VOL. 2, NO. 5, June 2012 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

Bulk Arrival Single Server, Bernoulli Feedback Queue with Fuzzy Vacations and Fuzzy Parameters

[1] M.Jeeva, [2] E.Rathnakumari
[1, 2] Department of Mathematics, Ethiraj College for Women, Chennai, India

ABSTRACT

This papers deals with a mathematical non-linear programming method to construct the membership function of the system characteristics of a M/G/1, bulk arrival queues with server vacations and feedback facility, in which arrival rate, service time, batch size, departure probability, and vacation time are all fuzzy numbers. A customer after getting a service, if requires another service, is taken immediately for the next service and this can be repeated any number of times. The server leaves for a vacation as soon as the system becomes empty and returns after a vacation of random duration. Both single and multiple vacation polices are considered. The α – cut approach is used to transform a fuzzy queue into a family of conventional crisp queues in this context. By means of membership functions of the system characteristics, a set of parametric nonlinear programs is developed to describe the family of crisp single server bulk - arrival queues with fuzzy feedback probability and fuzzy vacation time. A man power planning problem is solved successfully to illustrate the validity of the proposed approach. Because the system characteristics are expressed and governed by the membership functions, the fuzzy bulk - arrival queues with single server are represented more accurately and analytic result are more useful for system designers and practitioners.

Keywords: *Batch arrival, vacation policy, α-cut, membership function, performance measures.*

1. INTRODUCTION

We consider here a M^X/G/1Queue with unlimited waiting space. As soon as the system is empty, the server leaves the system and return after a vacation of random duration. The vacation policy may be of two types-single or multiple. In single vacation policy, the server takes exactly one vacation as soon as the system becomes empty and on return waits idle until the first customer arrives. In multiple vacation policy the server keeps on taking vacations until on return from a vacation, at least one customer is present in the queue. Such queuing models without vacations and their behavior are discussed by many authors[5,7,15,17] such models with vacations are considered in [1,6,12,13,14]. In [11] Bernoulli feedback probability in the bulk queue model with server vacation time is considered. The single server bulk queues are extensively studied by many researchers like [2, 3, 4, and 5]. Bulk arrival queuing models are extensively used in many practical situations such as production/manufacturing systems, communication systems and computer networks [5,8]. For example in production/manufacturing systems will not begin until a specified number of raw materials are accumulated during an idle period, we often analyse this system by a bulk arrival queuing model which provides a powerful tool for evaluating the system performance. Within the context of traditional queuing theory, the inter arrival times and service times are required to follow certain probability distributions. However in many practical applications, the statistical information may be obtained subjectively i.e. the arrival pattern and service pattern are more suitably described by linguistic terms such as fast, slow (or) moderate, rather than by probability distributions. Thus the fuzzy queues are much more realistic than the commonly used crisp queues.

In [9] parametric programming is adopted to construct the membership functions of the performance measure for fuzzy queues and successfully applied to four simple fuzzy queues with one or two fuzzy variables namely M/F/1, F/M/1 , F/F/1 , FM/FM/1, where F denotes fuzzy time and FM denotes fuzzified time. It seems that their approach is applicable to the fuzzy bulk arrival queues. However since the fuzzy bulk arrival queuing systems are much more complicated than the above four fuzzy queues, the solution procedure for bulk arrival queue is not explicitly known and deserve further investigation. Fuzzy probabilities are discussed in [19].

We are motivated by group arrival models occurring in many situations such as data communication systems, people arriving at the customer counter of an airport, travel agents queuing in booking offices, receipt of order to the executed by company.

In this paper, we develop a method that is able to provide fuzzy performance measure for bulk arrival queues with fuzzified exponential arrival rate, service time, batch size, departure probability and vacation time. The basic idea is to apply the α-cuts and Zadeh's extension principle [10,20] to transform the fuzzy bulk arrival queues to a family of crisp bulk arrival queues. As the α value varies, the crisp bulk arrival queues are then described and solved by a pair of mixed integer nonlinear programming (MINLP) technique. The solution from the MINLP completely and successfully yield the membership functions of the system characteristics, including the expected number of customers in the system and waiting time in the queue for single and multiple vacations.

VOL. 2, NO. 5, June 2012 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

The remainder of the paper is organized as follows: Section 2 represents the system characteristics of standard and fuzzy batch-arrival queuing models with single server. In section 3, a mathematical programming approach is developed to derive the membership functions of these system characteristics. To demonstrate the validity of the proposed approach, man power planning problem is described and solved. Discussion is provided in section 4, and conclusions are drawn in section 5.

2. FUZZY BULK ARRIVAL QUEUE WITH FUZZY FEEDBACK PROBABILITY AND FUZZY VACATION TIME

We consider the $M^X/G/1$ queue. The arrival rate is assumed to be Poisson while the distribution of group size, service time and vacation time are assumed to be general. Each customer after getting the first service, if requires another service, is served immediately. This can be repeated any number of times, till he finally departs the service. The total service time(TS) is the combination of all these repeated service times of the customer. Let f be the feed back probability that a customer returns for another service and d be the departure probability, so that d + f = 1. After completion of one full service to a customer and his departure if there is no customer in queue, then the server takes a vacation of random length. If the server, on return from a vacation, finds no customer waiting, he immediately takes another vacation and this can be repeated until he fined at least one waiting customer upon return from the vacation. This is known as multiple vacation policy

Let L_S (MV) and L_S (SV) represent the expected number of customers in the system in multiple vacation and single vacation respectively. Then from [11]

$$L_S(MV) = \frac{\lambda E(C)E(S)}{d} + \frac{\lambda E(C)E(V^2)}{2E(V)} + \frac{\lambda^2E^2(C)[dE(S^2)+2(1-d)E^2(S)]+\lambda E(S)d[E(C^2)-E(C)]}{2d[d-\lambda E(C)E(S)]} \quad \text{.........(1)}$$

$$L_S(SV) = \frac{\lambda E(C)E(S)}{d} + \frac{\lambda E(C)E(V^2)}{2V^*(\lambda)+\lambda E(V)} + \frac{\lambda^2E^2(C)[dE(S^2)+2(1-d)E^2(S)]+\lambda E(S)d[E(C^2)-E(C)]}{2d[d-\lambda E(C)E(S)]} \quad \text{............(2)}$$

Where E(C), E(S), and E(V) represents expected group size, expected service time and expected vacation time respectively.

Let W_q(MV) and W_q(SV) represents the expected waiting time of a customer in the queue in multiple and single vacation respectively.

$$W_q(MV) = \frac{E(V^2)}{2E(V)} + \frac{\lambda E(C)[dE(S^2)+2(1-d)E^2(S)]}{2d(d-\lambda E(C)E(S))} + \frac{E(S)[E(C^2)-E(C)]}{2E(C)[d-\lambda E(C)E(S)]} \quad \text{..............(3)}$$

$$W_q(SV) = \frac{E(V^2)}{2(V^*(\lambda)+\lambda E(V))} + \frac{\lambda^2E^2(C)[dE(S^2)+2(1-d)E^2(S)]+\lambda dE(S)[E(C^2)-E(C)]}{2d\lambda E(C)[d-\lambda E(C)E(S)]} \quad \text{................(4)}$$

Where $V^*(\lambda)$ represents probability of zero arrival in a vacation time. Here the steady state condition is $\lambda E(C)E(S) < d$.

To extend the applicability of the bulk arrival queuing model with feedback probability and vacation time, we allow for fuzzy specifications of system parameters. Suppose the batch arrival rate λ for customers and service time S for each customer departure probability d vacation time V and group size C are approximately known and can be represented by the fuzzy sets $\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$.

Using α-cuts the trapezoidal arrival rate, service time, arrival size, vacation time, and departure probability are represented by different levels of intervals of confidences.

Let this interval of confidence be represented by $[t_{1\alpha}, t2\alpha]$. Since probability distributions for the α-cuts can be represented by uniform distributions, we have

$$P(t_\alpha) = \frac{1}{t_{2\alpha}-t_{1\alpha}}, t_{1\alpha} \le t_\alpha \le t_{2\alpha}$$

Thus the mean of the distribution is

$$E(T_\alpha) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha}-t_{1\alpha}} t_\alpha dt_\alpha = \frac{1}{2}(t_{2\alpha}+t_{1\alpha})$$

Similarly for the second moment, we have

$$E(T_\alpha^2) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha}-t_{1\alpha}} t_\alpha^2 dt_\alpha = \frac{t_{2\alpha}^3 - t_{1\alpha}^3}{3(t_{2\alpha}-t_{1\alpha})}$$

VOL. 2, NO. 5, June 2012 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

Using the well-known formula Var (T_α) = E(T^2_α) – $[E(T_\alpha)]^2$, the variance can now be obtained as Var (T_α) = $\frac{1}{12}(t_{2\alpha} - t_{1\alpha})^2$

Let $\phi_{\tilde{\lambda}}(x)$, $\phi_{\tilde{S}}(y)$, $\phi_{\tilde{d}}(p)$, $\phi_{\tilde{V}}(v)$, $\phi_{\tilde{C}}(c)$ Denote the membership functions of $\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$. We have the following fuzzy sets

$$\tilde{\lambda} = \{(x, \phi_{\tilde{\lambda}}(x)) / x \in X\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

$$\tilde{S} = \{(y, \phi_{\tilde{S}}(y)) / y \in Y\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (6)$$

$$\tilde{d} = \{(p, \phi_{\tilde{d}}(p)) / p \in P\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (7)$$

$$\tilde{V} = \{(v, \phi_{\tilde{V}}(v)) / v \in V\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (8)$$

$$\tilde{C} = \{(c, \phi_{\tilde{C}}(c)) / c \in C\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

Where X,Y, P,V,C are the crisp universal sets of the batch-arrival, service time, departure probability, vacation time and group size respectively.

Let f(x, y,p,v,c) denote the system characteristics of interest. Since $\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$ are fuzzy numbers , f($\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$) is also a fuzzy number. Following Zadeh's extension principle [18, 19] the membership function of the system characteristics f($\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$) is defined as

$$\phi_{f(\tilde{\lambda},\tilde{S},\tilde{d},\tilde{V},\tilde{C})}(z) = \sup_{\substack{x\in X, y\in Y, p\in P, v\in V, c\in C \\ \lambda E(C)E(S)<1}} \min \{\phi_{\tilde{\lambda}}(x), \phi_{\tilde{S}}(x), \phi_{\tilde{d}}(x), \phi_{\tilde{V}}(x), \phi_{\tilde{C}}(c) / z = f(x,y,p,v,c)\} \quad \ldots\ldots\ldots(10)$$

Assume that the system characteristics of interest are the expected number of customers in the system. It follows from (1) that the expected number of customers in the system for multiple vacations is

$$f(x,y,d,v,c) = \frac{xE(c)E(v^2)}{p} + \frac{xE(c)E(v^2)}{2E(v)} + \frac{x^2E^2(c)[p\,E(y^2) + 2(1-p)E^2(y)] + xE(y)p[E(c^2) - E(c)]}{2p[p - xE(c)E(y)]} \quad \ldots\ldots\ldots..(11)$$

The membership function for the expected number of customers in the system is

$$\phi_{\tilde{L}_S(MV)}(z) = \sup_{\substack{x\in X, y\in Y, p\in P, v\in V, c\in C \\ \lambda E(C)E(S)<1}} \min \{\phi_{\tilde{\lambda}}(x), \phi_{\tilde{S}}(y), \phi_{\tilde{d}}(p), \phi_{\tilde{V}}(v), \phi_{\tilde{C}}(c) / z = \frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[p\,E(y^2) + 2(1-p)E^2(y)] + xE(y)p[E(c^2) - E(c)]}{2p[p - xE(c)E(y)]} \quad \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Unfortunately, the membership function is not expressed in the usual form, making it very difficult to imagine its shape. In this paper we approach the representation problem using a mathematical programming technique. Parametric NLP's are developed to find the α –cuts of f($\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$) based on the extension principle.

3. PARAMETRIC NON LINEAR PROGRAMMING

To express the membership function $\phi_{\tilde{L}_S(MV)}(z)$ of $\tilde{L}_S(MV)$ in an understandable and usable form, we adopt Zadeh's approach, which relies on α-cuts of $\tilde{\lambda}$, $\tilde{S}$, $\tilde{d}$, $\tilde{V}$, $\tilde{C}$ as crisp intervals are as follows:

$$\lambda(\alpha) = [x^L_\alpha, x^U_\alpha] = [\min_{x\in X}\{x / \phi_{\tilde{\lambda}}(x) \ge \alpha\}, \max_{x\in X}\{x / \phi_{\tilde{\lambda}}(x) \ge \alpha\}] \quad \ldots\ldots\ldots\ldots\ldots.(13a)$$

$$S(\alpha) = [y^L_\alpha, y^U_\alpha] = [\min_{y\in Y}\{y / \phi_{\tilde{S}}(y) \ge \alpha\}, \max_{y\in Y}\{y / \phi_{\tilde{S}}(y) \ge \alpha\}] \quad \ldots\ldots\ldots\ldots\ldots(13b)$$

$$d(\alpha) = [p^L_\alpha, p^U_\alpha] = [\min_{p\in P}\{p / \phi_{\tilde{d}}(p) \ge \alpha\}, \max_{p\in P}\{p / \phi_{\tilde{d}}(y) \ge \alpha\}] \quad \ldots\ldots\ldots\ldots\ldots.(13c)$$

$$V(\alpha) = [v^L_\alpha, v^U_\alpha] = [\min_{v\in V}\{v / \phi_{\tilde{V}}(v) \ge \alpha\}, \max_{v\in V}\{v / \phi_{\tilde{V}}(v) \ge \alpha\}] \quad \ldots\ldots\ldots\ldots\ldots\ldots(13d)$$

$$C(\alpha) = [c^L_\alpha, c^U_\alpha] = [\min_{c\in C}\{c / \phi_{\tilde{C}}(c) \ge \alpha\}, \max_{c\in C}\{c / \phi_{\tilde{C}}(c) \ge \alpha\}] \quad \ldots\ldots\ldots\ldots\ldots..(13e)$$

The batch arrival rates service time departure probability vacation time group size are shown as intervals when the membership functions are no less than a given possibility level for α. As a result, the bounds of these intervals can be described as functions of α and can be obtained as,

$$x^L_\alpha = \min \phi^{-1}_{\tilde{\lambda}}(\alpha) \quad x^U_\alpha = \max \phi^{-1}_{\tilde{\lambda}}(\alpha)$$

$$y^L_\alpha = \min \phi^{-1}_{\tilde{S}}(\alpha) \quad y^U_\alpha = \max \phi^{-1}_{\tilde{S}}(\alpha)$$

$$p^L_\alpha = \min \phi^{-1}_{\tilde{d}}(\alpha) \quad p^U_\alpha = \max \phi^{-1}_{\tilde{d}}(\alpha)$$

$$v^L_\alpha = \min \phi^{-1}_{\tilde{V}}(\alpha) \quad v^U_\alpha = \max \phi^{-1}_{\tilde{V}}(\alpha)$$

VOL. 2, NO. 5, June 2012 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

$$c_\alpha^L = \min \phi_{\tilde{C}}^{-1}(\alpha) \quad c_\alpha^U = \max \phi_{\tilde{C}}^{-1}(\alpha)$$

Therefore, we use the α-cuts of $\tilde{L}_S(MV)$ to construct its membership functions since the membership function defined in (12) is parameterized by α.

Using Zadeh's extension principle, $\phi_{\tilde{L}_S(MV)}$ is the minimum of $\phi_{\tilde{\lambda}}(x)$, $\phi_{\tilde{S}}(y)$, $\phi_{\tilde{d}}(p)$, $\phi_{\tilde{V}}(v)$, $\phi_{\tilde{C}}(c)$. To derive the membership function of $\phi_{\tilde{L}_S(MV)}$(z), we need atleast one of the following cases to hold such that

$$z = \frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\}$$

Satisfies $\phi_{\tilde{L}_S(MV)}$(z) = α.

Case (i)

$$(\phi_{\tilde{\lambda}}(x) = \alpha, \phi_{\tilde{S}}(y) \ge \alpha, \phi_{\tilde{d}}(p) \ge \alpha, \phi_{\tilde{V}}(v) \ge \alpha, \phi_{\tilde{C}}(c) \ge \alpha)$$

Case (ii)

$$(\phi_{\tilde{\lambda}}(x) \ge \alpha, \phi_{\tilde{S}}(y) = \alpha, \phi_{\tilde{d}}(p) \ge \alpha, \phi_{\tilde{V}}(v) \ge \alpha, \phi_{\tilde{C}}(c) \ge \alpha)$$

Case (iii)

$$(\phi_{\tilde{\lambda}}(x) \ge \alpha, \phi_{\tilde{S}}(y) \ge \alpha, \phi_{\tilde{d}}(p) = \alpha, \phi_{\tilde{V}}(v) \ge \alpha, \phi_{\tilde{C}}(c) \ge \alpha)$$

Case (iv)

$$(\phi_{\tilde{\lambda}}(x) \ge \alpha, \phi_{\tilde{S}}(y) \ge \alpha, \phi_{\tilde{d}}(p) \ge \alpha, \phi_{\tilde{V}}(v) = \alpha, \phi_{\tilde{C}}(c) \ge \alpha)$$

Case (v)

$$(\phi_{\tilde{\lambda}}(x) \ge \alpha, \phi_{\tilde{S}}(y) \ge \alpha, \phi_{\tilde{d}}(p) \ge \alpha, \phi_{\tilde{V}}(v) \ge \alpha, \phi_{\tilde{C}}(c) = \alpha)$$

This can be accomplished using parametric Non Linear programming technique. The NLP to find the lower and upper bounds of the α – cuts of case (i) are

$$[L_S(MV)]_\alpha^{L_1} = \min z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………..(14a)

$$[L_S(MV)]_\alpha^{U_1} = \max z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

....……….(14b)

For case (ii) are

$$[L_S(MV)]_\alpha^{L_2} = \min z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………(14c)

$$[L_S(MV)]_\alpha^{U_2} = \max z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………(14d)

For case (iii) are

$$[L_S(MV)]_\alpha^{L_3} = \min z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………(14e)

$$[L_S(MV)]_\alpha^{U_3} = \max z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

……………(14f)

For case (iv) are

$$[L_S(MV)]_\alpha^{L_4} = \min z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

………………(14g)

$$[L_S(MV)]_\alpha^{U_4} = \max z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………….(14h)

For case (v) are

$$[L_S(MV)]_\alpha^{L_5} = \min z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

…………….(14i)

$$[L_S(MV)]_\alpha^{U_5} = \max z\left(\frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[pE(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p - xE(c)E(y)]}\right)$$

……………..(14j)

From the definition of λ(α), S(α), d(α), V(α), C(α) can be replaced by $x \in [x_\alpha^L, x_\alpha^U]$ $y \in [y_\alpha^L, y_\alpha^U]$, $d \in [d_\alpha^L, d_\alpha^U]$, $v \in [v_\alpha^L, v_\alpha^U], c \in [c_\alpha^L, c_\alpha^U]$. The α-cuts form a nested structure with

$$[x_{\alpha_1}^L, x_{\alpha_1}^U] \subseteq [x_{\alpha_2}^L, x_{\alpha_2}^U]$$
$$[y_{\alpha_1}^L, y_{\alpha_1}^U] \subseteq [y_{\alpha_2}^L, y_{\alpha_2}^U]$$
$$[p_{\alpha_1}^L, p_{\alpha_1}^U] \subseteq [p_{\alpha_2}^L, p_{\alpha_2}^U]$$
$$[v_{\alpha_1}^L, v_{\alpha_1}^U] \subseteq [v_{\alpha_2}^L, v_{\alpha_2}^U]$$

$$[c_{\alpha_1}^L, c_{\alpha_1}^U] \subseteq [c_{\alpha_2}^L, c_{\alpha_2}^U]$$

Therefore (14a), (14c), (14e), (14g), (14i) have the same smallest element and (14b), (14d), (14f), (14h), (14j) have the same largest element. To find the membership function of $\phi_{\tilde{L}_S(MV)}$(z), which is equivalent to finding the lower bound $L_S(MV)_\alpha^L$ and upper bound $L_S(MV)_\alpha^U$ which can be written as

$$(L_S(MV))_\alpha^L = \min_{\substack{x\in X, y\in Y, p\in P, v\in V, c\in C \\ \lambda E(C)E(S)<1}} z = \frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[p\,E(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p-xE(c)E(y)]}\}$$

……………(15a)

$$(L_S(MV))_\alpha^U = \max_{\substack{x\in X, y\in Y, p\in P, v\in V, c\in C \\ \lambda E(C)E(S)<1}} z = \frac{xE(c)E(y)}{p} + \frac{xE(c)E(v^2)}{2E(V)} + \frac{x^2E^2(c)[p\,E(y^2)+2(1-p)E^2(y)] + xE(y)p[E(c^2)-E(c)]}{2p[p-xE(c)E(y)]}\}$$

……………(15b)

At least one of x and y must hit the boundaries of their α-cuts to satisfy

$$\phi_{\tilde{L}_S(MV)} = \alpha .$$

This model is a set of mathematical programs with boundary constraints and lends itself to the systematic study of how the optimal solutions change with $x_\alpha^L, x_\alpha^U, y_\alpha^L, y_\alpha^U, p_\alpha^L, p_\alpha^U, v_\alpha^L, v_\alpha^U, c_\alpha^L, c_\alpha^U$ as α varies over (0.1]. This model is a special case of parametric NLPs.

The crisp interval $[L_S(MV)_\alpha^L, L_S(MV)_\alpha^U]$ obtained from (15) represents the α-cuts of $\tilde{L}_S(MV)$, we have and $L_S(MV)_{\alpha_1}^U \le L_S(MV)_{\alpha_2}^U$, $0 < \alpha_2 < \alpha_1 \le 1$. In other words, $L_S(MV)_\alpha^L$ increases and $L_S(MV)_\alpha^U$ decreases. Consequently, the membership function $\phi_{\tilde{L}_S(MV)}(z)$, cab be found from (15). If both $L_S(MV)_\alpha^L$ and $L_S(MV)_\alpha^U$ in (15) are invertible with respect to α, then a left shape function $L(z) = [L_S(MV)_\alpha^L]^{-1}$ and the right $R(z) = [L_S(MV)_\alpha^U]^{-1}$ can be derived, from which the membership function $\phi_{\tilde{L}_S(MV)}(z)$, is constructed:

$$\phi_{\tilde{L}_S(MV)} = \begin{cases} L(z)(L_S(MV))_{\alpha=0}^L \le z \le (L_S(MV))_{\alpha=1}^L \\ 1(L_S(MV))_{\alpha=1}^L \le z \le (L_S(MV))_{\alpha=1}^U \\ R(z)(L_S(MV))_{\alpha=1}^U \le z \le (L_S(MV))_{\alpha=0}^U \end{cases}$$

In most cases, the values of $\{[L_S(MV)_\alpha^L, L_S(MV)_\alpha^U] / \alpha \in [0,1]\}$ and $L_S(MV)_\alpha^U$ cannot be solved analytically. Consequently a closed form membership function for $\phi_{\tilde{L}_S(MV)}(z)$ cannot be obtained. However the numerical solution for $L_S(MV)_\alpha^L$ and at different possibility levels can be collected to approximate the shapes of L(z) and R(z) , that is, the set of intervals $\{[L_S(MV)_\alpha^L, L_S(MV)_\alpha^U] / \alpha \in [0,1]\}$ shows the shape of $\phi_{\tilde{L}_S(MV)}(z)$, although the exact function is not known explicitly. Note that the membership functions for the expected waiting time in the queue can be expressed in the similar manner.

4. NUMERICAL EXAMPLE

In order to illustrate how the method discussed in the paper can be applied to the fuzzy $M^X/G/1$ queue, let us consider a following illustration. A company recruits its employees based on their performance in a training program, if an employee is not performed well in the training, he will be given training, and it will continue any number of times. If there is no employee for training then the trainer will go for vacation. Both single and multiple vacations are allowed. Arrival rate, service time group size departure probability and vacation time are trapezoidal fuzzy numbers and described by $\tilde{\lambda}$ = [2,3,4,5] $\tilde{S}$ = [7,8,9,10] $\tilde{C}$ = [1,2,3,4] $\tilde{V}$ = [3,4,5,6] $\tilde{d}$ = [110,111,112,113] respectively. The system manager wants to evaluate performance measures of the system such as the expected number of employees in system for multiple vacations.
Following (1)

$$L_S(MV) = \frac{\lambda E(C)E(S)}{d} + \frac{\lambda E(C)E(V^2)}{2E(V)} + \frac{\lambda^2E^2(C)[d\,E(S^2)+2(1-d)E^2(S)] + \lambda E(S)d[E(C^2)-E(C)]}{2d[d-\lambda E(C)E(S)]}$$

It is clear that in this example the steady state condition ρ = λE (S)E(C) < d is satisfied, thus the performance measure of interest can be constructed. First it is easy to find

VOL. 2, NO. 5, June 2012 ISSN 2225-7217
ARPN Journal of Science and Technology

http://www.ejournalofscience.org

$$[x_\alpha^L, x_\alpha^U] = [2+\alpha, 5-\alpha]\ [y_\alpha^L, y_\alpha^U] = [7+\alpha, 10-\alpha]$$
$$[p_\alpha^L, p_\alpha^U] = [110+\alpha, 113-\alpha]$$
$$[v_\alpha^L, v_\alpha^U] = [3+\alpha, 6-\alpha]\ [c_\alpha^L, c_\alpha^U] = [1+\alpha, 4-\alpha]$$

Table I: The α-cuts of the performance measures at 11α values for multiple vacations

α	$[L_S(MV)]^L[L_S(MV)]^U$		$[W_q(MV)]^L[W_q(MV)]^U$	
0.0	23.0187	694.1886	4.5265	55.4578
0.1	25.4142	519.1404	4.7636	42.3016
0.2	28.1239	409.9177	5.0363	34.0827
0.3	31.2201	335.3395	5.3525	28.4625
0.4	34.7985	281.2325	5.7228	24.3780
0.5	38.9887	240.2265	6.1613	21.2765
0.6	43.9704	208.1097	6.6878	18.8422
0.7	50.0011	182.3008	7.3308	16.8814
0.8	57.4623	161.1294	8.1322	15.2689
0.9	66.9456	143.4665	9.1572	13.9201
1.0	79.4200	128.5222	10.5128	12.7756

Table II: The α-cuts of the performance measures at 11α values for single vacations

α	$[L_S(SV)]^L[L_S(SV)]^U$		$[W_q(MV)]^L[W_q(MV)]^U$	
0.0	16.8783	670.7285	19.9368	469.5102
0.1	18.7353	496.3664	22.0741	353.4501
0.2	20.9082	387.8173	24.5231	280.9701
0.3	23.4689	313.9007	27.3530	231.4254
0.4	26.5128	260.4439	30.6557	195.4321
0.5	30.1690	220.0771	34.5561	168.1107
0.6	34.6168	188.5894	39.2281	146.6726
0.7	40.1132	163.3996	44.9209	129.4091
0.8	47.0392	142.8380	52.0038	115.2142
0.9	55.9860	125.7764	61.0510	103.3409
1.0	67.9220	111.4251	73.0026	93.2661

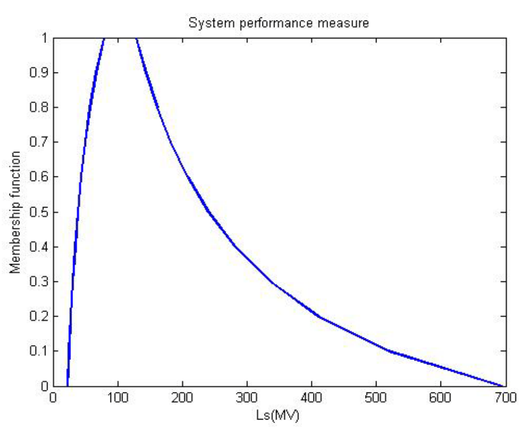

Fig 1: Expected waiting time of an employee in the queue for single vacation

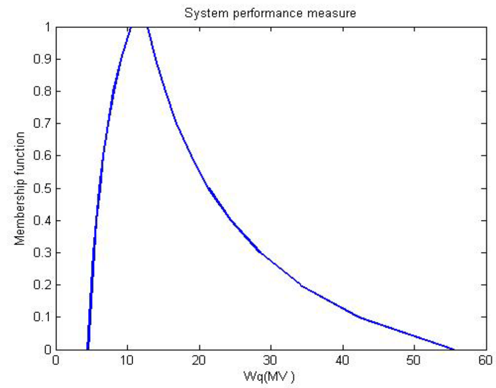

Fig 2: Expected waiting time of an employee in the queue for multiple vacation

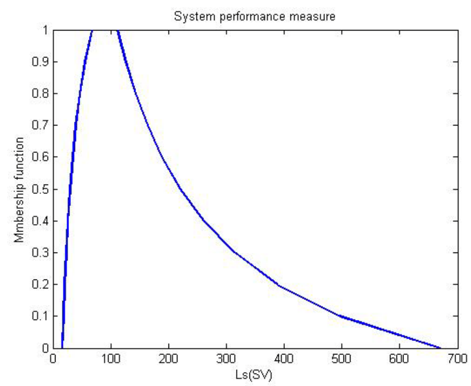

Fig 3: Expected length of the system for multiple vacation

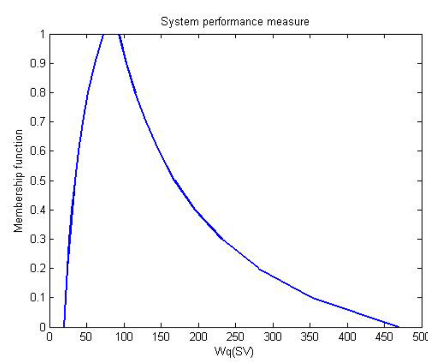

Fig 4: Expected waiting time of an employee in the queue for multiple vacation

With the help of Mat lab 7.04, we perform α-cuts of batch arrival rate, service time, vacation time, departure probability, group size and fuzzy expected number of employees in the system for multiple vacation at eleven distinct α levels 0 , 0.1, 0.2, …..1. Crisp intervals for fuzzy expected number of employees in system for multiple vacations at different possibilities α levels are presented in table I. Similarly other performance measure such as expected number of employees in the system for single vacation, expected waiting time of an employee in the queue for single and multiple vacation are also derived in table I. Fig 1 depicts the rough shape of L_S(MV) constructed from α-values. The rough shape turns out rather fine and looks like a continuous function. Other performance measures are depicted by remaining figures.

The α-cut represent the possibility that these four performance measure will lie in the associated range. Specially, α = 0 the range, performance measures could appear and for α =1 the range, the performance measures are likely to be. For example, while these four performance measures are fuzzy, the most likely value of the expected length of the system for multiple vacation L_S (MV) falls between 79.4200 and128.5222, and its value is impossible to fall outside the range of 23.0887 and 694.1886, it is definitely possible that the expected waiting time of an employee in the queue for multiple vacation W_e (MV) falls between 10.5128and 12.7756approximately, and it will never fall before 4.5265 or exceed 55.4578. The above information will be very useful for designing a queuing system.

5. CONCLUSION

Batch arrival Bernoulli feed back with server vacations queuing models have wider applications in operations and service mechanism for evaluation system performance. This paper develops a method to find the membership function of the system performance measure where the batch arrival size, arrival rate, service time, departure probability, vacation time are fuzzy. The idea is based on Zadeh's extension principle to transform the bulk arrival fuzzy queue to a family of bulk arrival crisp queues that can be described by two pairs of MINLP models. Since the performance measure is expressed by the membership function rather than by a crisp value, it maintains the fuzziness of input information, and the results can be used to represent the fuzzy system more accurately.

REFERENCES

[1] Baba.Y, On the $M^X/G/1$ queue with vacation time, Operations Research Letters 5(1986) 93-98.

[2] T.J. Bailey, On queuing process with bulk service, Journal of the Royal Statistical Society 16 (1954) 80-87.

[3] U.N.Bhatt, Embedded Markov Chain Analysis of bulk queues, Journal of the Australian Mathematical Society 244-263.

[4] A. Borthakur, A Poisson queue with general bulk service rule, Journal of the Assam Scientific Society 14(1971) 162-167

[5] Choudry M.L. and Templeton J.G.C. , A first Course in bulk queues, Wiley, New York,1983.

[6] Doshi B.T., Queuing systems with vacations- a survey, Queuing system I (1986) 29-66,1986.

[7]. Easton, Glen, Chaudry M.L. and Posner M.J.M., Some numerical results for the queing system , $GI^X/M/1$, European
Journal of Operations Research 18 (1984) 133-135.

[8] D.Gross,C.M. Harris, Fundamentals of Queuing Theory, Third edn.,Wiley, New York,1988.

[9] C.Kao,C.C. Li,S.P. Chen, Parametric Programming to the analysis of fuzzy queues, Fuzzy sets and systems 107 (1999) 93- 100.

[10] Kaufmann.A., An Introduction to the theory of fuzzy subsets, Vol I. Academic press, New York 1975.

[11] Lakshmi Srinivasan, N. Renganathan, R. Kalyanaraman, Single Server ,Bulk arrival, Bernoulli feed back queue with
vacations – Some Performance Measures,Information and Management services 13 (2002) 45-54.

[12] Lee,H.W., Bulk arrival queues with server vacations, Appl. Math, Modelling, Vol 13 (1989) 374-377.

[13] Levy,Y. Yechiali,U., Utilization of idle time in a M/G/1 queuing systems,Management Sc., Vol 22, (1975) 202-211.

[14] Levy,Yeuaton and Yechiali, Uri, An M/M/S queue with servers vacations, INFOR,Vol 14, (1976) 153-163,1976

[15] Powell,W.B., Iterative algorthims for bulk service queues with Poisson and non Poisson arrivals, Trans.SciVol 20 (1986) 60-79.

[16] R.R. Yager, A Characterization ofthe extension principle, Fuzzy Sets and Systems 18 (1986) 205-217.

[17] Yao, David D.W., Chaudry,M.L.andTempleton,J.G.C., On bounds for bulk arrival queues,

[18] L.A.Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, 1(1978) 3-28.

[19] L.A.Zadeh, Fuzzy probabilities,Information processing and management 20, No. 3(1984) 363-372..

[20] Zimmermann, H.J., Fuzzy Set Theory and its applications, fourth ed., Klnwer Academic, Boston 2001.