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A Similarity Solution of the Convective Boundary Layer Flow with Thermal Radiation

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ABSTRACT

The combined free convective dynamic boundary layer and thermal radiation boundary layer at a semi-infinite vertical plate has been studied with variable suction. The fluid is considered to be gray absorbing emitting. The coupled unsteady non-linear momentum and energy equations of the combined layer are then reduced to similarity equations by introducing a time dependent similarity parameter. The similarity equations are solved numerically by adopting shooting method using the Nachtsheirm - Swigert iteration technique. The resulting velocity and temperature distributions are shown graphically for different values of the parameters. The numerical values of the wall shear stress and the heat flux are also shown in tabular form. Finally these results are discussed.

Keywords: *Thermal radiation, Velocity distribution, Temperature, distribution, Similarity analysis, Convection, Boundary layer, Numerical analysis.*

1. INTRODUCTION

The heating of rooms and building by the use of radiators is a familiar example of heat transfer by free convection. This is due to the significant role of thermal radiation in surface heat transfer when convection heat transfer is small particularly in free convection problems involving absorbing emitting fluids. One of the initiators of the problem, Goody [1] considered a neutral fluids.. Cess [2] however, considered absorbing emitting gray fluids with a black vertical plate. His solution was based on perturbation technique and was applicable for small values of the conduction-radiation interaction parameter.

Recently, in light of the work of Cess[2] the study of the interaction of natural convection with thermal radiation in laminar boundary layer with isothermal horizontal surface in a gray gas made by Ali et al.. Sound al gekar and Takhar [3] studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley-Vincntine-Giles equilibrium model .. Flowing, Mansour[4] studied the interaction of mixed convection with thermal radiation in laminar boundary-layer flow over horizontal, continuous moving sheets with suction and injection. He then studied the interaction of thermal radiation at a semi-infinite plate longitudinally streamlined by visco-elastic fluid. More recently Alabraba et al [5] studied the same problem of free convection interaction with thermal radiation in a hydromagnetic boundary layer taking into account the binary chemical reaction and the less attended Soret and Dufour effects.

Very recently Hossain and Takhar[6] analyzed the effect of the radiation using the Ross land diffusion approximation that leads to non similar boundary layer equations governing the mixed convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with a uniform free stream velocity and surface temperature. As in the case of Hossian and Takhar, earlier Sattar and Kalim [7] studied the effects of unsteady free-convection interaction with thermal radiation in a boundary layer flow. In other work locally similar solutions were obtained. In the presents work, following the work of Sattar and Kalim [7] the problem of unsteady free-convection interaction with thermal radiation of an absorbing emitting fluid along a vertical plate with variable suction has been investigated. The similarity solutions are then obtained numerically for very small values of conduction radiation parameter, which are of practical interest from physical point of view.

2. MATHEMATICAL FORMULATION

We consider a two dimensional unsteady flow in a combine dynamic boundary layer and thermal radiation boundary layer over a vertical plate. The plate is maintained at a uniform temperature T_w, and placed vertically in a quiescent fluid of infinite extent at a constant temperature T. There is variable suction at the plate taken to be a function of time. The fluid is assumed to be gray, emitting and absorbing, but non-scattering medium. The physical co-ordinates (x, y) are chosen such that the $x-axis$ lies in the plane of the plate and be oriented in the direction of the flow and the $y-axis$ be normal to it. The radioactive heat flux in the $x-$ direction is considered negligible in comparison in the $y-$ direction. Since the plate is considered to be of infinite extent all derivatives with respect to x vanish. Now considering the boussinesque approximation the two dimensional boundary layer equation related to our problem can be put forward as

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right) \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y} \qquad (3)$$

Where u and v are the longitudinal and normal components of velocity, T is the field temperature, g is the acceleration due to gravity, β is the volumetric co-efficient of thermal expansion, α is the thermal diffusivity, k is the thermal conductivity, v is the dynamic viscosity and q_r is the local radioactive heat flux. The boundary conditions relevant to the problem are

$$\begin{aligned} &u=0,\quad v=0,\quad T=T_w \\ &at \quad y=0 \\ &u\to 0,\quad v\to 0,\quad T\to T_\infty \\ &as \quad y\to\infty \end{aligned} \qquad (4)$$

the radioactive heat flux term is simplified by making use of the Ross land Approximation as

$$q_r=-\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y} \qquad (5)$$

where σ and k are the Stefan-Boltzman constant and the mean absorption co-efficient there. Since the plate is considered to be of infinite extent all derivatives with respect to x vanish. Here the governing equations relevant to our problem reduces to

$$\frac{\partial v}{\partial y}=0 \qquad (6)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right) \qquad (7)$$

and

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}= \alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y} \qquad (8)$$

Our aim is to obtain a similarity solution. For this purpose we introduce a similarity parameter δ defined as

$$\delta=\delta(t) \qquad (9)$$

Such that δ is a time dependent length scale. The continuity equation (1) can then be satisfied in terms of this length scale δ as

$$v=-\frac{v}{\delta}v_0 \qquad (10)$$

Here the constant v_0 represents the dimensionless suction or injection parameter.
We now introduce the following dimensionless variable

$$\begin{aligned} &\eta=\frac{y}{\delta} \\ &u=U_0\delta_* f(\eta) \\ &\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty} \end{aligned} \qquad (11)$$

Where $\delta_*=\frac{\delta}{\delta_0}$ is the value of δ at $t=t_0$ and U_0 is the uniform constant velocity.

From the equations (9) to (11) we get

$$\frac{\partial u}{\partial t}=U_0\frac{2\delta}{\delta_0^2}f\frac{d\delta}{dt}-U_0\frac{\eta\delta}{\delta_0}f\frac{d\delta}{dt}$$

$$\frac{\partial^2 u}{\partial y^2}=U_0 f''\frac{1}{\delta_0^2}$$

$$\frac{\partial T}{\partial t}=-\theta'(T_w-T_\infty)\frac{\eta}{\delta}\frac{d\delta}{dt}$$

$$\frac{\partial^2 T}{\partial y^2}=\left(T_w-T_\infty\right)\theta''\frac{1}{\delta^2}$$

$$\frac{\partial q_r}{\partial y}= -\frac{4\sigma}{3k}\left[(T_w-T_\infty)^4 12\theta^2\theta'\frac{1}{\delta^2}+(T_w-T_\infty)4\theta^3\theta''\frac{1}{\delta}\right]$$

Substitute the above values in the equation (2) and (3) we get respectively

$$-\frac{\eta\delta f'}{v}\frac{d\delta}{dt}+\frac{2\delta f}{v}\frac{d\delta}{dt}-v_0 f'= f''+G_r\theta \qquad (12)$$

$$-\frac{\delta}{v}\frac{d\delta}{dt}\eta\theta'-v_0\theta'= \frac{1}{P_r}\theta''+\frac{R}{P_r}\left[3(C_T+\theta)^2\theta'^2+(C_T+\theta)^3\theta''\right] \qquad (13)$$

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Where $\frac{\delta}{v}\frac{d\delta}{dt}$ is the dimensionless quantity? $G_r = \frac{g\beta}{vU_0}\left(T_w - T_\infty\right)\delta_0^2$ is the local Grashof number. $P_r = \frac{v}{\alpha}$ is the Prandtl number $R = \frac{16\sigma}{3K\kappa}\left(T_w - T_\infty\right)$ is the conduction radiation parameter. $C_T = \frac{T_\infty}{T_w - T_\infty}$ is the temperature difference parameter.

The equation (12) and (13) are similar expect the dimensionless quantity $\frac{\delta}{v}\frac{d\delta}{dt}$ where t appears explicitly. Thus the similarity condition requires that this quantity $\frac{\delta}{v}\frac{d\delta}{dt}$ must be a constant quantity. We therefore suppose that

$$\frac{\delta}{v}\frac{d\delta}{dt} = C \text{ (Constant)} \qquad (14)$$

Now integrated (14) with the constant that when t = 0,

$$\delta = 0 \text{ we obtain } \delta = \sqrt{2cvt} \qquad (15)$$

It appears from (15) that the length scale δ is consistent with the usual length scale considered for various non-steady flows.
Thus taking a realistic value of C to be 2 in (14), the equation (12) and (13) finally reduces to

$$f'' + 2\left(\eta + \frac{v_0}{2}\right)f' - 4f + G_r\theta = 0 \qquad (16)$$

$$\theta'' + 2P_r\left(\eta + \frac{v_0}{2}\right)\theta' = -R\left[3\left(C_T + \theta\right)^2\theta'^2 + \left(C_T + \theta\right)^3\theta''\right] \qquad (17)$$

Subject to the equation (16) and (17) the boundary conditions (4) now transfer to

$$\left\{\begin{array}{l} f = 0, \theta = 1 \quad \text{at } \eta = 0 \\ f = 0, \theta = 0, \quad \text{as } \eta \to \infty \end{array}\right\} \qquad (18)$$

The above equation (16) and (17) thus describe the basic ordinary differential equations of our problem of which are now sought subject to the boundary conditions (18).

3. METHOD OF SOLUTION

Equation (16) and (17) with boundary conditions (18) are solved numerically using standard initial value solver the shooting method.

For the purpose of this method, we have used Nacthsheim-Swigert iteration technique (Nachtsheim & Swigert, 1965). In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition. The Nachtsheim-Swigert iteration technique thus needs to be discussed elaborately.

The boundary conditions (18) associated with the linear ordinary differential equations (16) and (17) of the boundary layer type are of the two-points asymptotic class. Two-point boundary conditions have values of the dependent variable specified at two different values of the independent variable. Specification of an asymptotic boundary condition implies the value of velocity approaches to unity and the value of temperature approaches to zero as the outer specified value of the independent variable is approached. The method of numerically integrating two-point asymptotic boundary value problem of the boundary layer type, the initial value method, requires that it be recast as an initial value problem. Thus if is necessary to estimate as many boundary conditions at the surface as were given at infinity. The governing differential equations are then integrated with these assumed surface boundary conditions. If the required outer boundary condition is satisfied, a solution has be achieved. However, this is not generally the case. Hence a method must be devised to logically estimate the new surface boundary conditions for the next trial integration. Asymptotic boundary value problems such as their governing the boundary layer equations we further complicated by the fact that the outer boundary condition is specified at infinity. In the trial integration's infinity is numerically approximated by some large value of the independent variable. There is no a priori general method of estimating this value. Selecting too small a maximum value for the independent variable may not allow to solution to asymptotically commerce the required accuracy. Selecting large a value may result in divergence of the trial integration or in slow convergence of surface

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boundary conditions required satisfying the asymptotic outer boundary condition. Selecting too large a value of the independent variable is expensive inters of computer time.

Nachtsheim-Swigert developed an iteration method, which overcomes these difficulties. Extension of the Nachtsheim-Swigert iteration shell to above equation system of differential equations (16) and (17) are straightforward. In equation (18) there are two asymptotic boundary conditions and hence two unknown surface conditions $f'(0)$ and $\theta'(0)$. Within the context of the initial-value method and the Nachtsheim-Swigert iteration technique the outer boundary conditions may be functionally represented as

$$f(\eta_{\max}) = f[f'(0), \theta'(0)] = \delta_1 \quad (19)$$

$$\theta(\eta_{\max}) = \theta[f'(0), \theta'(0)] = \delta_2 \quad (20)$$

with the asymptotic convergence criteria given by

$$f'(\eta_{\max}) = f'[f'(0), \theta'(0)] = \delta_3 \quad (21)$$

$$\theta'(\eta_{\max}) = \theta'[f'(0), \theta'(0)] = \delta_4 \quad (22)$$

Expanding in a first order Taylor series using equation (19) to (22) yields

$$f(\eta_{\max}) = f_c(\eta_{\max}) + f_x \Delta x + f_y \Delta y = \delta_1 \quad (23)$$

$$\theta(\eta_{\max}) = \theta_c(\eta_{\max}) + \theta_x \Delta x + \theta_y \Delta y = \delta_2 \quad (24)$$

$$f'(\eta_{\max}) = f'_c(\eta_{\max}) + f'_x \Delta x + f'_y \Delta y = \delta_3 \quad (25)$$

$$\theta'(\eta_{\max}) = \theta'_c(\eta_{\max}) + \theta'_x \Delta x + \theta'_y \Delta y = \delta_4 \quad (26)$$

Where $x = f'(0), y = \theta'(0)$ the x and y subscripts Indicate partial differentiation, e.g.

$$f'_x = \frac{\partial f'(\eta_{\max})}{\partial f'(0)},\ f'_y = \frac{\partial f'(\eta_{\max})}{\partial \theta'(0)}$$

The subscript "c" indicates the value of the function at $\eta_{\max}$ determined from the trial integration. Solutions of these equations in a least square sense requires determining the minimum value of

$$E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 \quad (27)$$

Differentiating E with respect to x

$$\left(f_x^2 + \theta_x^2 + f_x'^2 + \theta_x'^2\right)\Delta x + \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)\Delta y = -\left(f_c f_x + \theta_c \theta_x + f'_c f'_x + \theta'_c \theta'_x\right)$$
(28)

Differentiating E with respect to y

$$\left(f_y^2 + \theta_y^2 + f_y'^2 + \theta_y'^2\right)\Delta y + \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)\Delta x = -\left(f_c f_y + \theta_c \theta_y + f'_c f'_y + \theta'_c \theta'_y\right)$$
(29)

We can write (28) and (29) in system of linear equations in the following form as

$$A_{11}\Delta x + A_{12}\Delta y = d_1 \quad (30)$$

$$A_{21}\Delta x + A_{22}\Delta y = d_2 \quad (31)$$

Where

$$A_{11} = \left(f_x^2 + \theta_x^2 + f_x'^2 + \theta_x'^2\right)$$
$$A_{22} = \left(f_y^2 + \theta_y^2 + f_y'^2 + \theta_y'^2\right)$$
$$A_{12} = A_{21} = \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)$$
$$d_1 = -\left(f_c f_x + \theta_c \theta_x + f'_c f'_x + \theta'_c \theta'_x\right)$$
$$d_2 = -\left(f_c f_y + \theta_c \theta_y + f'_c f'_y + \theta'_c \theta'_y\right)$$

From the equation (30) and (31) we have

$$\Delta x = \frac{\det A_1}{\det A}, \Delta y = \frac{\det A_2}{\det A}$$

Where,

$$\det A = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}, \det A_1 = \begin{vmatrix} d_1 & A_{12} \\ d_2 & A_{22} \end{vmatrix}$$
$$\det A_2 = \begin{vmatrix} A_{11} & d_1 \\ A_{21} & d_2 \end{vmatrix}$$

This adopting the numerical technique aforementioned the solutions of the linear ordinary differential equation (13) and (14) with boundary conditions (15) are obtained together with sixth order implicit Runge-kutta initial value solver. Here we applied the shooting method for different values of pertinent parameters. In the process of integration the skin friction coefficient $f'(0)$ and the heat transfer rate $\theta'(0)$ also calculated out.

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The numerical solutions thus obtained are displaced in figure 1-5. It is now important to calculate the physical quantities of primary interest which are the wall shear stress and the surface heat flux. These quantities are obtained from the process of the numerical calculations and the values are displayed in Table 1.

Table 1: Numerical values of the dimensionless shear stress τ_w and the surface heat flux q_w are

ν	G_r	P_r	R	C_T	τ_w	q_w
0.5	10	0.71	0.1	0.1	3.0634	-1.0698
1.0	10	0.71	0.1	0.1	2.9790	-1.2947
1.5	10	0.71	0.1	0.1	2.8739	-1.5339
0.5	05	0.71	0.1	0.1	1.5319	-1.0696
0.5	08	0.71	0.1	0.1	2.4508	-1.0697
0.5	10	1.00	0.1	0.1	2.7539	-1.3183
0.5	10	2.00	0.1	0.1	2.1404	-2.0496
0.5	10	0.71	0.0	0.1	2.9988	-1.1884
0.5	10	0.71	0.2	0.1	3.1242	-0.9756
0.5	10	0.71	0.1	0.5	3.1780	-0.9433
0.5	10	0.71	0.1	1.0	3.4129	-0.7686

4. RESULTS

For the purpose of discussing the numerical solution, the effects of various parameters on the flow behavior have been carried out for different values of suction parameter ν_0, the condition radiation interaction parameter R, temperature difference C_T, Prandtl number P_r, and Grashof number G_r. The effects of temperature difference C_T and suction ν_0 on the velocity profiles are shown in Fig. 1. It can be seen from this figure that the velocity profiles increase with the increasing values of C_T which it decreases with increasing values of ν_0. Thus from Fig.1, we conclude that as the free convection increase velocity also increase.

In Figure 2, the effects of the condition radiation interaction parameter R on the velocity profiles are shown from this figure; it is observed that as the interaction of thermal radiation intensifies (increase in R), the velocity increases with an accompanying increase in the velocity gradient at the wall.

In Figure 3, the effects of the Prandlt number on the temperature profiles are shown. It can be seen from this Fig that the temperature profiles decrease due to increasing values of the Prandlt number. In Figure 4, the effects of the Grashof number G_r and the Prandlt number P_r on the velocity profiles are displayed. It is apparent from this figure that the velocity profiles increase when Grashof number increases and velocity profiles decrease when P_r increases.

Fig. 5 shows that the effects of C_T and ν_0 on the temperature profiles. The temperature profile is found to increases when C_T increase and decrease when ν_0 increases. Finally the numerical values of the skin fiction co-efficient τ_w and the rate of heat transfer q_w for various values of R, C_T, ν_0, P_r and G_r are shown in the table 1. From this table it can be seen that for fixed values of Prandlt number P_r, Temperature difference C_T, Grashof number G_r, and thermal radiation interaction parameter R, the skin fiction co-efficient decrease and the local heat flux q_w increase when the suction parameter ν_0 increases.

For the fixed values of suction parameter ν_0, Radiation interaction parameter R, Prndlt number P_r, and temperature difference C_T, the skin fiction τ_w increases and q_w decreases when grashof number G_r, increases. It is also observed that for fixed values of the parameters ν_0, G_r, R, and C_T, the skin fiction τ_w and local heat flux q_w decreases when prandlt number P_r increases.

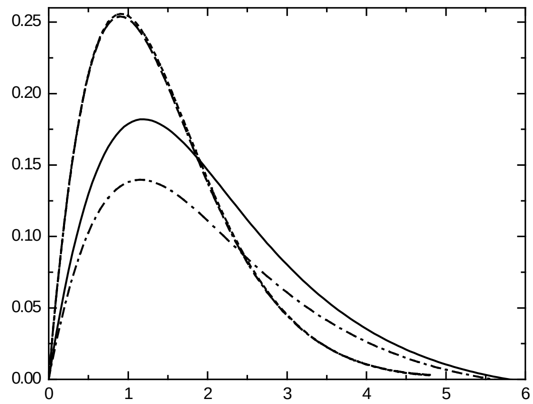

Fig 1: Velocity profile for different values of C_T And ν_0.

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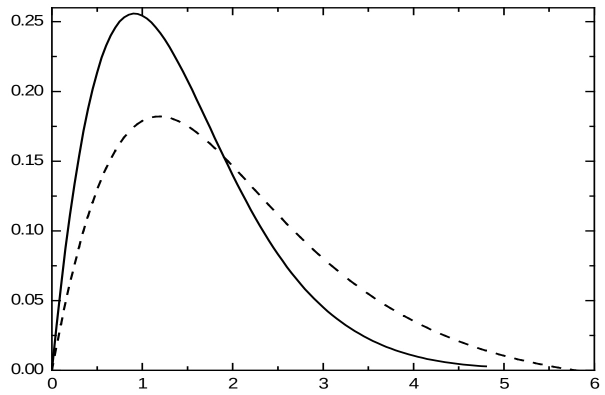

Fig 2: Velocity profile for different values of R

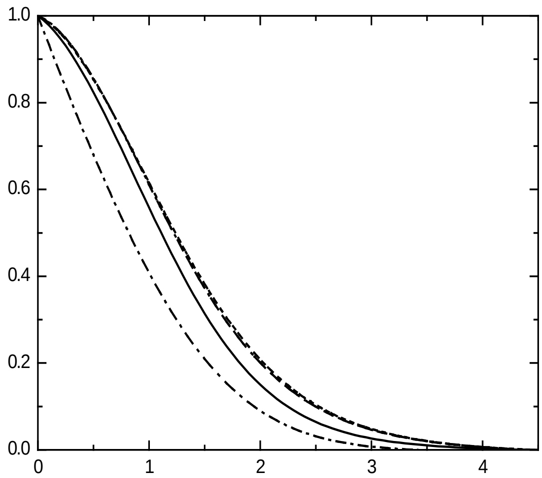

Fig 3: Temperature profile for different values of P_r

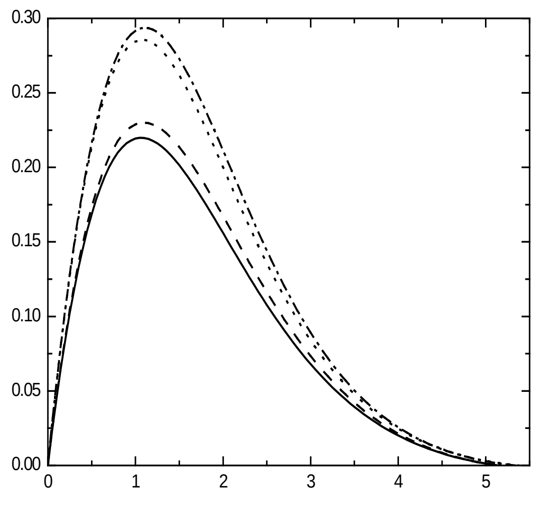

Fig 4: Velocity profiles for different values of P_r and G_r

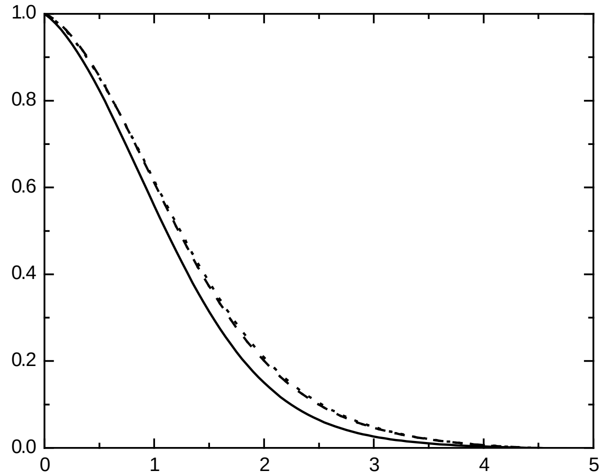

Fig 5: Temperature profiles for different values of C_T and ν_0

REFERENCES

[1]. Goody R.M., J.Fluid Mech., 1965, 424.

[2]. Cess R.D., The interaction of thermal radiation with free convection heat transfer, Int.J.Heat Mass Trans., 9, 1966, 1269-1277.

[3]. Soundalgekar, Patil, M.R and Takher., MHD flow past a vertical oscillating plate, Nucl. Eng. Des 64(1), (1981), 43-48.

[4]. Monsur M.A. (1990) Astrophysics and space science 166,1990, 269-275.

[5]. Alabraba M.A., Bestman, A.R., and Ogulu, A., Astrophysics and space science 195,1992, 431.

[6]. Hossain M.A. and Takhar H.S., Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass transfer, 31 (4), 1996, 243-248.

[7]. M.Ferdows,M.A.Sattar and M.N.A.Siddiki, Numerical approach on parameter of the thermal radiation interaction with convection in aboundary layer flow at a vertical plate with variable suction.ThammasaInt t.J . Sc.T ech.V, ol.9 ,N o. 3,J uly-September 2004.

[8]. N C Roy and M A Hossain;; Proc.IMechE Vol.224 PartC: J.Mechanical.

[9]. M. Sultana, M.M. Haque, M.M. Alam, M. Ferdows and A. Postelnicu European Journal of Scientific Research, Vol.53, No. 3, P. 477-490, 2011.

ARPN Journal of Science and Technology

http://www.ejournalofscience.org

[10]. M.A. Sattar and M. Ferdows, Dufour and Soret effects, Chemical Engineering Communication, Vol. 198, No.9, pp. 1146-1167, 2011.

[11]. M A Hossain and R S R Gorla; Canadian Jour. Chem. Engn. Vol 87, (2009), pp. 534-540.

[12]. M. A. Hossain, S. Asghar, R. S. R Gorla; Mixed Convection Boundary-Layer Flow Along a Vertical Stretched Surface with Uniform Surface Mass Transfer, International Journal of Fluid Mechanics Research, Vol. 35(4), (2008), pp. 326-339.

[13]. M. Ferdows and Masahiro OTA, Unsteady heat transfer boundary layer solutions of polar fluid, Int. J. of Mathematics and Computation, Vol. 4, No. S09, P. 111-123, 2009

[14] M. Ferdows, Koji Kaino and S. Sivasankaran , Free convection flow in an inclined porous surface, Journal of Porous Media, Vol. 12, No.10, P. 997-1003, 2009

[15] M. Ferdows and B.I Olajuwon, On the Similarity solution of Micropolar Power law fluid over a vertical plate, International Journal of contemporary Mathematical sciences, Vol. 6, No. 3, P. 133 ¨C 143, 2011