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Quantum Black Hole and the Origin of the Universe

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ABSTRACT

Classically a black hole absorbs everything that comes too close and does not emit anything. However, when quantum effects are considered black holes create and emit particles. If the whole mass of the universe oscillates through its centre of gravity, the gravity becomes so strong that the universe will collapse into a quantum black hole. Treating the motion of quantum black hole as simple harmonic and radiation emitted by it as black body radiation we show that the universe was originated from a singular point consisting of one quantum black hole in infinite dimensions with infinite temperature. The amount of energy radiated per unit area was infinite and the total power radiated by the singular point was 1.3922×10^{-47} W.

Keywords: *Big bang; singularity; black body radiation; quantum black hole*

1. INTRODUCTION

Black holes are one of the most fascinating predictions of Einstein's general theory of relativity [1], which predicts that if matter is sufficiently compressed, its gravity becomes so strong that it carves out a region of space from which nothing can escape. Classically a black hole absorbs everything that comes too close and does not emit anything. However, when quantum effects are considered black holes create and emit particles [2]. Research on the physics of black holes over the last few decades has helped us for understanding some very fundamental and unanswered questions of physics [3,4]. One of them is "Was the origin of our universe started from a singular point?". The most important ingredient to answer this question involves a quantum black hole. A quantum black hole [5–7] is an intrinsically semi-classical object rather than fully quantum object. It has a microscopic size but a macroscopic mass. It can be described with the help of quantum mechanics but not necessarily quantum field theory. It obeys the laws of thermodynamics and decays into other particles. It may possess a spin which is integer or half-integer. It may also possess an electric charge. Recently, Majumder [3] has studied quantum black hole with generalized uncertainty principle (GUP) [8]. He found that the leading contribution to black hole's mass comes from the square root of the quantum number n which coincides with Bekenstein's proposal [9–14] that the mass of a black hole is proportional to the square root of the quantum number n. He also found that when quantum gravity effects are considered via the GUP the mass of black hole is directly proportional to the quantum number but its reduces the value of mass for a particular value of the quantum number. Although many theoretical work have been done to mathematically model quantum black holes, their exact nature remains elusive [15,16]. Progress in physics usually requires some guidance from experiment. Now physicists are expected that the LHC is a black hole factory [17]. Quantum black holes will be produced and studied there. The purpose of this paper is to consider the whole universe as a quantum black hole, the motion of the quantum black hole as simple harmonic and the radiation emitted by it as black body radiation. One can regard this idea as associated in some sense with quantum gravity. However, the approach here is simpler and much more elementary than the usual works on quantum gravity.

This paper is organized as follows. In section 2, we assume that if the whole mass of the universe oscillates through its centre of gravity with very high frequencies ~ $1/t_{pl}$, the gravity becomes so strong that the universe will collapse into a quantum black hole. Then we evaluate the mass, Schwarzschild radius and time period of the quantum black hole in terms of Planck mass, Planck length and Planck time respectively. In section 3, we discuss quantum black hole in n dimensions. In section 4, we assume quantum black hole as a black body. Then we calculate the total radiation energy per unit volume, per unit surface area and the total power radiated by the surface in 3 dimensions and n dimensions. In section 5, we discuss the explosion of quantum black hole and creation of particles in the universe. In section 6, we present our conclusions.

2. QUANTUM BLACK HOLE AS A SIMPLE HARMONIC OSCILLATOR

Suppose a hole is drilled from one end of the earth to the other end through its centre. Then the points on the earth's surface on both sides where hole has been drilled can be treated as singular points. If a stone is dropped in this hole, it will oscillate from one end of the earth to the other end in a certain time period performing simple harmonic motion (SHM) [18]. The amplitude of this simple harmonic motion will be R, where R is the radius of the earth. The equation of simple harmonic motion will be given by x = R sinωt, where ω is angular frequency of the oscillating stone. The velocity and

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acceleration can be written as v = Rω cosωt and a = -Rω²sinωt respectively. The maximum acceleration of the stone on the surface of the earth will be $g = R\omega^2 = R(2\pi/t)^2$ and the corresponding time period will be $t = 2\pi\sqrt{R/g}$. The frequency of oscillating particle,

$$\nu = 1/t = 1/2\pi \sqrt{g/R}\ , \quad (1)$$

I.e. the frequency of the oscillating particle is independent of its mass.

Gravitational force on the particle by the earth is given by

$$F = GMm/R^2, \quad (2)$$

Where 'M' is mass of the earth and 'm' is the mass of the particle. On earth's surface,

$$mg = GMm/R^2 \Rightarrow g = GM/R^2 \quad (3)$$

Substituting the value of g from eq. (3) in eq. (1), we get

$$\nu = 1/2\pi \sqrt{GM/R^3} \quad (4)$$

Hence, energy of the oscillating particle

$$E = h/2\pi \sqrt{GM/R^3} = \hbar\sqrt{GM/R^3} \quad (5)$$

Let us apply the same idea to our universe. If the whole mass of the universe oscillates through its centre of gravity with very high frequencies $\sim 1/t_{pl}$, the gravity becomes so strong that the universe will collapse into a quantum black hole and will execute a SHM. Let us assume the mass of quantum black hole is M_B which oscillates through its centre of gravity. Then the energy of the quantum black hole can be written as

$$E_B = \hbar\sqrt{\frac{GM_B}{R_S^3}} \quad (6)$$

where, Schwarzschild radius [6] is given by

$$R_S = 2GM_B/c^2 \quad (7)$$

According to Mass-Energy Equivalence relation the energy of quantum black hole can also be written as:

$$E_B = M_Bc^2 \quad (8)$$

From eqs. (6) and (8),

$$M_Bc^2 = \hbar\sqrt{\frac{GM_B}{R_S^3}} \quad (9)$$

Substituting the value of R_S from eq. (7) in eq. (9) and solving we get

$$M_B = (\hbar c/8^{1/2}\,G)^{1/2} \quad (10)$$

$$= M_{pl}/8^{1/4} = 0.5946\,M_{pl}, \quad (11)$$

Where M_{pl} is the
Planck mass = $(\hbar c/G)^{1/2} = 2.2\times10^{-5}$ gm [6,19].

The time period of quantum black hole is given by,

$$t = 2\pi[(2GM_B/c^2)^3/GM_B]^{1/2} \quad (12)$$

Substituting the value of MB from eq. (10) in eq. (12) and solving we get

$$t = 4\pi/2^{1/4}\,t_{pl} = 10.567\,t_{pl}, \quad (13)$$

Where t_{pl} = Planck time = $(\hbar G/c^5)^{1/2} = 5\times10^{-44}$ sec.

Schwarzschild radius of quantum black hole can be written as

$$\begin{aligned} R_S &= 2GM_B/c^2 = 2GM_{pl}/8^{1/4}\,c^2 \\ &= 2G(\hbar c/G)^{1/2}/8^{1/4}\,c^2 \\ &= (2/8^{1/4})(\hbar G/c^3)^{1/2} \\ &= (2/8^{1/4})\,L_{pl} = 1.1892\,L_{pl}, \end{aligned} \quad (14)$$

Where L_{pl} = Planck length = $(\hbar G/c^3)^{1/2} = 1.6\times10^{-33}$ cm [19, 20].

Acceleration due to gravity on the surface of quantum black hole is given by,

$$\begin{aligned} g_B &= GM_B/R_s^2 = GM_B/(2GM_B/c^2)^2 \\ &= c^4/4GM_B = 2.233\times10^{51}\,m/s^2 \end{aligned} \quad (15)$$

From eq. (15), it is clear that the acceleration due to gravity on the surface of quantum black hole is very high in comparison to the acceleration due to gravity on the surface of earth.

3. QUANTUM BLACK HOLE IN n DIMENSIONS

Quantum black hole can be expressed in n dimensions. Let us consider a hyper sphere in n dimensions. When the number of dimensions n is even, volume is given by

$$V = \pi^{n/2}r^n/(n/2)! \quad (16)$$

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When the number of dimensions n is odd, volume is given by

$$V = 2\pi^{(n-1)/2} [(n+1)/2]!\, (2r)^n/(n+1)! \qquad (17)$$

The hyper surface of a hyper sphere of n dimensions is a curved "surface" figure of n-1 dimensions. As with the volume of the hyper sphere, the hyper surfaces "content" can be expressed by a formula that has a different form for odd and even number of dimensions. When the number of dimensions, n, is even, the surface content is:

$$S = 2\pi^{n/2} r^{n-1}/(n/2 - 1)! \qquad (18)$$

When the number of dimensions, n, is odd, the surface content is:

$$S = 2\pi^{(n-1)/2} [(n-1)/2]!(2r)^{n-1}/(n-1)! \qquad (19)$$

Using the above formulae we have evaluated the volume and surface area formulae for different dimensional hyper sphere (up to n = 7) and are shown in Table I.

Table I: Volume and surface area of hyper sphere

Dimensions (n)	Volume	Surface Area
2	$V_2 = \pi r^2$	$S_2 = 2\pi r$
3	$V_3 = 4/3\ \pi r^3$	$S_3 = 4\pi r^2$
4	$V_4 = \pi^2 r^4/2$	$S_4 = 2\pi^2 r^3$
5	$V_5 = (8/15)\pi^2 r^5$	$S_5 = (8/3\pi^2)\, r^4$
6	$V_6 = \pi^3 r^6/6$	$S_6 = \pi^3 r^5$
7	$V_7 = (16/105)\ \pi^3 r^7$	$S_7 = (16/15)\ \pi^3 r^6$

Volume and surface area of quantum black hole depends upon the number of dimensions considered. The volume and surface area of quantum black hole in different dimensions up to n = 7 are evaluated and are shown in Table II.

Table II: Volume and surface area of quantum black hole

Dimensions (n)	Volume	Surface Area
2	$V_2 = 4.446\ L_{pl}^2$	$S_2 = 7.472\ L_{pl}$
3	$V_3 = 7.0445\ L_{pl}^3$	$S_3 = 17.77\ L_{pl}^2$
4	$V_4 = 9.8694\ L_{pl}^4$	$S_4 = 33.1967\ L_{pl}^3$
5	$V_5 = 12.519\ L_{pl}^5$	$S_5 = 52.6366\ L_{pl}^4$
6	$V_6 = 14.6165\ L_{pl}^6$	$S_6 = 73.7436\ L_{pl}^5$
7	$V_7 = 9.449\ L_{pl}^7$	$S_7 = 93.5422\ L_{pl}^6$

Now let us consider quantum black hole in n dimensions.

CASE I: When n is even,

$$\text{Volume, } V_n = \pi^{n/2} R_S^n/(n/2)! = \pi^{n/2} (2L_{pl}/8^{1/4})^n/(n/2)!$$
$$= (\pi/2\sqrt{2})^{n/2}\, 2^n L_{pl}^n/(n/2)! \qquad (20)$$

Surface area,

$$S_n = 2\pi^{n/2} R_S^{n-1}/(n/2 - 1)!$$
$$= 2\pi^{n/2} (2L_{pl}/8^{1/4})^{n-1}/(n/2 - 1)!$$
$$= 2^{(n+3)/4} \pi^{n/2} L_{pl}^{n-1}/(n/2 - 1)! \qquad (21)$$

CASE II: When n is odd,

$$\text{Volume, } V_n = 2\pi^{(n-1)/2}[(n+1)/2]!\ (2R_S)^n/(n+1)!$$
$$= 2\pi^{(n-1)/2} [(n+1)/2]!\, (4L_{pl}/8^{1/4})^n/(n+1)!$$
$$= 2^{(5n/4)+1} \pi^{(n-1)/2} [(n+1)/2]!\ L_{pl}^n/(n+1)! \qquad (22)$$

Surface area,

$$S_n = 2\pi^{(n-1)/2}[(n-1)/2]!\ (2R_S)^{n-1}/(n-1)!$$
$$= 2\pi^{(n-1)/2} [(n-1)/2]!\, (4L_{pl}/8^{1/4})^{n-1}/(n-1)!$$
$$= 2^{(5n-1)/4} \pi^{(n-1)/2} [(n-1)/2]!\ L_{pl}^{n-1}/(n-1)! \qquad (23)$$

4. QUANTUM BLACK HOLE AS A BLACK BODY

If we assume one quantum black hole as a black body, its total radiation energy per unit volume in the cavity is given by Stefan-Boltzmann expression as [21]

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$$U(T) = \sigma T^4, \qquad (24)$$

where Stefan's constant σ = 7.5662 ×10^{-16} J/m^{3}K^4.

The amount of energy radiated per unit area is given by

$$E(T) = c\, U(T)/4 \qquad (25)$$

Considering quantum black hole in three dimensions we get,

$$U(T) = E_B/V_3 = M_B c^2/V_3$$
$$= (M_{pl}c^2/8^{1/4})/(8\pi L_{pl}^3/3\sqrt[4]{2})$$
$$= 3M_{pl}c^2/(8\sqrt{2}\,\pi\, L_{pl}^3)$$
$$= 4.08 \times 10^{112}\ \text{J/m}^3 \qquad (26)$$

From eq. (24) and (26) we calculate the temperature of quantum black hole in three dimensions as:

$$T = 7.3433 \times 10^{31}\ \text{K} \approx \text{Planck temperature} \qquad (27)$$

The amount of energy radiated per unit area in one second,

$$E(T) = cU(T)/4$$
$$= 3.06 \times 10^{120}\ \text{W/m}^2 \qquad (28)$$

We know the surface area of quantum black hole in three dimensions, $S_3 = 4\sqrt{2}\ \pi L_{pl}^2$

Hence, total power radiated by the surface

$$P_3 = E(T)\, S_3 = \frac{3M_{pl}c^3}{8L_{pl}} = 1.3922 \times 10^{52}\ \text{W} \qquad (29)$$

Now let us see what happens in radiation if the number of dimensions is increased. For n = 4, volume and surface area of quantum black hole are

$$V_4 = \pi^2 L_{pl}^4 \qquad (30)$$

$$S_4 = 4\pi^2 L_{pl}^3 / \sqrt[4]{2} \qquad (31)$$

Total energy radiated per unit volume is,

$$U_4(T) = E_B/V_4 = (M_{pl}c^2/8^{1/4})/\pi^2 L_{pl}^4$$
$$= M_{pl}c^2/(8^{1/4}\pi^2 L_{pl}^4) \qquad (32)$$

From eq. (26) we know that

$$U_3(T) = 3M_{pl}c^2/(8\sqrt{2}\,\pi L_{pl}^3)$$

$$U_4(T)/U_3(T) \approx 1/L_{pl} \approx 10^{33}\ \text{cm}^{-1} \qquad (33)$$

Hence, $U_4(T) \approx 10^{33}\ \text{cm}^{-1}\ U_3(T)$ (34)

Similarly,
$$U_5(T) \approx 10^{66}\ \text{cm}^{-2}\ U_3(T)$$
$$U_6(T) \approx 10^{99}\ \text{cm}^{-3}\ U_3(T)$$
$$U_7(T) \approx 10^{132}\ \text{cm}^{-4}\ U_3(T)$$
$$U_8(T) \approx 10^{165}\ \text{cm}^{-5}\ U_3(T)$$

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In general, $U_n(T) \approx 10^{33(n-3)}\ \text{cm}^{-(n-3)}\ U_3(T)$ (35)

Now $U_3(T) = \sigma\ T_3^4$ and $U_4(T) = \sigma\ T_4^4$

$$U_4(T)/U_3(T) = (T_4/T_3)^4$$
$$T_4/T_3 = [U_4(T)/U_3(T)]^{1/4}$$
$$= [10^{33}\ \text{cm}^{-1}]^{1/4}$$
$$= 10^{33/4}\ \text{cm}^{-1/4} \qquad (36)$$
$$T_4 = 10^{33/4}\ \text{cm}^{-1/4} T_3$$

Similarly,
$$T_n = [10^{33(n-3)}\text{cm}^{-(n-3)}]^{1/4} T_3$$
$$= 10^{33(n-3)/4}\text{cm}^{-(n-3)/4} T_3 \qquad (37)$$

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} 10^{33(n-3)/4}\text{cm}^{-(n-3)/4} T_3 = \infty\ \text{K} \qquad (38)$$

The amount of energy radiated per unit area

$$E(T) = \frac{c}{4}\, U(T)$$

$$E_n(T) = \frac{c}{4}\, U_n(T)$$
$$= \frac{c}{4} \times 10^{33(n-3)}\ \text{cm}^{-(n-3)}\ U_3(T) \qquad (39)$$

Thus, If only one dimension is increased, the amount of energy radiated per unit area by the quantum black hole increases 10^{33} times. For $n \to \infty$ dimensions,

$$\lim_{n\to\infty} E_n(T) = \lim_{n\to\infty} \frac{c}{4}\, U_n(T)$$
$$= \lim_{n\to\infty} \frac{c}{4} \times 10^{33(n-3)}\ \text{cm}^{-(n-3)}\ U_3(T)$$
$$= \infty\ \text{W/m}^2 \qquad (40)$$

The total power radiated by quantum black hole can be calculated as follows:

$$P_n = E_n(T)\, S_n = \frac{c}{4}\, U_n(T)\, S_n$$

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$$= \frac{c}{4} \times 10^{33(n-3)}\ \mathrm{cm}^{-(n-3)}\ U_3(T) \times 10^{-33n}\ S_3$$

$$= \frac{c}{4} \times 10^{-99}\ \mathrm{cm}^{-(n-3)}\ U_3(T)\ S_3 \qquad (41)$$

For $n \to \infty$ dimensions,

$$\lim_{n\to\infty} P_n = \lim_{n\to\infty} \frac{c}{4} \times 10^{-99}\ \mathrm{cm}^{-(n-3)}\ U_3(T)\ S_3$$

$$= \lim_{n\to\infty} 10^{-99}\ \mathrm{cm}^{-(n-3)}\ P_3$$

$$= 10^{-99} P_3 = 10^{-99} \times 1.3922 \times 10^{52}\ \mathrm{W}$$

$$= 1.3922 \times 10^{-47}\ \mathrm{W} \qquad (42)$$

5. EXPLOSION OF QUANTUM BLACK HOLE AND CREATION OF PARTICLES

It is now generally believed that particle creation can be studied in strong gravitational fields [22]. This has led to the prediction of Hawking radiation in which particles are emitted from a black hole with a thermal spectrum [2] corresponding to a temperature that increase rapidly as the mass of the black hole decreases. Black hole explosion is discussed by Hawking in [20]. A black hole will create and emit particles such as neutrinos or photons if the black hole is a body with a temperature of $\approx 10^{-6}(M_{\odot}/M)$ K, where $M_{\odot}$ is the mass of the sun and M is the mass of the black hole [23]. A black hole having mass only a billion tons (e.g. a primordial black hole roughly the size of a proton) would have a temperature of 120 billion degrees Kelvin. At such a temperature a black hole would be able to emit $e^- - e^+$ pairs, photons, neutrinos and gravitons. As a black hole emits thermal radiation it loses its mass. This would increase the surface gravity and so increase the rate of emission. Therefore, the black hole would have a finite life $\approx 10^{71}(M_{\odot}/M)^{-3}$ s. Near the end of its life the rate of emission would be very high $\approx 10^{30}$ erg in the last 0.1s. This is considered as black hole explosion. A decaying black hole radiates a large number of particles in all directions with very high energies [17]. The decay products include all the species found in the universe. Recently [5], it is also discussed that quantum black hole will decay into a large number of elementary particles. Thus, the observation of quantum black hole explosion would provide very important information on elementary particle physics, information that might not be available in any other way [24].

According to our view, the oscillation of whole mass of quantum black hole through its centre of gravity leads to binding of quantum black hole to a certain number of dimensions, can be said as critical dimension, from which the number of dimensions cannot be increased further. The quantum black hole attains a very high temperature and pressure, can be said as critical temperature and pressure, from which temperature and pressure cannot be increased further. At this point the quantum black hole explodes which leads to the creation of particles. The particles created may be protons, photons, neutrinos, electrons, quarks, leptons etc. Atoms are created due to arrangement of these particles. These atoms combine to form molecules. Hence, quantum black hole can be said as the generator of atoms and molecules available in the universe. In fact, already it is noted [25–27] that a Planck mass particle decays via Bekenstein radiation within a Planck time.

6. CONCLUSIONS

According to big bang theory [28], our universe was created from a singular point having infinite energy density and infinite temperature. Then as time passed on, temperature cooled down and the universe was expanding. In 1999, from supernovae observations, it is found that the universe is not simply expanding but is accelerating too [29–35]. Recently [36], it is explained that in a cosmological context, an expanding universe must result from a single tiny brick of Planck cube. In this paper, we consider the motion of quantum black hole as simple harmonic and radiation emitted by it as black body radiation. We find that (i) $\lim_{n\to\infty} T_n = \infty$ K, (ii) $\lim_{n\to\infty} E_n(T) = \infty$ W/m^2, and (iii) $\lim_{n\to\infty} P_n = 1.3922 \times 10^{-47}$ W. Hence, we have come to conclusion that the universe was originated from a singular point consisting of one quantum black hole in infinite dimensions with infinite temperature. The amount of energy radiated per unit area was also infinite and the total power radiated by the singular point was 1.3922×10^{-47} W. Indeed Rosen showed that a Planck mass particle at the Planck scale can be considered to be a universe in itself [27,37]. Again El Naschie [38] has shown that if the universe is a black hole, then all informational particles have wavelength equal to the Planck length. Recently in [39,40] the idea that the universe is a quantum black hole is also adopted. (iv) The big bang resembles a black hole explosion but on a vastly larger scale [22]. Understanding of creation of particles from black holes will lead to a similar understanding of how the big bang created particles in the universe. It is expected that there was an earlier phase of the universe in which matter collapsed into a quantum black hole. The explosion of the quantum black hole created all the particle species found in the universe.

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Our results are not conclusive but they are tantalizing. These might be due to properties of quantum black hole that we can not quite yet estimate precisely. These predictions will become amenable to experimental check if quantum black holes are ever found. The investigation of quantum black hole can further shed light on the nature of gravity and space-time at the smallest scale [5,7]. It may also explain the origin of the ultra-high cosmic rays and dark matter in cosmology. It may possible to test quantum gravity with quantum black holes well above Planck scale. If we are fortunate enough to detect the creation and almost instantaneous evaporation of quantum black holes, we could be almost certain of the existence of hidden dimensions [41,42]. In the next few years, these ideas will be tested at the LHC and at other experiments.

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