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# Man Power Training Model with Bulk Arrival, Bulk Service and Trainer Vacation

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## ABSTRACT

The paper deals with a single trainer who offers training programme and takes optional vacation after a training programme of batch size M is completed. Trainees who undergo training arrive in batches according to a compound Poisson process with batch arrival rate λ. Training is offered to batches of fixed size M (M$\geq$1) and the service times of successive batches follows a general distribution with density function A(x). The Laplace transforms of the probability generating function of different states of the system are obtained using supplementary variable technique. The corresponding steady state results have been derived. A particular case is discussed explicitly.

**Keywords:** *Batch Arrival, Batch Service, General Service Times, Trainer Vacations, Probability Generating Function, Laplace Transforms, Idle State, Steady State.*

## 1. INTRODUCTION

In several practical situations, the server may be unavailable to the customers (primary customers) over certain periods of time. The server may then be doing other work such as maintenance work or servicing secondary customers. The periods for which the server is unavailable are said to be server vacation periods. Vacation models have interesting theoretical properties which have their applicability in manpower planning, production, communication and computer design. Starting with Gaver [5] vacation queues have been receiving attention from researchers. Miller [8] was the first to study such a model where the server is unavailable during some random length of time (vacation) for the M/G/1 queuing system. Medhi [7] has also extensively studied on vacation models. Vacation models exhibit the stochastic decomposition result which allows the system to be analyzed by considering separately the distribution of the queue size with no vacations and the additional queue size due to vacations. This important result was first established by Fuhrmann and Cooper [3] for generalized vacation as well as multiple vacation models. Doshi [1,2] extended this result for the single vacation model through a different approach, where the server takes exactly one vacation at the end of each busy period. Gautam Choudhury [4] studied the steady state behavior of $M^X/G/1$ queuing system. Motivated by Madan [6] we have developed a manpower model which considers bulk arrival, bulk service and single trainer with optional vacation following Bernoulli schedule. The transient state solution of the trainer while offering training program and when he is on vacation is obtained using supplementary variable technique. The corresponding steady state results are obtained. A numerical illustration is given to validate the model.

## 2. MODEL DESCRIPTION

The efficient and smooth running of an organization depends on the employees with up-to-date knowledge and technical skills, for which training is required. An organization has a single trainer who offers training programs to the employees based on their grades, experience and qualifications. The trainer takes optional vacation following Bernoulli schedule after a training program of batch size M (M≥1) is completed. The trainer takes a vacation with probability p after a training program is over, or he may stay in the system with probability (1-p) and offer training if he finds a batch of M trainees waiting to undergo training. Trainees who undergo training arrive in batches according to a compound Poisson process with the arrival rate being λ. The service times of successive batches follows a general distribution with density function A (x).

The model is based on the following assumptions

### 2.1 Assumptions

a. Trainees arrive for the training program in batches according to a compound Poisson process.
b. The trainer offers training to batches of fixed size M (M$\geq$1) with service times of successive batches following a general distribution.

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c. The trainers may or may not take vacation as soon as the training program of batch of size M is finished. The duration of the vacation period follows an exponential distribution. The trainer may go on vacation with probability p or may continue offering training program with the probability (1-p) in the system if there is a batch of M trainees waiting.

d. On returning from vacation, the trainer instantly starts the training program if there is a batch of size M or he remains idle in the system.

e. Inter arrival times; service times and vacation times are independent of each other.

## 2.2 Notations

$\lambda$ : rate of arrival of a single trainee

M : fixed Size of the batch ($M \geq 1$)

X : random Variable which represents the size of arrival

$C_k$ : probability of [X=k], $k \geq 1$, that is the probability that a batch of k arrives in infinitesimal interval $(t, t + \Delta t)$ is $\lambda C_k \Delta t$

X (z) : probability generating function of X;

$$X(z) = \sum_{k=0}^{\infty} C_k z^k$$

A(x) : distribution function of service time

$\mu(x)\,\Delta x$ : conditional probability that the service of a batch will be completed in time $(x, x + \Delta x)$ that is the elapsed service time is x

$$\mu(x) = \frac{dA(x)}{1 - A(x)}$$

$$A(x) = \mu(x)\, e^{-\int_0^x \mu(t)dt}$$

Q(t) : probability that at time t, the trainer is idle but available in the system

$P_n(x,t)$ : probability that at time t the trainer is offering training program and there are n trainees waiting to undergo training excluding a batch of M trainers in service with elapsed service time, x, ($n \geq 0$)

$P_n(t)$ : probability that at time t there are n trainees waiting to undergo the training program excluding a batch of M trainees undergoing the training program irrespective of the value of x

$$P_n(t) = \int_0^{\infty} P_n(x,t)dx$$

b : proportional vacation time. The vacation period is exponentially distributed with inter-vacation time 1/b

$V_n(t)$ : probability that at time t, there are n, ($n \geq 0$) trainees waiting to undergo training and the trainer is on vacation

## 3. MATHEMATICAL FORMULATION AND ANALYSIS

The Chapman-Kolmogrov equations governing the system are

$$\frac{\partial}{\partial x}P_n(x,t) + \frac{\partial}{\partial t}P_n(x,t) = -[\lambda + \mu(x)]\, P_n(x,t) + \mu \sum_{k=1}^{n} C_k P_{n-k}(x,t)$$

$$\frac{\partial}{\partial x}P_n(x,t) + \frac{\partial}{\partial t}P_n(x,t) + [\lambda + \mu(x)]\, P_n(x,t) = \lambda \sum_{k=1}^{n} C_k P_{n-k}(x,t) \quad (1)$$

$$\frac{\partial}{\partial x}P_0(x,t) + \frac{\partial}{\partial t}P_0(x,t) + [\lambda + \mu(x)]\, P_0(x,t) = 0 \quad (2)$$

$$\frac{d}{dt}V_n(t) = -(\lambda + b)V_n(t) + \lambda \sum_{k=1}^{n} C_k V_{n-k}(t) + p\int_0^{\infty} P_n(x,t)\mu(x)dx$$

$$\frac{d}{dt}V_n(t) + (\lambda + b)V_n(t) = \lambda \sum_{k=1}^{n} C_k V_{n-k}(t) + p\int_0^{\infty} P_n(x,t)\mu(x)dx \quad (3)$$

$$\frac{d}{dt}V_0(t) + (\lambda + b)V_0(t) = p\int_0^{\infty} P_0(x,t)\mu(x)dx \quad (4)$$

$$\frac{d}{dt}Q(t) = -\lambda Q(t) + b\,V_0(t) + (1-p)\int_0^{\infty} P_0(x,t)\mu(x)dx$$

$$\frac{d}{dt}Q(t) + \lambda Q(t) = b\,V_0(t) + (1-p)\int_0^{\infty} P_0(x,t)\mu(x)dx \quad (5)$$

The above equations are to be solved subject to the boundary conditions

$$P_n(0,t) = b\,V_{n+M}(t) + (1-p)\int_0^{\infty} P_n(x,t)\mu(x)dx \quad (6)$$

$$P_0(0,t) = \lambda Q(t) + b$$

$$\sum_{r=1}^{M} V_r(t) + (1-p)\int_0^{\infty} P_0(x,t)\mu(x)dx \quad (7)$$

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We assume that initially there is no trainee in the system and the trainer is not under vacation.

## 3.1 Initial Conditions

$P_n(0) = 0,\ V_n(0) = 0,\ V_0(0) = 0,\ Q(0) = 1$ for $n = 0,1,2,\ldots$ (8)

## 3.2 Transient Solution

The following probability generating functions are used to solve the above differential difference equations

$$\left.\begin{aligned} P(x,z,t) &= \sum_{n=0}^{\infty} z^n P_n(x,t) \\ P(z,t) &= \sum_{n=0}^{\infty} z^n P_n(t) \\ V(z,t) &= \sum_{n=0}^{\infty} z^n V_n(t) \\ P_n(t) &= \int_0^{\infty} P_n(x,t)\,dx \end{aligned}\right\} \quad (9)$$

$$P_n(z,t) = \int_0^{\infty} P_n(x,z,t)\,dx$$

The Laplace transform of a function f(t) is defined as

$$\left.\begin{aligned} &\bar{F}(s) = L[f(t)] = \int_0^{\infty} e^{-st} f(t)\,dt,\ \mathrm{Re}(s)>0 \\ &\text{Also } L\left[\tfrac{d}{dt} f(t)\right] = s\bar{f}(s) - f(0) \end{aligned}\right\} \quad (10)$$

Taking Laplace Transform of equations (1) - (7) and using (8) and (10) we get,

$$\frac{\partial}{\partial x}\bar{P}_n(x,s) + (s+\lambda+\mu(x))\bar{P}_n(x,s) = \lambda\sum_{k=1}^{n} C_k \bar{P}_{n-k}(x,s) \quad (11)$$

$$\frac{\partial}{\partial x}\bar{P}_0(x,s) + (s+\lambda+\mu(x))\bar{P}_0(x,s) = 0 \quad (12)$$

$$(s+\lambda+b)\bar{V}_n(s) = \lambda\sum_{k=1}^{n} C_k \bar{V}_{n-k}(s) + p\int_0^{\infty}\bar{P}_n(x,s)\mu(x)\,dx \quad (13)$$

$$(s+\lambda+b)\bar{V}_0(s) = p\int_0^{\infty}\bar{P}_0(x,s)\mu(x)\,dx \quad (14)$$

$$(s+\lambda)\bar{Q}(s) = 1 + b\bar{V}_0(s) + (1-p)\int_0^{\infty}\bar{P}_0(x,s)\mu(x)\,dx \quad (15)$$

$$\bar{P}_n(0,s) = b\bar{V}_{n+M}(s) + (1-p)\int_0^{\infty}\bar{P}_n(x,s)\mu(x)\,dx \quad (16)$$

$$\bar{P}_0(0,s) = \lambda\bar{Q}(s) + b\sum_{r=1}^{M}\bar{V}_r(s) + (1-p)\int_0^{\infty}\bar{P}_0(x,s)\mu(x)\,dx \quad (17)$$

Multiplying (11) by $z^n$, summing over n = 1 to ∞, adding with (12) and using (9) we get

$$\begin{aligned} \frac{\partial}{\partial x}\bar{P}(x,z,s) + (s+\lambda+\mu(x))\bar{P}(x,z,s) &= \lambda\sum_{n=1}^{\infty}\sum_{k=1}^{n} C_k P_{n-k} z^n \\ &= \lambda\sum_{k=1}^{\infty} C_k z^k \left\{\sum_{n=k}^{\infty} P_{n-k} z^{n-k}\right\} \\ &= \lambda X(z)\bar{P}(x,z,s) \end{aligned}$$

$$\frac{\partial}{\partial x}\bar{P}(x,z,s) + [s+\lambda-\lambda X(z)+\mu(x)]\bar{P}(x,z,s) = 0 \quad (18)$$

Multiplying (13) by $z^n$, summing over n=1 to ∞, adding with (14) and using (9) we get

$$(s+\lambda-\lambda X(z)+b)\bar{V}(z,s) = p\int_0^{\infty}\bar{P}(x,z,s)\mu(x)\,dx \quad (19)$$

Multiplying (16) by $z^n$, summing over n=1 to ∞, adding with (17) and using (9) we get

$$\bar{P}(0,z,s) = bz^{-M}\bar{V}(z,s) + (1-p)\int_0^{\infty}\bar{P}(x,z,s)\mu(x)\,dx + \lambda\bar{Q}(s) - b\bar{V}_0(s) + b\sum_{r=1}^{M-1}(z^M - z^r)\bar{V}_r(s)$$

$$z^M\bar{P}(0,z,s) = b\bar{V}(z,s) + z^M(1-p)\int_0^{\infty}\bar{P}(x,z,s)\mu(x)\,dx + \lambda z^M\bar{Q}(s) + b\sum_{r=1}^{M-1}(z^M - z^r)\bar{V}_r(s) - b\bar{V}_0(s) \quad (20)$$

Using (15) we get

$$\bar{P}(0,z,s) = bz^{-M}\bar{V}(z,s) + (1-p)\int_0^{\infty}\bar{P}(x,z,s)\mu(x)\,dx + [\lambda-(s+\lambda)z^{-M}]\bar{Q}(s) + b\sum_{r=1}^{M-1}\bar{V}_r(s)(1-z^{-M+r}) + z^{-M} \quad (21)$$

Solving (11) we get

$$\bar{P}(x,z,s) = \bar{P}(0,z,s)\exp\left\{-(s+\lambda-\lambda X(z))x - \int_0^x \mu(x)\,dx\right\} \quad (22)$$

where $\bar{P}(0,z,s)$ is given by equation (21).

Integrating (22) with respect to x, we get,

$$\bar{P}(z,s) = \bar{P}(0,z,s)\left[\frac{1-\bar{A}(s+\lambda-\lambda X(z))}{s+\lambda-\lambda X(z)}\right] \quad (23)$$

Where $\bar{A}(s+\lambda-\lambda X(z)) = \int_0^{\infty} e^{-(s+\lambda-\lambda X(z))x} A(x)\,dx$

From (22) we get

$$\int_0^{\infty}\bar{P}(x,z,s)\mu(x)\,dx = \bar{P}(0,z,s)\bar{A}(s+\lambda-\lambda X(z)) \quad (24)$$

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Also

$$\int_0^{\infty} \bar{P}(x,s)\mu(x)dx = \bar{P}(0,s)\bar{A}(s+\lambda) \quad (25)$$

From (19) and (24)

$$\bar{V}(z,s) = p\,\frac{\bar{P}(0,z,s)\bar{A}(s+\lambda-\lambda X(z))}{[s+\lambda-\lambda X(z)+b]} \quad (26)$$

Using (21) and (24) we get

$$\bar{P}(0,z,s) = \frac{bz^{-M}\bar{V}(z,s)+b\sum_{r=1}^{M-1}(1-z^{-M+r})\bar{V}_r(s)+z^{-M}+[\lambda-(s+\lambda)z^{-M}]\bar{Q}(s)+(1-p)\bar{P}_0(0,s)\bar{A}(s+\lambda)}{[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))]} \quad (27)$$

Using equations (26) and (27) we get,

$$\bar{V}(z,s) = \frac{\{b\sum_{r=1}^{M-1}(1-z^{-M+r})\bar{V}_r(s)+[\lambda-(s+\lambda)z^{-M}]\bar{Q}(s)+z^{-M}\}p\bar{A}[s+\lambda-\lambda X(z)]}{[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))][s+\lambda-\lambda X(z)+b]-bz^{-M}p\bar{A}[s+\lambda-\lambda X(z)]}$$

$$= \frac{\{b\sum_{r=1}^{M-1}(z^{M}-z^{r})\bar{V}r(s)+[\lambda(z^{M-1})-s]\bar{Q}(s)+(1-p)\bar{P}_0(0,s)\bar{A}(s+\lambda)+1\}p\bar{A}\,[s+\lambda-\lambda X(z)]}{[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))][s+\lambda-\lambda X(z)+b]z^{M}-bp\bar{A}[s+\lambda-\lambda X(z)]} \quad (28)$$

There are M unknowns, $\bar{Q}(s)$ and $\bar{V}_r(s)$, r = 1, 2… M-1 in (28). In order to determine these we apply Rouche's Theorem as follows:

Let $f(z) = z^M$ and $g(z) = \frac{bp\bar{A}(s+\lambda-\lambda X(z))}{[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))][s+\lambda-\lambda X(z)+b]}$

Both f(z) and g(z) are differentiable inside and continuous on the contour $|z|$=1.

On the contour |z| = 1, |f (z)| = 1.

Moreover for Re s≥0, $|e^{-\sigma x}| \le 1$ and since $\int_0^{\infty} A(x)dx = 1$ we have

$$|\bar{A}(s+\lambda-\lambda X(z))| \le 1, \quad \text{Re } s \ge 0 \quad (29)$$

$$|g(z)| = \frac{|bp\bar{A}(s+\lambda-\lambda X(z))|}{|[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))][s+\lambda-\lambda X(z)+b]|}$$

$$= \frac{|bp\bar{A}(s+\lambda-\lambda X(z))|}{|[1-(1-p)\bar{A}(s+\lambda-\lambda X(z))]||[s+\lambda-\lambda X(z)+b]|}$$

Using (29), we get

$$|g(z)| < 1 = |f(z)|$$

Thus on $|z|$ = 1, $|f(z)| > |g(z)|$

Hence by Rouche's theorem $f(z) - g(z)$ has the same number of zeroes as that of $f(z)$ inside the circle $|z|$=1. Clearly $f(z)$ has M zeroes inside the unit circle and hence $f(z) - g(z)$ that is the denominator on the right hand side of (28) has M zeroes inside the unit circle $|z|$=1. Since $\bar{V}$(s,z) is regular in the unit circle $|z|$=1, the numerator on the right hand side of equation (28) must vanish for these zeroes of the denominator and hence the M unknowns can be found.

Thus $\bar{V}$ (z,s) & $\bar{P}$(z,s) can be determined.

## 4. PARTICULAR CASE

### 4.1 Arrival Pattern

Let X has geometric (decapitated) distribution

$$C_k = P(X = k) = c\,(1-c)^{k-1},$$

$$0 < c < 1, k=1, 2, 3\ldots. \quad (30)$$

With the probability generating function

$$X(z) = \frac{cz}{[1-(1-c)z]} \quad (31)$$

$$X'(1) = E(X) = \bar{C} = 1/c \quad (32)$$

$$X''(1) = \frac{2(1-c)}{c^2} \quad (33)$$

### 4.2 Service Pattern

Let the service time distribution follows a exponential distribution with parameter Mμ

$$A(x) = M\mu e^{-M\mu x}, x > 0 \quad (34)$$

$$\bar{A}(s+\lambda-\lambda X(z)) = \int_0^{\infty} e^{-(s+\lambda-\lambda X(z))t} M\mu e^{-M\mu t}dt$$

$$= \frac{M\mu}{s+\lambda-\lambda X(z)+M\mu}$$

$$= \frac{M\mu}{s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu} \quad (35)$$

Also, $\bar{A}(s+\lambda) = \frac{M\mu}{s+\lambda+M\mu}$ (36)

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Substituting (35) and (36) in (28), we get

$\bar{V}(z,s) =$

$$\frac{\{b\sum_{r=1}^{M-1}(z^M - z^r)\bar{V}_r(s) + [\lambda(z^M-1)-s]\bar{Q}(s) + (1-p)\bar{P}_0(0,s)(\frac{M\mu}{s+\lambda+M\mu}) + 1\}pM\mu/(s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu)}{\left[1-(1-p)M\mu/(s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu)\right]\left[s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+b\right]z^M - bp\frac{M\mu}{s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu}}$$

$$=\frac{\{b\sum_{r=1}^{M-1}(z^M - z^r)\bar{V}_r(s) + [\lambda(z^M-1)-s]\bar{Q}(s) + (1-p)\bar{P}_0(0,s)(\frac{M\mu}{s+\lambda+M\mu}) + 1\}pM\mu/(s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu)}{\left[s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+Mp\mu\right]\left[s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+b\right]z^M - bp\frac{M\mu}{[s+\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu]}}$$

(37)

$$\bar{P}(z,s) = \frac{\bar{P}(0,z,s)}{[s+\lambda-\lambda\frac{cz}{[1-(1-c)z]}+M\mu]}$$

(38)

## 4.3 Steady State Results

The corresponding steady state results can be obtained by using the Tauberian property

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t)$$

(39)

if the limit on the right exists.

## 4.4 Theorem

The steady state probability distribution for the manpower training model with bulk arrival, bulk service following general service distribution subject to optional trainer vacation based on Bernoulli schedule with single vacation policy are given by

$$P(1) = \frac{b[b(\sum_{r=1}^{M-1}(M-r)V_r + \lambda MQ]}{[M^2bp\mu - \lambda\bar{C}(Mp\mu+b)]}$$

$$V(1) = \frac{[b(\sum_{r=1}^{M-1}(M-r)V_r + \lambda MQ]Mp\mu}{M^2bp\mu - \lambda\bar{C}(pM\mu+b)}$$

where

Q =

$$\frac{1-\left(1+\frac{b}{Mp\mu}\right)\left[\frac{bMp\mu\sum_{r=1}^{M-1}(M-r)V_r}{M^2bp\mu-\lambda\bar{C}(Mp\mu+b)}\right]}{\left[1+\frac{\lambda M^2p\mu}{M^2bp\mu-\lambda\bar{C}(Mp\mu+b)}\left(1+\frac{b}{Mp\mu}\right)\right]}$$

Provided $M^2bp\mu > \lambda\bar{C}(Mp\mu+b)$

Where

P (1) is the steady state probability that the trainer is offering training program,

V (1) is the steady state probability that the trainer is on vacation,

Q is the probability that the trainer is idle,

b is the proportional vacation time,

M is the batch size,

p is the probability that the trainer is on vacation,

λ is the rate of arrival,

μ is the rate at which the trainer offers the training program,

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$V_r$ is the probability that there are r trainees waiting to undergo training program, r =1, 2, … and

$\overline{C}$ is the mean of X, that is $\overline{C} = \frac{1}{c}$.

## 4.5 Proof

From (37) we have

$$\overline{V}(z,s) = \frac{\{b\sum_{r=1}^{M-1}(z^M - z^r)\overline{V}_r(s) + \left[\lambda(z^M-1) - s\right]\overline{Q}(s) + (1-p)\overline{P}_0(0,s)\frac{M\mu}{s+\lambda+M\mu} + 1\}\frac{pM\mu}{(s+\lambda - \frac{\lambda cz}{[1-(1-c)z]} + M\mu)}}{\left[s+\lambda - \frac{\lambda cz}{[1-(1-c)z]} + Mp\mu\right]\left[s+\lambda - \frac{\lambda cz}{[1-(1-c)z]} + b\right]z^M - \frac{bpM\mu}{[s+\lambda - \frac{\lambda cz}{[1-(1-c)z]} + M\mu]}}$$

Multiplying both sides by s taking limit as s→0and applying property (39) we get

$$V(z) = \lim_{s\to 0} s\overline{V}(z,s)$$

$$= \frac{[b\sum_{r=1}^{M-1}(z^M - z^r)V_r + \lambda(z^M-1)Q]\frac{pM\mu}{(\lambda - \frac{\lambda cz}{[1-(1-c)z]} + M\mu)}}{\left[\lambda - \frac{\lambda cz}{[1-(1-c)z]} + Mp\mu\right]\left[\lambda - \frac{\lambda cz}{[1-(1-c)z]} + b\right]z^M - \frac{bpM\mu}{[\lambda - \frac{\lambda cz}{[1-(1-c)z]} + M\mu]}}$$

(40)

For z = 1, the right hand side of (40) is indeterminate of the form$\left(\frac{0}{0}\right)$.

Using 'L' Hospitals rule we have,

$$V(1) = \lim_{z\to 1} V(z)$$

$$= \frac{[b(\sum_{r=1}^{M-1}(M-r)V_r + \lambda MQ]p\mu}{M^2 bp\mu - \lambda\overline{C}(Mp\mu + b)}, \quad M^2 bp\mu > \lambda\overline{C}(Mp\mu + b)$$

(41)

Where $\overline{C} = X'(1) = \frac{1}{c}$ (the mean of X).

This is the steady state probability that the trainer is on vacation irrespective of the number of trainees waiting to undergo the training program.

Using (35) and (36) in (27) we get

$\overline{P}(0,z,s) =$

$$\frac{bz^{-M}\overline{V}(z,s) + b\sum_{r=1}^{M-1}(1 - z^{-M+r})\overline{V}_r(s) + z^{-M} + [\lambda - (s+\lambda)z^{-M}]\overline{Q}(s) + (1-p)\overline{P}_0(0,s)\frac{M\mu}{s+\lambda+M\mu}}{[1-(1-p)\frac{M\mu}{[s+\lambda - \frac{\lambda cz}{[1-(1-c)z]}] + M\mu}]}$$

(42)

Multiplying both sides by s taking limit as s→0 and applying property (39) we get

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$$P(0,z)=\frac{bz^{-M}V(z)+b\sum_{r=1}^{M-1}(1-z^{-M+r})\overline{V}_r+\lambda(z^M-1)Q}{[1-(1-p)\frac{M\mu}{[\lambda-\frac{\lambda cz}{[1-(1-c)z]}+M\mu]}]} \quad (43)$$

$$P(z)=\lim_{s\to 0} s\overline{P}(z,s)$$
$$=\frac{P(0,z)}{[\lambda-\frac{\lambda cz}{[1-(1-c)z]}]+M\mu} \quad (44)$$

Substituting (43) in (44) yields

$$P(1)=\lim_{z\to 1} P(z)=\frac{bV(1)}{pM\mu}$$

$$=\frac{b[b(\sum_{r=1}^{M-1}(M-r)V_r+\lambda MQ]}{M^2bp\mu-\lambda\overline{C}(pM\mu+b)},\ M^2bp\mu>\lambda\overline{C}(Mp\mu+b) \quad (45)$$

This is the steady state probability that the trainer is offering training program irrespective of the number of trainees waiting to undergo training.
The normalizing condition is $P(1) + V(1) + Q = 1$ (46)

$$Q=\frac{1-\left(1+\frac{b}{pM\mu}\right)\left[\frac{bMp\mu\sum_{r=1}^{M-1}(M-r)V_r}{M^2bp\mu-\lambda\overline{C}(pM\mu+b)}\right]}{\left[1+\frac{\lambda M^2bp\mu\left(1+\frac{b}{pM\mu}\right)}{M^2bp\mu-\lambda\overline{C}(pM\mu+b)}\right]} \quad (47)$$

This is the probability that the trainer is idle. From this theorem we infer the following:

The stability condition under which the steady state exists is that the utilization factor $\rho < 1$.

The utilization factor is $\rho = 1 - Q$.

Using (46) and (47) we get

$$\rho=1-\frac{\left[\left(1+\frac{b}{pM\mu}\right)\left[\frac{bMp\mu\sum_{r=1}^{M-1}(M-r)V_r}{M^2bp\mu-\lambda\overline{C}(pM\mu+b)}\right]\right]}{\left[1+\frac{M^2bp\mu\left(1+\frac{b}{pM\mu}\right)}{M^2bp\mu-\lambda\overline{C}(pM\mu+b)}\right]}<1 \quad (48)$$

## 5. SPECIAL CASE

In the model developed, if the arrival of trainees is one by one instead of bulk arrival, single service instead of bulk service (that is M = 1) and if the trainer takes compulsory vacation after offering a training program, the corresponding steady state results obtained coincide with that of Madan [6].

Thus the validity of the model is confirmed.

### 5.1 Numerical Illustration

Assuming an 8 hour working day we take the following values for the different parameters

$M=10,\ \lambda=\frac{2}{8}=0.125,\ \mu=\frac{3}{8}=0.375,\ b=\frac{1}{8}=0.125,$

$\overline{C}=\frac{1}{C}=\frac{1}{0.7}=1.1$

Let $V_r$ (the probability of having r trainees waiting to undergo training when the trainer is on vacation) follow a Poisson distribution.

$V_1= 0.1947$ $V_2=0.02433$
$V_3=0.002028$
$V_4= 0.0001267$ $V_1=0.633\times10^{-5}$
$V_6= 0.264\times10^{-7}$ $V_7=9.43\times10^{-9}$
$V_8= 2.94\times10^{-10}$ $V_9=8.8169\times10^{-12}$

$$b\sum_{r=1}^{M-1}(M-r)V_r=0.24346$$

P (1) =0.21706, V (1) =0.65118, Q =0.1327

This illustration shows that the trainer apart from conducting training program is also involved in managing other administrative affairs of the organization (during vacation period). If the training programs are offered frequently more costs are incurred and this may affect the turnover of the organization. This is evident

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from the illustration. Thus this model matches with practical situation and it can be applied in any industry with appropriate, above-mentioned assumptions.

## 6. CONCLUSION

The model developed can be applied in any organization involving human resources and also in diversified fields. The rare combination of bulk arrival, bulk service and vacation of queuing theory is incorporated into the manpower training model. This specified manpower queuing model thus developed also has a broad range of applications in human resource management in any organization, machine breakdowns, maintenance in production, computer and communication systems.

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