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Notes on Soft Matrices Operations

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ABSTRACT

In this paper, we describe soft sets, soft matrices, soft sub matrices, soft matrix operations and their basic properties. We also recall the definition of operations on soft sets and show that they are equivalent to the corresponding operations on their soft matrices.

Keywords :*Soft sets, soft matrices, soft matrix operations.*

1.INTRODUCTION

In 1999, soft sets were introduced by Molodtsov [1] with an objective to help solving problems involving uncertainties which other extant modern theories dealing with uncertainties such as probability theory, fuzzy set theory, rough set theory, etc., cannot successfully solve mainly due to their inherent parameterization inadequacies.

In the past ten years or so, progress towards systematization of foundation for soft set theory and its applications has been perceptible.

Maji *et al*.,[2,3] developed many new operations on soft sets, established their properties and presented an application to a decision making problem.

Cagman and Enginoglu [4,5] developed a theory of soft matrices and successfully applied it to a decision-making problem. They also constructed a uni-int decision making method.

In this paper, we will be largely drawing definitions from [5]. In sections 2 and 3, we describe soft sets, soft matrices, soft submatrices and operations on them. In section 4, we recall the definition of operations on soft sets and show that they are equivalent to the corresponding operations on soft matrices representing them.

We have included only those references which pertain to the aim of this paper. However, as the subject is still in its infancy stage, inclusion of a plethora of definitions and examples may be unavoidable.

2.SOFT SETS, SOFT MATRICES AND SOFT SUBMATRICES

We describe representations of soft sets by matrices, called *soft matrices,* which are found useful as they can be efficiently stored in a computer.

Definition 2.1 [1]

Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all possible parameters with respect to U, and $A \subseteq E$. A pair (F_A, E) on the universe U, also denoted (F,A), is called a *soft set* defined as the set of ordered pairs viz.,

$(F_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$ where $f_A : E \to P(U)$ such that $f_A(e) = \varnothing$ (the empty set) if $e \notin A$.

Thus a soft set on the universe U is a parameterized family of subsets of U. Here f_A is called an approximate function of the soft set (f_A, E). Every set $f_A(e)$, $e \in E$ may be understood as the set of *e-elements* or *e-approximate elements* of the soft set or *e-approximate value set* or simply *e-approximate set*. In other words, a soft set may be considered as a collection of approximations or a *value- class.* Henceforth, we denote subsets of E by $A,B,C, \ldots$

Example 2.1

Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$.

Let $A = \{e_1, e_3, e_4\}$, $f_A(e_1) = \{u_2, u_4\}$, $f_A(e_3) = U$ and $f_A(e_4) = \{u_1, u_3, u_5\}$. Then we can view the soft set (f_A, E) as consisting of the following collection of approximations:

$(f_A, E) = \{(e_1, \{u_2, u_4\}), (e_3, U), (e_4, \{u_1, u_3, u_5\})\}$.

Also, $f_A(e_2) = \varnothing = f_A(e_5)$ since $e_2, e_5 \notin A$.

Definition 2.2 [4]

Let (f_A, E) be a soft set over U. Then a subset R_A of $U \times E$ is uniquely defined as

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$R_A = \{(u,e) : e \in A, u \in f_A(e)\}$, called a *relation form* of the soft set (f_A, E).

The *characteristic function* of R_A is defined as

$$\chi_{R_A} : U \times E \to \{0,1\},$$

where

$$\chi_{R_A}(u,e) = \begin{cases} 1, & (u,e) \in R_A; \\ 0, & (u,e) \notin R_A. \end{cases}$$

Let $U = \{u_1, u_2, \ldots, u_m\}, E = \{e_1, e_2, \ldots, e_n\}$ and $A \subseteq E$. Then R_A can be represented by a matrix as follows:

Let $a_{ij} = \chi_{R_A}(u_i, e_j)$. We can represent a matrix

$$\left[a_{ij}\right]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

and call it an $m \times n$ *soft matrix* of the soft set (f_A, E) over *U*. In other words, a soft set is uniquely represented by its corresponding soft matrix. For convenience, while writing soft matrices, we may suppress their dimensions unless necessary and also represent them by capital letters *M, N, P*, etc. Let the set of all $m \times n$ soft matrices over *U* be denoted $SM(U)_{m\times n}$ or just *SM*(*U*), henceforth considered to represent the universe of soft matrices.

It may be noted that, similar to matrix representations of relations in set theory, the tabular form of representing a soft set was already depicted in [3], which is exactly being represented as a matrix.

Example 2.2

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universe set, and $E = \{e_1, e_2, e_3, e_4\}$ be a set of all parameters with respect to *U*.

Let $A = \{e_1, e_3, e_4\},\ f_A(e_1) = \{u_3, u_4\},\ f_A(e_3) = \varnothing$, and $f_A(e_4) = \{u_1, u_3, u_5\}$. Then the soft set (f_A, E) is given by

$(f_A, E) = \{(e_1, \{u_3, u_4\}),\ (e_4, \{u_1, u_3, u_5\})\}$.

The relation form R_A of (f_A, E) is given by

$R_A = \{(u_3, e_1), (u_4, e_1), (u_1, e_4), (u_3, e_4), (u_5, e_4)\}$.

Hence the soft matrix $[a_{ij}]$ of the soft set (f_A, E) is given by

$$\left[a_{ij}\right] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ i = 1, 2, \ldots, 5;\ j = 1, 2, \ldots, 4.$$

As noted earlier, $f_A(e_3) = \varnothing$, since there is no element in *U* related to the parameter $e_3 \in A$, it does not appear in the aforesaid description of the soft set (f_A, E).

Definition 2.3 [4]

Let $[a_{ij}] \in SM(U)$. Then $[a_{ij}]$ is called

(a) A *zero soft matrix,* denoted $[\tilde{0}]$, if

$a_{ij} = 0 \quad \forall \quad i$ and j;

(b) An *A-universal soft matrix,* denoted $[a_{ij}]$, if

$a_{ij} = 1 \quad \forall \quad j \in I_A = \{j : e_j \in A\}$ and *i*.

(Note that it is so called, since $a_{ij} = 1$ only for the parameters appearing in the set $A \subset E$); and

(c) A *universal soft matrix* denoted $[\tilde{I}]$, if

$a_{ij} = 1 \quad \forall \quad i$ and j.

Example 2.3

Let $U = \{u_1, u_2, u_3, u_4\},\ E = \{e_1, e_2, e_3, e_4\}$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in SM(U)_{4\times 4}$. If $A = \{e_1, e_2, e_3\},\ f_A(e_1) = f_A(e_2) = f_A(e_3) = \varnothing$, then $[a_{ij}] = [\tilde{0}]$ is a zero soft matrix given by

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$$\left[\tilde{0}\right]=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}.$$

If $B=\{e_2,e_4\}$, $f_B(e_2)=U=f_B(e_4)$, then $\left[b_{ij}\right]$ is a *B*-universal soft matrix given by

$$\left[\tilde{b}_{ij}\right]=\begin{bmatrix}0&1&0&1\\0&1&0&1\\0&1&0&1\\0&1&0&1\end{bmatrix}.$$

If $C=E,\ f_e(e_i)=U$ for each *i*, then $\left[\tilde{c}_{ij}\right]=\left[\tilde{I}\right]$ is a universal soft matrix given by

$$\left[\tilde{I}\right]=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}.$$

Definition 2.4 [4]

Let $M=\left[a_{ij}\right], N=\left[b_{ij}\right]\in SM(U).$ Then we define the following:

(i) M is a *soft submatrix* of *N*, denoted $M\tilde{\subseteq}N$, if $a_{ij}\le b_{ij}$ for each *i* and *j*.
In this case, we also say that *M* is *dominated* by *N* or *N dominates M*. Note that, similar to $R^k(k>1)$, the *k*-dimensional real space, $\le$ holds without the holding of either < or =.
We define *M* and *N comparable*, denoted $M\approx N$, iff $M\tilde{\subseteq}N\ or\ N\tilde{\subseteq}M;$

(ii) *M* is a *proper soft submatrix* of *N* denoted $M\tilde{\subset}N$, if $\left[a_{ij}\right]\tilde{\subset}\left[b_{ij}\right]$ and for at least one term $a_{ij}<b_{ij}$ for all *i* and *j*. In this case, we say that *M* is *properly dominated* by *N*.s

(iii) *M* is a *strictly proper soft submatrix* of *N*, denoted $M\tilde{\subset}N$, if $M\tilde{\subseteq}N$ and $a_{ij}<b_{ij}$ for each *i* and *j*. In this case we say that *M* is *strictly dominated* by *N*.

(iv) *M* and *N* are *soft equal matrices,* denoted $M\tilde{=}N$, if $a_{ij}=b_{ij}$ for each *i* and *j*. Equivalently, if $M\tilde{\subseteq}N$ and $N\tilde{\subseteq}M$, then $M\tilde{=}N$. It is immediate to see that $\tilde{\subseteq}$ is a partial ordering (reflexive, antisymmetric and transitive) on the class of soft matrices.

3.OPERATIONS ON SOFT MATRICES

We discuss the operations of union, intersection complement, difference and products of soft matrices and their basic properties.

Definition 3.1 [4]

Let $M=\left[a_{ij}\right], N=\left[b_{ij}\right]\in SM(U)$. Then a soft matrix $P=\left[c_{ij}\right]\in SM(U)$ is called the

(i) *union* of *M* and *N*, denoted $M\tilde{\cup}N$, if
$$c_{ij}=\max\left\{a_{ij},b_{ij}\right\} \text{ for all } i \text{ and } j;$$

(ii) *intersection* of *M* and *N*, denoted $M\tilde{\cap}N$, if
$$c_{ij}=\min\left\{a_{ij},b_{ij}\right\} \text{ for all } i \text{ and } j;$$

(iii) *complement* of *M*, denoted M^0 , if
$$c_{ij}=1-a_{ij} \text{ for all } i \text{ and } j;$$

(iv) *difference* of *N* from *M*, also called the *relative complement* of *N* in *M*, denoted *M* – *N* or *M**N* ,if
$$P=M\tilde{\cap}N^0.$$

In view of the (ii) above, *M* and *N* are said to be *disjoint* if $M\tilde{\cap}N=\left[\tilde{0}\right].$

Example 3.1

Let

$$M=\begin{bmatrix}0&0&1&1\\1&0&1&0\\0&1&0&1\\0&0&0&0\\0&0&0&1\end{bmatrix} \text{ and } N=\begin{bmatrix}1&0&0&0\\0&1&0&1\\1&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix}.$$

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Then,

(i) $M \tilde{\cup} N = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;

(ii) $M \tilde{\cap} N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = [\tilde{0}]$ which implies that *M* and *N* are disjoint;

(iii) $M^0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

(iv) $N^0 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$;

(v) $M - N = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M \tilde{\cap} N^0$; and

Proposition 3.1: Properties of Soft Matrix Operations

Let $M = [a_{ij}],\ N = [b_{ij}],\ P = [c_{ij}] \in SM(U)$.

(i) $M \tilde{\cup} M = M; \quad M \tilde{\cap} M = M$
(Idempotent laws)

(ii) $M \tilde{\cup} [\tilde{0}] = M; \quad M \tilde{\cap} [\tilde{I}] = M$
(Identity laws)

(iii) $M \tilde{\cup} [\tilde{I}] = [\tilde{I}]; \quad M \tilde{\cap} [\tilde{0}] = [\tilde{0}]$
(Domination laws)

(iv) $[\tilde{0}]^0 = [\tilde{I}]\ ; \quad [\tilde{I}]^0 = [\tilde{0}]$
(De Morgan's laws)

(v) $M \tilde{\cup} M^0 = [\tilde{I}]\ ; \quad M \tilde{\cap} M^0 = [\tilde{0}]$
(De Morgan's laws)

(vi) $(M \tilde{\cup} N)^0 = M^0 \cap N^0\ ; \quad (M \tilde{\cap} N)^0 = M^0 \tilde{\cup} N^0$
(De Morgan's laws)

(vii) $(M^0)^0 = M$ for all M
(Involution law/double complement)

(viii) $M \tilde{\cup} N = N \tilde{\cup} M\ ; \quad M \tilde{\cap} N = N \tilde{\cap} M$
(Commutative laws)

(ix) $M \tilde{\cup} (N \tilde{\cup} P) = (M \tilde{\cup} N) \tilde{\cup} P\ ; \quad M \tilde{\cap} (N \tilde{\cap} P) = (M \tilde{\cap} N) \tilde{\cap} P$ (Associative laws)

(x) $M \tilde{\cup} (N \tilde{\cap} P) = (M \tilde{\cup} N) \tilde{\cap} (M \tilde{\cup} P);$

$M \tilde{\cap} (N \tilde{\cup} P) = (M \tilde{\cap} N) \tilde{\cup} (M \tilde{\cap} P).$
(Distributive laws)

Proof: Most of the proofs follow from definitions.
Let us, for example, prove the first parts of (vi), (ix) and (x).

(vi) For each *i* and *j*,

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$$\left(M \tilde{\cup} N\right)^0 = \left(\left[a_{ij}\right] \tilde{\cup} \left[b_{ij}\right]\right)^0$$
$$= \left[\max\left\{a_{ij}, b_{ij}\right\}\right]^0$$
$$= \left[1 - \max\left\{a_{ij}, b_{ij}\right\}\right]$$
$$= \left[\min\left\{1 - a_{ij},\ 1 - b_{ij}\right\}\right]$$
$$= \left[a_{ij}\right]^0 \tilde{\cap} \left[b_{ij}\right]^0$$
$$= M^0 \tilde{\cap} N^0.$$

(ix)

$$M \tilde{\cup} \left(N \tilde{\cup} P\right) = \left[a_{ij}\right] \tilde{\cup} \left(\left[b_{ij}\right] \tilde{\cup} \left[c_{ij}\right]\right)$$
$$= \left[\max\left\{a_{ij}, \max\left(b_{ij}, c_{ij}\right)\right\}\right]$$
$$= \left[\max\left\{\max\left(a_{ij}, b_{ij}\right), c_{ij}\right\}\right]$$
$$= \left(\left[a_{ij}\right] \tilde{\cup} \left[b_{ij}\right]\right) \tilde{\cup} \left[c_{ij}\right]$$
$$= \left(M \tilde{\cup} N\right) \tilde{\cup} P.$$

(x)

$$M \tilde{\cup} \left(N \tilde{\cap} P\right) = \left[a_{ij}\right] \tilde{\cup} \left(\left[b_{ij}\right] \tilde{\cap} \left[c_{ij}\right]\right)$$
$$= \left[\max\left\{a_{ij}, \min\left(b_{ij}, c_{ij}\right)\right\}\right]$$
$$= \left[\min\left\{\max\left\{a_{ij}, b_{ij}\right\}, \max\left\{a_{ij, c_{ij}}\right\}\right\}\right]$$
$$= \left(\left[a_{ij}\right] \tilde{\cup} \left[b_{ij}\right]\right) \tilde{\cap} \left(\left[a_{ij}\right] \tilde{\cup} \left[c_{ij}\right]\right)$$
$$= \left(M \tilde{\cup} N\right) \tilde{\cap} \left(M \tilde{\cup} P\right).$$

Remark: Proof of (ix) for example can also be seen by using two-valued truth table of a well-formed formula containing three atomic propositions.

Definition 3.2 [4]: **Product of Soft Matrix**

Let $M = \left[a_{ij}\right], N = \left[b_{ik}\right] \in SM(U)_{m\times n}$. Then

(i) *AND-product* of *M* and *N*, denoted $M \wedge N$, is defined $\wedge : SM(U)_{m\times n} \times SM(U)_{m\times n} \to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right] \wedge \left[b_{ik}\right] = \left[c_{ip}\right]$, where $c_{ip} = \min\left\{a_{ij}, b_{ik}\right\}$ and $P = n(j-1) + k$.

(ii) *OR-product* of *M* and *N*, denoted $M \vee N$, is defined $\vee : SM(U)_{m\times n} \times SM(U)_{m\times n} \to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right] \vee \left[b_{ik}\right] = \left[c_{ip}\right]$, where $c_{ip} = \max\left\{a_{ij}, b_{ik}\right\}$ and $P = n(j-1) + k$.

(iii) *AND-NOT-product* of *M* and *N*, denoted $M \overline{\wedge} N$, is defined $\overline{\wedge} : SM(U)_{m\times n} \times SM(U)_{m\times n} \to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right] \overline{\wedge} \left[b_{ik}\right] = \left[c_{ip}\right]$, where $c_{ip} = \min\left\{a_{ij}, 1 - b_{ik}\right\}$ and $P = n(j-1) + k$.

(iv) *OR-NOT-product* of *M* and *N*, denoted $M \underline{\vee} N$, is defined $\underline{\vee} : SM(U)_{m\times n} \times SM(U)_{m\times n} \to SM(U)_{m\times n^2}$ such that $\left[a_{ij}\right] \underline{\vee} \left[b_{ik}\right] = \left[c_{ip}\right]$, where $c_{ip} = \max\left\{a_{ij}, 1 - b_{ik}\right\}$ and $P = n(j-1) + k$.

Proposition 3.2 [4]

Let $M = \left[a_{ij}\right],\ N = \left[b_{ik}\right] \in SM(U)$. Then the following hold:

(i) $(M \vee N)^0 = M^0 \wedge N^0;\ (M \wedge N)^0 = M^0 \vee N^0$ (De Morgan's laws)

(ii) $(M \underline{\vee} N)^0 = M^0 \overline{\wedge} N^0;\ (M \overline{\wedge} N)^0 = M^0 \underline{\vee} N^0$ (De Morgan's laws)

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Proof: The proofs follow from definitions.

Example 3.2

Let $M = [a_{ij}],\ N = [b_{ik}] \in SM(U)_{4\times 4}$ given by

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } N = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Then

$$M \wedge N = [a_{ij}] \wedge [b_{ik}] = [c_{ip}]_{4\times 16} = \begin{bmatrix} 0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&1 \\ 0&0&0&0&1&0&0&1&1&0&0&1&1&0&0&1 \\ 0&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

Similarly, the other products $M \vee N,\ M \,\overline{\wedge}\, N$ and $M \,\underline{\vee}\, N$ can be found.

Also, $$(M \wedge N)^{\circ} = \begin{bmatrix} 1&1&1&1&1&1&0&0&1&1&1&1&1&1&0&0 \\ 1&1&1&1&0&1&1&0&0&1&1&0&0&1&1&0 \\ 1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1 \\ 1&1&1&1&1&1&0&0&1&1&1&1&1&1&1&1 \end{bmatrix},$$

$$M^{\circ} \vee N^{\circ} = \begin{bmatrix} 1&0&1&0 \\ 1&0&0&0 \\ 1&0&0&1 \\ 1&0&1&1 \end{bmatrix} \vee \begin{bmatrix} 1&1&0&0 \\ 0&1&1&0 \\ 1&1&0&0 \\ 1&1&0&0 \end{bmatrix} = \begin{bmatrix} 1&1&1&1&1&1&0&0&1&1&1&1&1&1&0&0 \\ 1&1&1&1&0&1&1&0&0&1&1&0&0&1&1&0 \\ 1&1&1&1&1&1&0&0&1&1&0&0&1&1&1&1 \\ 1&1&1&1&1&1&0&0&1&1&1&1&1&1&1&1 \end{bmatrix}.$$

Thus $(M \wedge N)^0 = M^0 \vee N^0$.

Note that the commutative laws are not valid for products of soft matrices.

4.COMPARING OPERATIONS ON SOFT SETS WITH OPERATIONS ON THEIR CORRESPONDING SOFT MATRICES

We recall from section 2 that a soft set can be uniquely characterized by a soft matrix and vice versa. Thus a soft set (f_A, E) over U is formally equal to its soft matrix $[a_{ij}]_{m\times n}$ where $m = |U|$ and $n = |E|$, the cardinality of U and E respectively.

In order to show that the operations on soft sets and that on their corresponding soft matrices are equivalent, we first recall the following definitions on soft sets.

Definition 4.1 [5]

Let (f_A, E) and (f_B, E) be two soft sets over *U*. We define the

(i) *union* of (f_A, E) and (f_B, E), denoted $(f_A, E) \tilde{\cup} (f_B, E)$, as a soft set (f_C, E), defined by the approximate function $f_C(e) = f_A(e) \cup f_B(e)\ \ \forall\ \ e \in E$.

(ii) *intersection* of (f_A, E) and (f_B, E), denoted $(f_A, E) \tilde{\cap} (f_B, E)$, as a soft set

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$\left(f_D, E\right)$,defined by the approximate function
$f_D(e) = f_A(e) \cap f_B(e) \;\; \forall \;\; e \in E$.

(iii) *complement* of $\left(f_A, E\right)$ denoted
$\left(f_A, E\right)^C = \left(f_A^C, E\right)$ or $\left(f_{A^C}, E\right)$,as a soft set $\left(f_{A^C}, E\right)$,defined by the approximate function $f_{A^C}(e) = U - f_A(e) \;\; \forall \;\; e \in E$.

(iv) *difference* of $\left(f_A, E\right)$ and $\left(f_B, E\right)$, denoted
$\left(f_A, E\right) \backslash \left(f_B, E\right)$,as a soft set
$\left(f_K, E\right)$,defined by the approximate function
$f_K(e) = f_A(e) - f_B(e) = f_A(e) \cap f_{B^C}(e) \;\; \forall \;\; e \in E$.

(v) *AND-product* of $\left(f_A, E\right)$ and $\left(f_B, E\right)$, denoted
$\left(f_A, E\right) \wedge \left(f_B, E\right)$,as a soft set
$\left(f_G, E \times E\right)$,defined by the function
$f_G(x, y) = f_A(x) \cap f_B(y) \;\; \forall \;\; (x, y) \in E \times E$.

(vi) *OR-product* of $\left(f_A, E\right)$ and $\left(f_B, E\right)$, denoted
$\left(f_A, E\right) \vee \left(f_B, E\right)$,as a soft set
$\left(f_H, E \times E\right)$,defined by the function
$f_H(x, y) = f_A(x) \cup f_B(y) \;\; \forall \;\; (x, y) \in E \times E$.

We illustrate them as follows:

Example 4.1

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the universe set and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters.
Let $A = \{e_1, e_2\}$ and $B = \{e_2, e_3, e_4\}$. Suppose that the soft sets $\left(f_A, E\right)$ and $\left(f_B, E\right)$ are given by

$$\left(f_A, E\right) = \left\{\left(e_1, \{u_2, u_4\}\right), \left(e_2, \{u_1, u_2\}\right)\right\}$$

and

$$\left(f_B, E\right) = \left\{\left(e_2, \{u_1, u_2\}\right), \left(e_3, \{u_1, u_4\}\right), \; \left(e_4, U\right)\right\},$$

respectively.

Then, their soft matrices are respectively given by

$$\left[a_{ij}\right] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \left[b_{ij}\right] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

From Definition 4.1:

(i) $\left(f_A, E\right) \tilde{\cup} \left(f_B, E\right) = \left(f_C, E\right)$, where
$f_C(e) = f_A(e) \cup f_B(e), \;\; \forall \;\; e \in E$.
Then
$\left(f_C, E\right) = \left\{\left(e_1, \{u_2, u_4\}\right), \left(e_2, \{u_1, u_2, u_3\}\right), \left(e_3 \{u_1, u_4\}\right), \left(e_4, U\right)\right\}$
and its soft matrix $\left[c_{ij}\right]$ is given by

$$\left[c_{ij}\right] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[a_{ij}\right] \tilde{\cup} \left[b_{ij}\right].$$

(ii) $\left(f_A, E\right) \tilde{\cap} \left(f_B, E\right) = \left(f_D, E\right)$, where
$f_D(e) = f_A(e) \cap f_B(e), \;\; \forall \;\; e \in E$

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Then $(f_D, E) = \{(e_2, \{u_1\})\}$ and its soft matrix $[d_{ij}]$ is given by

$$[d_{ij}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = [a_{ij}] \tilde{\cap} [b_{ij}].$$

(iii) $(f_A, E)^C = (f_{A^C}, E)$, where

$$f_{A^C}(e) = U - f_A(e) \ \ \forall \ \ e \in E.$$

Then

$$(f_{A^C}, E) = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_4, u_5\}), (e_3, U), (e_4, U)\}$$

and its soft matrix $[a_{ij^C}]$ is given by

$$[a_{ij^C}] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = [a_{ij}]^0.$$

(iv) $(f_A, E) \backslash (f_B, E) = (f_K, E)$, where

$$f_K(e) = f_A(e) - f_B(e) = f_A(e) \cap f_{B^C}(e) \ \ \forall \ \ e \in E$$

.
Then

$(f_K, E) = \{(e_1, \{u_2, u_4\}), (e_2, \{u_3\})\}$ and its soft matrix $[k_{ij}]$ is given by

$$[k_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = [a_{ij}] \tilde{\cap} [b_{ij}]^0.$$

(v) $(f_A, E) \wedge (f_B, E) = (f_G, E \times E)$, where

$$f_G(x, y) = f_A(x) \cap f_B(y) \ \ \forall \ \ (x, y) \in E \times E.$$

Then

$$(f_G, E \times E) = \{((e_1, e_2), \{u_2\}), ((e_1, e_3), \{u_4\}), ((e_1, e_4), \{u_2, u_4\}) \\ ((e_2, e_2), \{u_1\}), ((e_2, e_3), \{u_1\}), ((e_2, e_4), \{u_1, u_3\})\}$$

and its soft matrix $[g_{ip}]$ is given by

$$[g_{ip}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = [a_{ij}] \wedge [b_{ik}].$$

(vi) $(f_A, E) \vee (f_B, E) = (f_H, E \times E)$, where $f_H(x, y) = f_A(x) \cup f_B(y) \ \ \forall \ \ (x, y) \in E \times E$.

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Then

$$\begin{aligned}(f_H, E\times E) = \{&((e_1,e_1),\ \{u_2,u_4\}),((e_1,e_2),\{u_1,u_2,u_4\}),\\ &((e_1,e_3),\{u_1,u_2,u_4\}),((e_1,e_4),U),\\ &((e_2,e_1),\{u_1,u_3\}),((e_2,e_2),\{u_1,u_2,u_3\}),\\ &((e_2,e_3),\{u_1,u_3,u_4\}),((e_2,e_4),U),\\ &((e_3,e_2),\{u_1,u_2\}),((e_3,e_3),\{u_1,u_4\}),((e_3,e_4),U)\\ &((e_4,e_2),\{u_1,u_2\}),((e_4,e_3),\{u_1,u_4\}),((e_4,e_4),U)\}\end{aligned}$$

and its soft matrix $\left[h_{ip}\right]$ is given by

$$\left[h_{ip}\right]=\begin{bmatrix}0&1&1&1&1&1&1&1&0&1&1&1&0&1&1&1\\1&1&1&1&0&1&0&1&0&1&0&1&0&1&0&1\\0&0&0&1&1&1&1&1&0&0&0&1&0&0&0&1\\1&1&1&1&0&0&1&1&0&0&1&1&0&0&1&1\\0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1\end{bmatrix}=\left[a_{ij}\right]\vee\left[b_{ik}\right].$$

Similarly,the other products

$(f_A, E)\overline{\wedge}(f_B, E)=\left[a_{ij}\right]\overline{\wedge}\left[b_{ik}\right]$ and

$(f_A, E)\underline{\vee}(f_B, E)=\left[a_{ij}\right]\underline{\vee}\left[b_{ik}\right]$ can also be found.

5.CONCLUSION

After defining soft sets and soft matrices, we describe operations on both of them and show that they are equivalent. Despite having found some sporadic applications, one of the important areas of future research is concerned with constructing an efficient algorithm for solving a complex problem using soft matrices.

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