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Heat Generation Effects in an Inclined Enclosure under the Influence of Magnetic Field

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***Abstract*—The effects of heat generation in a natural convection flow in an inclined enclosure heated from one side and cooled from the adjacent side under the influence of a magnetic field using staggered grid finite-difference technique is studied. The governing equations are solved numerically for velocity and temperature fields at the mid-horizontal plane for various values of thermal radiation, magnetic field and heat generation parameters by considering three different inclination angles and magnetic field directions and keeping the aspect ratio fixed. The results indicate that the flow pattern and temperature field are significantly dependent on the above mentioned parameters.**

***Index Terms*—Natural convection; Inclined cavity; Magnetic field; Heat generation.**

I. Introduction

The study of magnetic field on natural convection in fluid-saturated porous medium has received considerable attention in recent years due to its wide variety of applications in engineering and technology such as solar energy collection, nuclear reactor insulation, cooling of electronic devices, furnaces, drying technologies and crystal growth in liquids, etc. Also the presence of an external magnetic field has increasing applications in material manufacturing industry as a control mechanism since the Lorentz force suppresses the convection currents by reducing the velocities when the fluid is electrically conducting and exposed to a magnetic field. Mansour et al. [1] studied the effects of an inclined magnetic field on the unsteady natural convection in an inclined cavity filled with a fluid saturated porous medium considering heat source in the solid phase. Al-Najem et al.[2] showed that the increase in the Hartmann number causes reduction in the heat transfer rate from cavity sidewalls. This phenomenon was related to the damping effect of the magnetic field which results in the domination of conduction over convection heat transfer. Ece and Büyük [3] studied numerical solutions for the velocity and temperature fields inside the rectangular enclosure and to determine the effect of the magnetic field strength and direction, the aspect ratio and the inclination of the enclosure on the transport phenomena. Later, Ece and Büyük [4] investigated the steady natural convection flow in an inclined square enclosure with differentially heated adjacent walls under the influence of magnetic field. They examined from the above two papers that circulation and convection become stronger with increasing Grashof numbers but it was suppressed by the presence of a strong magnetic field and clockwise inclination enhances the surface heat flux slightly almost along the bottom wall. When technology processes take place at high temperatures thermal radiation heat transfer become very important and its effects cannot be neglected. Recent developments in hypersonic flights, missile re-entry rocket combustion chambers and gas cooled nuclear reactors, have focused attention of researchers on thermal radiation as a mode of energy transfer and emphasized the need for inclusion of radiative transfer in these processes. Many studies have appeared concerning the interaction of radiative flux with thermal convection flows. Jordï¿œn [5] studied the effects of thermal radiation and viscous dissipation on MHD unsteady free-convection flow over a semi-infinite vertical porous plate. He examined the velocity, temperature, local skin-friction and local Nusselt number for various physical parameters, including the radiation parameter, Eckert number, magnetic number and suction (or injection). The effect of heat-generation/absorbtion in an enclosure in the presence of magnetic field also plays an important role in convective flows. Chamkha [6] investigated on an unsteady laminar combined forced and free convection flow and heat transfer of an electrically conducting and heat generating or absorbing fluid in a vertical lid-driven cavity in the presence of a magnetic field. Mahmud et al. [7] studied analytically combined free and forced convection flows of an electrically conducting and heat-generating/absorbing fluid in a vertical channel with two parallel plates under the action of transverse magnetic field. Grosan et al. [8] discussed the effects of magnetic field and internal heat generation on the free convection in a rectangular cavity filled with a porous medium. Not much attention has been given on the study of laminar natural convection flow in the presence of a magnetic field in an inclined enclosure in the presence of thermal radiation, heat generation with thermal boundary conditions (i.e. the square enclosure is heated from the left vertical wall and cooled from the top wall by keeping other walls in adiabatic state).

II. Governing equations and boundary conditions

We consider an unsteady laminar natural convection flow in the presence of a magnetic field in an inclined rectangular enclosure of length L and height H. The geometry and boundary conditions are schematically shown in Fig.1. The angle of inclination of the enclosure from horizontal in the counter-clockwise direction is denoted by ϕ. The magnetic

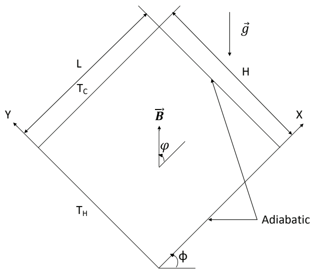

Figure 1. Geometry and the coordinate system.

field strength B_0 is applied at an angle φ with respect to the coordinate system. The right-side and the bottom walls are insulated and the fluid is isothermally heated and cooled at the left side and top walls with uniform temperature of T_H and T_C, respectively. The magnetic Reynolds number is assumed to be small and the induced magnetic field due to the motion of the electrically conducting fluid is neglected. The Joule heating of the fluid and the effect of viscous dissipation are also negligible. Dimensionless variables used in the analysis are defined as,

$$x=\frac{X}{L}, y=\frac{Y}{H}, \theta=\frac{T-T_C}{T_H-T_C}, u=\frac{HU}{\alpha}, v=\frac{LV}{\alpha},$$
$$\delta=\frac{H}{L}, p=\frac{\left[P+\rho_0 g(X sin\phi+Y cos\phi)\right]L^2}{\rho_0\alpha^2} \quad (1)$$

where δ is the aspect ratio of the enclosure. The governing equations in dimensionless variables take the following form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (2)$$

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+P_r\left(\frac{1}{\delta}\frac{\partial^2 u}{\partial x^2}+\frac{1}{\delta^3}\frac{\partial^2 u}{\partial y^2}\right)-\frac{1}{\delta^2}\left(\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}\right)+ GrP_r^2\theta\sin\phi+Ha^2P_r(v\sin\varphi\cos\varphi-\frac{1}{\delta}u\sin^2\varphi) \quad (3)$$

$$\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+P_r\left(\delta\frac{\partial^2 u}{\partial x^2}+\frac{1}{\delta}\frac{\partial^2 u}{\partial y^2}\right)-\left(\frac{\partial v^2}{\partial y}+\frac{\partial uv}{\partial x}\right) +\delta GrP_r^2\theta\cos\phi+Ha^2P_r(u\cos\varphi\sin\varphi-\delta v\cos^2\varphi) \quad (4)$$

$$\frac{\partial\theta}{\partial t}=\left(\delta\frac{\partial^2\theta}{\partial x^2}+\frac{1}{\delta}\frac{\partial^2\theta}{\partial y^2}\right)-\left(\frac{\partial u\theta}{\partial x}+\frac{\partial v\theta}{\partial y}\right) +\frac{4}{3\delta N_R}\frac{\partial^2\theta}{\partial y^2}+\delta He\ \theta. \quad (5)$$

where,

$$P_r=\frac{\mu}{\rho_0\alpha}, Ha=LB_0\sqrt{\frac{\sigma}{\mu}}, He=\frac{QL^2}{k},$$
$$Gr=\frac{\rho_0^2 g\beta L^3(T_H-T_C)}{\mu^2}, N_R=\frac{kk^*}{4\sigma T_c^3}. \quad (6)$$

Here P_r, Ha, Gr, N_R and He are Prandtl, Hartmann number, Grashof number, thermal radiation parameter and heat generation parameter, respectively. Radiation heat flux q_r is considered according to Rosseland approximation such that $q_r=-(4\sigma/3k^*)(\partial T^4/\partial Y)$, where σ and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. Boundary conditions are:

$$\theta=1 \quad on \quad x=0 \quad and \quad \theta=0 \quad on \quad y=1 \quad (7)$$

$$(\partial\theta/\partial x)\Big|_{x=1}=0 \quad and \quad (\partial\theta/\partial y)\Big|_{y=0}=0 \quad (8)$$

The heat transfer coefficient in terms of the local Nusselt number (Nu) and average Nusselt number are defined by, $Nu=-\delta\frac{\partial\theta}{\partial x}\Big|_{x=0}$ and $\overline{Nu}_H=\delta\int_0^1 Nu\ dy$.

III. Results and discussions

MAC cell control-volume based finite-difference discretization of the non-dimensional governing equations $(2)-(5)$ in staggered grid, are used in the present paper for numerical solution. The derivatives involved in the convective terms are discretized using a hybrid scheme, which is a convex combination of second-order central-difference and second-order upwind difference scheme whereas the derivatives involved in other terms are discretized using second-order central-difference scheme. Numerical results for contours of the streamlines and isotherms as well as the velocity and temperature profiles at mid-horizontal plane of the cavity for various values of radiation parameter, inclination angle ϕ, magnetic field angle φ and heat generation parameter are presented in this study. In addition, representative results for the Nusselt number and average Nusselt number are presented in graphically and in tabulated form. Here $P_r=1$, the aspect ratio $\delta=1$ (square enclosure), Hartmann number $Ha=100$ and Grashof number $Gr=10^4$ are fixed. The inclination angle ϕ of the enclosure is chosen as $-45^0, 0^0, 45^0$ and the angle of orientation of the magnetic field φ is taken $0^0, 45^0, 90^0$. All the numerical calculation are done using 80×80 grid size. Comparison of the present numerical results of the average Nusselt number at the hot wall $(x=0)$ is made with Ece and Büyük [11] for various values of ϕ and φ in Table 1 and a very good agreement has been obtained with their numerical results. Fig. 2 and 3 depicts the plot of vertical velocity profiles for different values of N_R and He, keeping the other parameters fixed for different values of ϕ and φ. From Fig. 2(a) it is seen that when $\phi=-45^0$ and $\varphi=90^0$, velocity profile at a point on the mid-horizontal plane decreases with increase in N_R near the left vertical wall whereas opposite effect is observed when $0.8<x<0.65$ and the profile again takes the same pattern near the right vertical wall as that observed near the left vertical plate. Similar effect is observed for $\varphi=0^0, 45^0$ but with a reduction as the peak vertical velocity profiles. The effect of N_R increases as the angle of orientation of the magnetic field φ increases. Again it is observed that when ϕ and N_R are fixed, vertical velocity profile increases with increase in φ near the left hot wall. Similar effects are observed for Figs. 2(b, c) for N_R. It is interesting to note that the similar effects of the parameters discussed in Figs. 2(a, b, c) for different N_R are valid for Figs. 3(d, e, f) for different values of He. It is seen from all these figures that when heating the enclosure from the left vertical wall, the vertical velocity produces a sinusoidal type with

Table I

Comparison of average Nusselt number $\overline{Nu}_H|_{x=0}$ of a square enclosure when $N_R = 0.0,\ He = 0.0$.

			$\overline{Nu}_H$					
			$\varphi = 0^0$		$\varphi = 45^0$		$\varphi = 90^0$	
δ	ϕ	Gr	present result	Ece and Büyük [11]	present result	Ece and Büyük [11]	present result	Ece and Büyük [11]
1	0^0	10000	3.535017	3.683067	3.535393	3.681906	3.535554	3.681277
	-45^0		3.536340	3.680572	3.536548	3.684567	3.536351	3.684299
	45^0		3.534034	3.678017	3.533959	3.682005	3.534064	3.680053

its maximum values near the left hot wall. Fig. 4 and 5 shows the temperature profiles for different values of N_R, He, ϕ and φ by keeping the other parameters fixed.

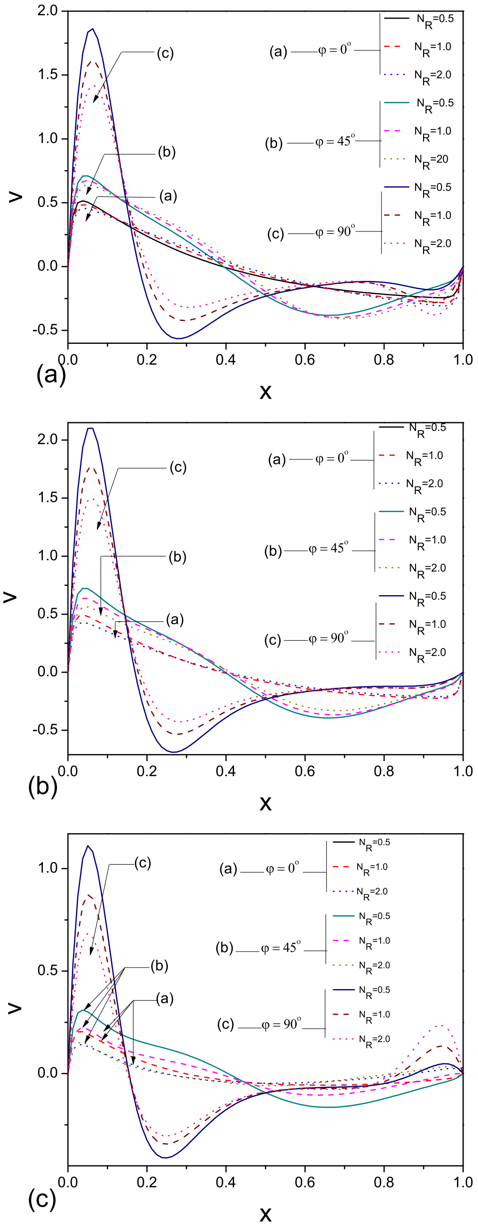

Figure 2. Vertical velocity profiles at mid-horizontal plane for different values of N_R.

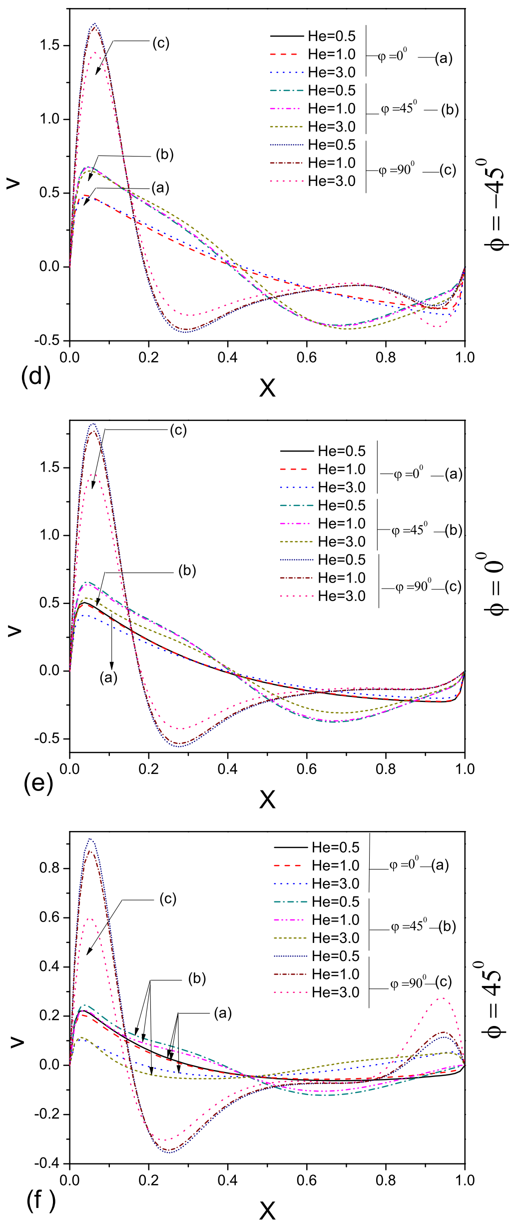

Figure 3. Vertical velocity profiles at mid-horizontal plane for different values of He.

It is observed from Fig. 4 (a) that when $\phi = -45^0$ the temperature profile on the mid-horizontal plane increases with increase in the value of N_R with its maximum values at the left hot wall and minimum value at right vertical wall, which indicates that the temperature decrease from the left wall to the right wall i.e the temperature reduces as the value of x increases for all the three values of angle of orientation of the magnetic field φ. Similar effects are observed for Figs. 4(b, c) for N_R when $\varphi = 0^0$. The same effects as mentioned in Figs. 4(a, b, c) for different N_R are seen in Figs. 5(d, e, f) for different values of He.

IV. Conclusion

In the present study we have studied the influence of magnetic field, thermal radiation and heat generation on unsteady two-dimensional laminar natural convection flow in

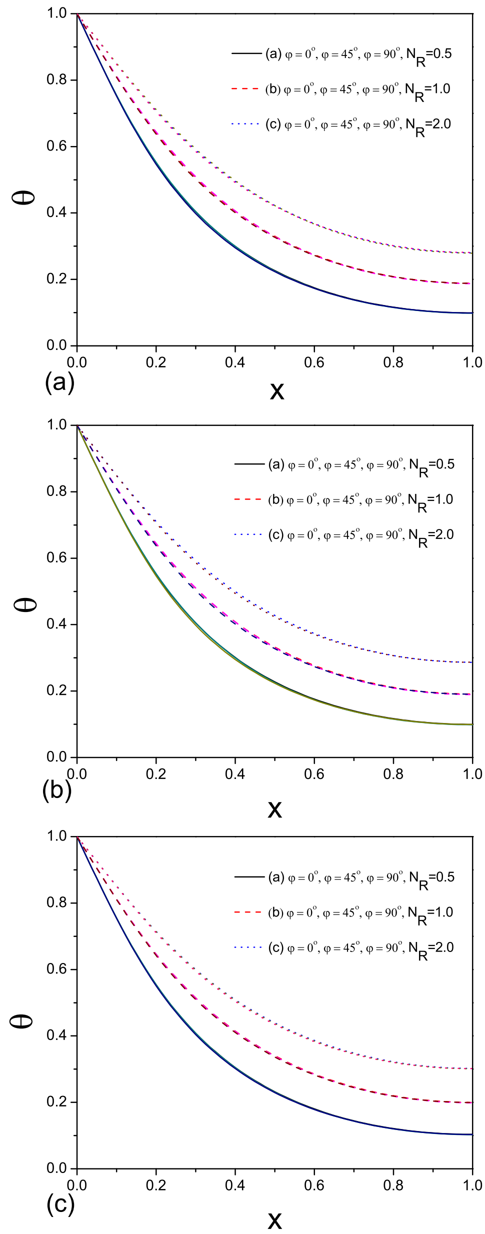

Figure 4. Temperature profiles at mid-horizontal plane for different values of N_R for $\varphi = 0^0, 45^0, 90^0$.

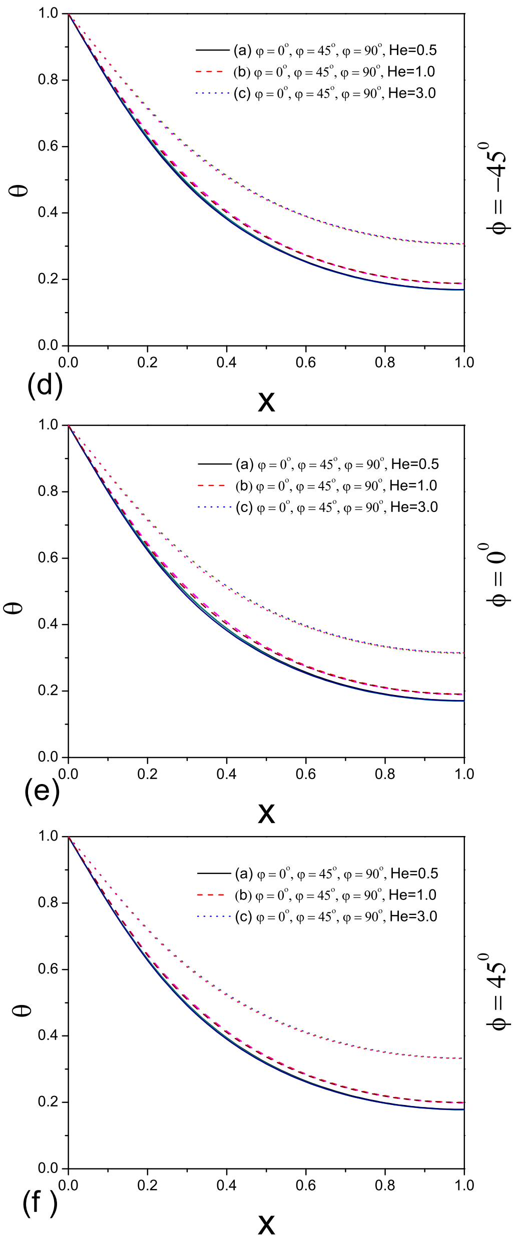

Figure 5. Temperature profiles at mid-horizontal plane for different values of He for $\varphi = 0^0, 45^0, 90^0$.

an inclined enclosure which is heated from the left vertical wall and cooled from the top wall while the other walls are kept adiabatic field using staggered grid finite-difference technique. The flow characteristics and the convection inside the tilted enclosure depend strongly upon the strength, direction of the magnetic field and the inclination of the enclosure. It is concluded that the temperature decreases with increase in the value of the thermal radiation and heat generation parameters.

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