frac{3}{n(m+1)}} \exp[c_2 F(t)] dy^2 + \frac{1}{c_1}(t^k e^t)^{\frac{3}{n(m+1)}} \exp[-c_2 F(t)] dz^2. \quad (31)$$

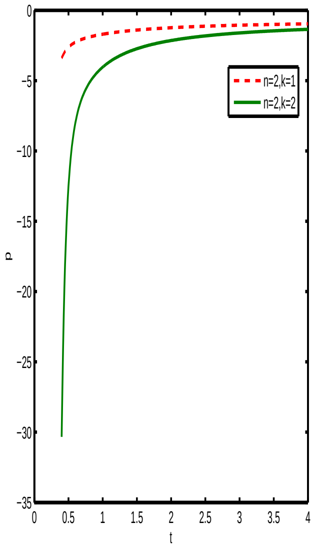

Figure 2. The plot of isotropic pressure p vs time t. Here $m = 0.3$, $c_2 = 1$.

IV. Some Physical and Geometric Properties

The expressions for the pressure (p), the energy density (ρ), the string tension density (λ) and the particle density (ρ_p) for the model (31) are given by

$$p = -\left[\frac{3(8m^2+1)}{4n^2(m+1)^2}\left(1+\frac{k}{t}\right)^2 + \left(\frac{c_2}{2(t^k e^t)^{\frac{3}{n}}}\right)^2 + \frac{3(1+2m)}{2n(m+1)}\left\{\frac{1}{n}\left((1+\frac{k}{t}\right)^2 - \frac{k}{t^2}\right\}\right], \quad (32)$$

$$\rho = \frac{4m+1}{(m+1)^2}\left\{\frac{3}{2n}\left(1+\frac{k}{t}\right)\right\}^2 - \left(\frac{c_2}{2(t^k e^t)^{\frac{3}{n}}}\right)^2, \quad (33)$$

$$\lambda = \frac{3(1-2m)}{2n^2(m+1)}\left[3\left(1+\frac{k}{t}\right)^2 - \frac{nk}{t^2}\right], \quad (34)$$

$$\rho_p = \frac{3(8m^2+16m-1)}{4n^2(m+1)^2} \times \left(1+\frac{k}{t}\right)^2 - \left(\frac{c_2}{2(t^k e^t)^{\frac{3}{n}}}\right)^2 + \frac{3(2m-1)}{2n(m+1)}\left[\frac{1}{n}\left(1+\frac{k}{t}\right)^2 - \frac{k}{t^2}\right]. \quad (35)$$

It is worth mentioned here that for $n \leq 1$, the models do not exist physically realistic as it provides negative energy density from early stage. Therefore, we have concentrated our analysis by considering two cases (i) $n = 2, k = 1$ and (ii) $n = 2, k = 2$ as representative cases.

Figure 2 depicts the variation of pressure versus time for both cases (i) and (ii) where the other parameters are taken to be $m = 0.3$, $c_2 = 1$ for a representative case. From the figure, we observe that pressure is a increasing function of time. In both cases the pressure starts from a large negative value and approaches to a small negative values near zero. It is worth mentioned here that for case (i), the pressure is increasing sharply in comparison to case(ii).

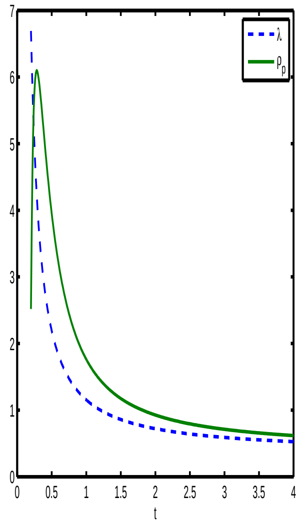

Figure 3. The plot of string tension density λ and particle density ρ_p vs time t for case(i). Here $m = 0.3$, $c_2 = 1$.

From Eq. (34), it is observed that λ for both cases i.e. ($n = 2, k = 1$) and ($n = 2, k = 2$), the string tension density is a decreasing function of time and it approaches to zero at present epoch and $\lambda > 0$ always. Figures 3 & 4 show the plots of string tension density versus time in both cases respectively. It is observed that λ decreases more sharply in case (i) compared to case (ii). It can also be seen from figures 3 & 4 that the particle density ρ_p varies from negative values to maximum positive value and then again decreases near to zero.

Figures 3 & 4 also show the comparative behaviour of particle energy density and string tension versus time in both cases (i) & (ii). Here we observe that $\rho_p/\lambda < 1$ near to big bang singularity but at the later time $\rho_p/\lambda > 1$. When $\rho_p > \lambda$, i.e., particle energy density remains larger than the string tension density during the cosmic expansion (see, Refs. Kibble [1]; Krori et al. [13]), which indicates that massive strings dominate the early universe evolving with decelerating and in later phase it will disappear which is in agreement with current astronomical observations. When $\rho_p < \lambda$ i.e. particle energy density remains smaller than the string tension density during the cosmic expansion indicates that strings dominate the universe evolving with acceleration. If this is so, we should have some signature of massive string at present epoch of the observations. However, it is not been seen so far. Further it is observed that for sufficiently large times, the ρ_p and λ tend to zero. Therefore, the strings disappear from the universe at late time (i.e. present epoch). The same is predicted by the current observations.

From Figures 3 & 4, we observe that the string tension density λ quickly falls as t increases. This λ plays crucial role for the anisotropic character of the universe. As λ falls very fast with time, so the universe quickly becomes isotropic. It is worth mention here that the corresponding solution does not blow up at any given epoch for the choice of the scale factor (22). Hence our derived model is physically acceptable.

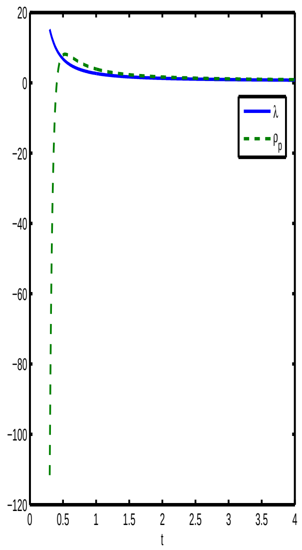

Figure 4. The plot of ρ_p and λ vs time t for case (ii). Here $m = 0.3$, $c_2 = 1$.

The left hand side of energy conditions have been depicted in Figures 5 & 6 for cases (i) and (ii) respectively. From these figures we observe that

$(i)\ \rho > 0$,

$(ii)\ \rho + p > 0$,

$(iii)\ \rho - p > 0$.

Therefore, we see that the weak energy condition (WEC) as well as the dominant energy condition (DEC) are satisfied in our model. It is also observed that $\rho + 3p > 0$ at initial time but on later time $\rho + 3p < 0$ which in turn imply that the strong energy condition (SEC) violates in the present model on later time. The violation of SEC gives anti gravitational effect. Due to this effect, the universe gets jerk and the transition from the earlier decelerated phase to the recent accelerating phase take place (see, Caldwell [51]). Hence the present model is turning out as a suitable model for describing the late time acceleration of the universe.

Figure 7 is a plot of the variation of anisotropy A_m versus time t for case(i) and case (ii), whereas the other parameters have been taken $m = 0.3$ and $c_2 = 1$ as a representative case. From the figure, we observe that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in the present model at present epoch for both cases.

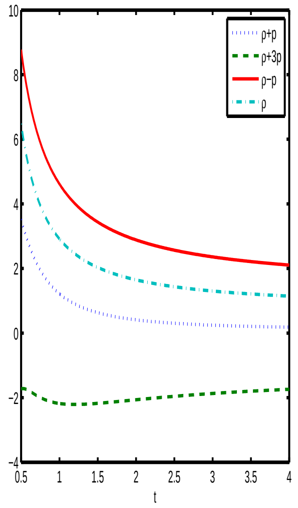

Figure 5. The plot of energy conditions vs time t for case(i). Here $m = 0.3$, $c_2 = 1$.

The physical parameters such as spatial volume (V), directional Hubble factors (H_i), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) are given by

$$V = (t^k e^t)^{\frac{3}{n}}, \tag{36}$$

$$H_1 = \frac{3m}{n(m+1)}\left(1 + \frac{k}{t}\right),$$

$$H_2 = \frac{3}{2n(m+1)}\left(1 + \frac{k}{t}\right) + \frac{c_2}{2(t^k e^t)^{\frac{3}{n}}},$$

$$H_3 = \frac{3}{2n(m+1)}\left(1 + \frac{k}{t}\right) - \frac{c_2}{2(t^k e^t)^{\frac{3}{n}}}, \tag{37}$$

$$\theta = 3H = \frac{3}{n}\left(1 + \frac{k}{t}\right), \tag{38}$$

$$\sigma^2 = 3\left(\frac{(2m-1)}{2n(m+1)}\left(1 + \frac{k}{t}\right)\right)^2 + \left(\frac{c_2}{2(t^k e^t)^{\frac{3}{n}}}\right)^2, \tag{39}$$

$$A_m = \frac{1}{2}\left(\frac{2m-1}{m+1}\right)^2 + \frac{1}{6}\left(\frac{nc_2}{\left(1+\frac{k}{t}\right)(t^k e^t)^{\frac{3}{n}}}\right)^2. \tag{40}$$

Therefore, the above solutions are exact solutions of Einstein's field equations (12)−(15). From Eqs. (36) and (38), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ which

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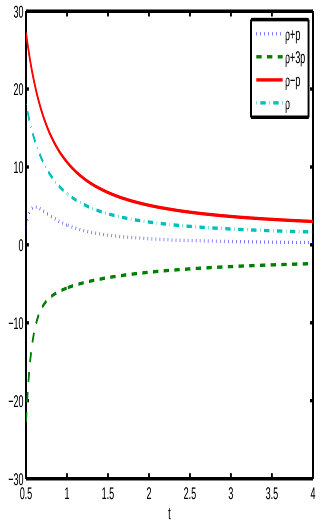

Figure 6. The plot of energy conditions vs time t for case (ii). Here $m = 0.3$, $c_2 = 1$.

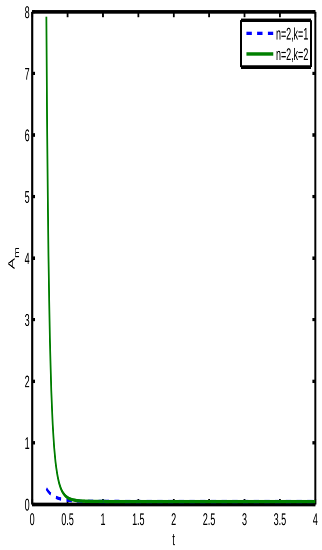

Figure 7. The plot of anisotropic parameter A_m vs time t. Here $m = 0.3$, $c_2 = 1$.

is big bang scenario. From Eqs. (28)−(30), we see that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity [52]. All the physical quantities isotopic pressure (p), proper energy density (ρ), string tension density (λ), particle density ρ_p, Hubble factor (H) and shear scalar (σ) diverge at $t = 0$. As $t \to \infty$, volume becomes infinite where as p, ρ, λ, ρ_p, H, θ approach to zero. Finally we conclude that the models represent shearing, non-rotating and expanding universe, which starts with a big bang and approaches to isotropy at present epoch.

V. Concluding Remarks

In this paper, spatially homogeneous and anisotropic Bianchi-I models representing massive string with variable deceleration parameter in general relativity have been studied. Generally models are expanding, shearing and rotation free. There is a point type singularity at $t = 0$ in the model. We remark that $t \to \infty$, $V \to \infty$ and $\rho \to 0$ i.e. spatial volume increases with time and proper energy density decreases with time as anticipated. The main features of the models are as follows:

- The models are based on exact solutions of Einstein's field equations for the anisotropic Bianchi-I space-time filled with string fluid as a source of matter.
- The proposed law of variation of scale factor as increasing function of time in Bianchi-I space-time generates a time dependent DP which represent models of the Universe evolving from early decelerated phase to present accelerating phase (see, Fig. 1). This is in good agreement with recent observations.
- With the different choice of pair (n, k) in Eq. (22), we can generate a class of cosmological models in Bianchi-I space-time. In derived model, the present value of DP is estimated as

$$q_0 = -1 + \frac{k}{nH_0^2 t_0^2},$$

where H_0 is the present value of Hubble's parameter and t_0 is the age of universe at present epoch. We observe that for $(n = 2, k = 2)$, $q_0 = 0$; for $(n = 2, k = 1)$, $q_0 = -0.5$; for $(n = 3, k = 1)$, $q_0 = -0.66$ and for $(n = 1, k = 1)$, $q_0 = -1$. Determination of the deceleration parameter from the count magnitude relation for galaxies is a difficult task due to the evolutionary effects. The present value q_0 of the deceleration parameter obtained from observations are $-1.27 \leq q_0 \leq 2$ (Schuecker et al. [53]). Studies of galaxy counts from redshift surveys provide a value of $q_0 = 0.1$, with an upper limit of $q_0 < 0.75$ (Schuecker et al. [53]). Recent observations show that the deceleration parameter of the universe is in the range $-1 \leq q \leq 0$ i.e $q_0 \approx -0.77$ (Cunha et al. [54]). A de Sitter universe is a cosmological solution to Einstein's field equations of General Relativity for which $q_0 = -1$. Thus we see that the value of q at present epoch in our generated models are very near to the value obtained by recent observations.

- If we put $n = 2$ in Eq. (22), we obtain $a(t) = \sqrt{t^k e^t}$ which is used by Pradhan and Hassan [37] in studying accelerating dark energy models in Bianchi type-V space-time. If we put $n = 2$ and $k = 1$, we obtain $a(t) = \sqrt{te^t}$ which is examined by Amirhashchi et al. [55] in studying an interacting two-fluid scenario for dark energy in FRW universe.
- The observed isotropy of the universe can be achieved in the derived models at present epoch.
- The WEC and DEC are satisfied in the derived model of the universe which in turn imply that the models are physically realistic while the violation of SEC is reproducible with current astrophysical observations.
- In the present study, the strings dominate in the early universe and eventually disappear from the universe for sufficiently large times. At early universe, the possible occupation of cosmic strings is not allowed to exceed over 10% due to constraints of latest CMB data. At

late time evolution, the strings become negligible even then still play an important role in astronomical experiments. Recent results from the PAMELA (Adriani et al. [56]) and ATIC (Chang et al. [57]) experiments have indicated an excess power of cosmic ray positron flux compared to what is predicted from astrophysical backgrounds alone. Recently, Brandenberger [58] have studied cosmic ray positron from cosmic strings showing that very few leptonic cosmic strings could decay into leptons and may be applied to explain recently discovered positron anomaly by Pamela data.

- Finally, the exact solutions presented in the paper can be one of the potential candidate to describe the observed universe. Thus, the solutions demonstrated in this paper may be useful for better understanding of the characteristic of anisotropic Bianchi-I model in the evolution of the universe.

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