

Multi-Item Multi-Objective Fuzzy Inventory Model with Possible Constraints

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ABSTRACT

This paper presents a mathematical model of inventory control problem for determining the minimum total cost of multi-item multi-objective fuzzy inventory model. Here, three constraints are taken; they are warehouse space constraint, investment amount constraint and the third constraint is the percentage of utilization of volume of the warehouse space. Warehouse maintenance is one of the essential parts of service operation. The warehouse space in the selling stores plays an important role in stocking the goods. In the proposed model, the warehouse space in the selling store is considered in volume. The demand is depending on the unit cost and the unit cost is taken in fuzzy environment. The model is illustrated with a numerical example.

Keywords: *Inventory model, membership function, triangular fuzzy number, volume of the warehouse*

1. INTRODUCTION

The literal meaning of the inventory is the stock of goods for future use (production/ sales). The control of inventories of physical goods is a problem common to all enterprises in any sector of an economy. In any industry, the inventories are essential but they mean lockup of capital. The excess inventories are undesirable, which calls for controlling the inventories in the most profitable way. The different types of costs (Purchasing cost, Setup cost, Holding cost, etc.) involved in inventory problems are affect the efficiency of an inventory control problem.

Warehouse space available in the selling store plays an important role in inventory model. Warehouse space can be considered in terms of area and/or volume, but most of the researchers consider only the area of the warehouse space. Here, the warehouse space in the selling store is considered in volume.

The classical inventory problem is designed by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature[1]. But in practical situation, unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers and fixed costs of production are spread over a large number of items. Hence the unit price of an item inversely relates to the demand of that item.

Initially, the fuzzy set theory is used in the decision-making problem [2]. The objectives are introduced as fuzzy goals over the α -cut of a fuzzy constraint set [3]. And, the concept of solving a multi-objective linear programming problem is introduced [4]. Now, the fuzzy set theory has made an entry into the inventory control systems. The EOQ formula is examined in the fuzzy set theoretic

perspective associating the fuzziness with the cost data[5]. Multi objective fuzzy inventory model is formulated with three constraints and solved by using geometric programming method [6]. Multi-item stochastic and fuzzy-stochastic inventory models formulated under imprecise goal and chance constraints [7]. In all of the above articles, the warehouse space available in the selling store is taken in terms of area. If the warehouse space is taken in terms of volume then less percentage of volume of the warehouse space will be consumed. Consequently, the maximum of the volume of the warehouse space can be utilized effectively.

To optimize the total expenditure of the organization by using multi objective fuzzy inventory model and warehouse location problem, the available warehouse space in the selling store has been taken in terms of area[8]. In this paper, multi objective fuzzy inventory model is developed under three constraints such as warehouse space constraint, investment amount constraint and the third constraint is the percentage of utilization of volume of the warehouse space. Here, the volumes of the unit items are taken for calculations. And, the demand is dependent on unit cost. The unit cost is taken in fuzzy environment. The unit cost and lot size are the decision variables.

2. FUZZY INVENTORY MODEL

a. Model and Assumptions

We use the following notations in proposed model:

- i. n = number of items
- ii. I = Total investment cost for replenishment
- iii. L - Inside length of the warehouse
- iv. B - Inside breadth of the warehouse

- v. M - Maximum height of the shelf
- vi. V - Volume of the warehouse space

For i^{th} item: ($i = 1, 2, \dots, n$)

- vii. $D_i = D_i(p_i)$ demand rate (function of cost price)
- viii. Q_i = lot size (a decision variable)
- ix. S_i = set up cost per cycle
- x. H_i = inventory holding cost per unit item
- xi. p_i = price per unit item (a fuzzy decision variable)
- xii. l_i - Length of the unit item i
- (xiii) b_i - Breadth of the unit item i
- (xiv) h_i - Height of the unit item i
- (xv) v_i - Volume of the unit item i
- (xvi) V_w - Percentage of utilization of volume of the warehouse

The basic assumptions about the model are:

- i. Replenishment is instantaneous
- ii. Shortage is not allowed
- iii. Lead time is zero
- iv. Demand is related to the unit price as:

$$D_i = \frac{C_i}{p_i^{e_i}}$$

Where $C_i (>0)$ and $e_i (0 < e_i < 1)$ are constants and real numbers selected to provide the best fit of the estimated price function. While $C_i > 0$ is an obvious condition since both D_i and p_i must be non-negative. Volume of the unit item is defined by $v_i = l_i \times b_i \times h_i$

To calculate the volume of the warehouse space, multiply the lengths of the dimensions of the inside of the warehouse, that is, multiply the inside length, inside breadth and maximum shelf height.

i.e., Volume of the warehouse space is defined by $V = L \times B \times M$

The Total Cost = Purchasing cost + Set up cost + Holding cost

$$\begin{aligned} \text{Min } Z &= D_i p_i + \frac{D_i S_i}{Q_i} + \frac{H_i Q_i}{2} \\ &= \frac{C_i}{p_i^{e_i}} p_i + \frac{C_i}{p_i^{e_i}} \frac{S_i}{Q_i} + \frac{H_i Q_i}{2} \end{aligned}$$

Therefore we get,

$$\begin{aligned} \text{Min } Z &= \sum_{i=1}^n \left[\frac{C_i}{p_i^{e_i}} p_i + \frac{C_i}{p_i^{e_i}} \frac{S_i}{Q_i} + \frac{H_i Q_i}{2} \right] \\ &= \sum_{i=1}^n \left[C_i p_i^{1-e_i} + \frac{C_i}{p_i^{e_i}} \frac{S_i}{Q_i} + \frac{H_i Q_i}{2} \right] \text{ for } i = 1, 2, 3, \dots, n \end{aligned}$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

First constraint: The limitation on the available warehouse space in the store, $\sum_{i=1}^n v_i Q_i \leq V$

Second constraint: The upper limit of the total amount investment: $\sum_{i=1}^n p_i Q_i \leq I$

Where $p_i, Q_i > 0, (i=1, 2, \dots, n)$

Third constraint: Percentage of utilization of volume of the warehouse:

$$\frac{V \times V_w}{\left(\sum_{i=1}^n v_i Q_i \right) \times 100} = 1, 0 \leq V_w \leq 100.$$

b. Inventory Model in Fuzzy Environment

When p_i s are fuzzy decision variables, the said crisp inventory model is transformed in fuzzy environment, therefore

$$\text{Min } Z = \sum_{i=1}^n \left[C_i \tilde{p}_i^{1-\beta_i} + \frac{C_i}{\tilde{p}_i^{\beta_i}} \frac{S_i}{Q_i} + \frac{H_i Q_i}{2} \right] \tag{2.1}$$

Subject to $\sum_{i=1}^n v_i Q_i \leq V$

$$\sum_{i=1}^n \tilde{p}_i Q_i \leq I$$

$$\frac{V \times V_w}{\left(\sum_{i=1}^n v_i Q_i \right) \times 100} = 1,$$

and $\tilde{p}_i, Q_i > 0 (i=1, 2, \dots, n), 0 \leq V_w \leq 100$ (Here cap ‘ \sim ’ denotes the fuzzification of the parameters)

c. Membership Function

The membership function for the triangular fuzzy number $\tilde{p}_i = (k_{u_i}, k_{m_i}, k_{o_i}), i = 1, 2, \dots, n$ is

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$$\mu_{p_i}(x) = \begin{cases} \frac{p_i - k_{u_i}}{k_{m_i} - k_{u_i}}, & k_{u_i} \leq p_i \leq k_{m_i} \\ \frac{k_{o_i} - p_i}{k_{o_i} - k_{m_i}}, & k_{m_i} \leq p_i \leq k_{o_i} \\ 0, & \text{otherwise} \end{cases}$$

fully rigid in the world. Here, the fuzzy inventory model is taken with three constraints, particularly; volumes of the unit items are taken in the warehouse space constraint. By solving the above fuzzy inventory model, the optimal result will be calculated. The result reveals the minimum expected annual total cost of the inventory model and also the optimal percentage of utilization of the volume of the warehouse. In the result, the percentage of utilization of the volume of the warehouse space is less; it can be increased by changing the values like volume of the warehouse space, investment cost, etc. We may develop the proposed model with many limitations, such as the shortage level, number of orders, etc.

3. NUMERICAL EXAMPLE

The model is illustrated for one item ($n = 1$) and also the common parametric values assumed for the given model are $n=1, C_1=113, S_1 = \$100, H_1 = \$1,$

$\tilde{p}_1 = \$(10,15,20),$ and $I = \$1400, l_1 = 2m,$
 $b_1 = 3m, h_1 = 4m, L = 10m, B = 12m$ and
 $M = 30m.$

Now, from the given values
 $v_1 = 24m^3$ and $V = 3600m^3.$

The proposed model (2.1) is solved by using LINGO software and the optimal results are presented in the

Table 3.1 Optimal solution of the proposed model

e_1	p_1	μ_{p_1}	Q_1	V_w	Z
0.850	10.051	0.0102	56.38	37.59	216.12
0.860	11.816	0.3633	51.99	34.66	211.66
0.865	12.867	0.5734	49.80	33.20	209.33
0.869	13.805	0.7610	48.05	32.03	207.43
0.870	14.055	0.8110	47.62	31.74	206.94
0.877	15.995	0.8010	44.58	29.72	203.49
0.880	16.944	0.6112	43.28	28.85	201.97
0.883	17.974	0.4052	41.99	27.99	200.43
0.885	18.711	0.2578	41.13	27.42	199.39
0.888	19.898	0.0204	39.85	26.56	197.80

In the above table 3.1, the values of p_1, μ_{p_1}, Q_1, V_w and Z are find out by applying different values for $e_1.$

4. CONCLUSION

Fuzzy set theoretic approach of solving an inventory control problem is realistic as there is nothing like

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