Viscous Fluid Cosmological Models in Bianchi Type-I Space-Time

[1]Anirudh Pradhan, Rashid Zia[2] and R. P. Singh[3]
[1]Department of Mathematics, Hindu Post-graduate College,
Zamania-232 331, Ghazipur, India
[1]acpradhan@yahoo.com

[2,3]Department of Mathematics, T. D. Post-graduate College, Jaunpur-232 331, U. P., India
[2]rashidzya@gmail.com,[3]rashidzya@gmail.com

Abstract—**Exact solutions of Einstein's field equations are obtained in a spatially homogeneous and anisotropic Bianchi type-I space-time in presence of a dissipative fluid. Following the technique (Pradhan & Amirhashchi in Mod. Phys. Lett. A 26: 2261 2011), we consider the scale factor $a(t) = \sqrt{t^n e^t}$, which yield time dependent DP. To get the deterministic solution a barotropic equation of state together with the shear viscosity is proportional to expansion scalar, is also assumed. It is observed that the initial nature of singularity is not changed due to the presence of viscous fluid. The basic equation of thermodynamics have been deduced and the thermodynamic aspects of the models have been discussed. For different option of n, we can generate a class of bulk viscous models in Bianchi type-I space-time. The physical and geometric properties of cosmological models are also discussed.**

Index Terms—**Bianchi type-I universe, Viscous fluid, Variable deceleration parameter**

I. Introduction

Anisotropic cosmological models play significant role in understanding the behaviour of the universe at its early stages of evolution. Observations by the Differential Radiometers on NASA's Cosmic Background Explorer registered anisotropy in various angle scales. The theoretical argument [1] and the modern experimental data support the existence of an anisotropic phase, which turns into an isotropic one. The anisotropy of the universe within the framework of Bianchi type space-times is found to be useful in generating arbitrary ellipsoidality of the Universe, and to fine tune the observed CMBR anisotropies. Koivisto and Mota [2], [3] have investigated cosmological models with anisotropic EoS and have also shown that the present SN Ia data allows large anisotropy. The motivation for this investigation comes from the hints of statistical anisotropy of our Universe that several observations seem to suggest. Recently, Akarsu and Kilinc [4] have described some features of the Bianchi type-I universes in presence of fluid that yields an anisotropic EoS. Hence, for a realistic cosmological model one should consider spatially homogeneous and anisotropic space-times and then show whether they can evolve to the observed amount of homogeneity and isotropy. The only spatially homogeneous but anisotropic models other than Bianchi type models are the Kantowski-Sachs locally symmetric family. See Ellis & van Elst [5] for generalized, particularly anisotropic, cosmological models and Ellis [6] for a concise review on Bianchi type models. Recently Pradhan and Pandey [7], [8], Pradhan and Singh [9], [10], Pradhan and Chouhan [11], Pradhan et al. [12], Pradhan [13], Bali et al. [14], Singh et al. [15], Singh and Kumar [16], [17], Kumar and Singh [18], Saha [19]– [21] and Yadav et al. [22] have studied Bianchi type-I cosmological models in different context.

The study of role of dissipative effects in the evolution of the universe during early stages has taken considerable interest of researchers as in the early universe viscosity may arise due to various processes such as decoupling of neutrinos during the radiation rea, creation of super-string during the quantum era, particle collision involving gravitation, particle creation process and the formation of galaxies [1], [23], [24]. To describe the relativistic theory of viscosity, Eckart [25] made the first attempt. The theories of dissipation in Eckart formulation suffers from serious short-coming viz., causality and stability regardless of the choice of equation of state. Israel and Stewart [26] and Pavon [27] developed a fully relativistic formulation of the theory taking into account second order deviation terms in the theory, which is termed as "transient" or "extended" irreversible thermodynamics (EIT). In irreversible thermodynamics, the entropy is no longer conserved, but grows according to the second law of thermodynamics. Bulk viscosity arises typically in the mixtures either of different species or of species but with different energies. The solution of the full causal theory are well behaved for all time. Several authors (Singh and Kale [28] and Yadav [29] and references therein) have investigated cosmological models with dissipative effect. Pradhan [30]– [33], Pradhan et al. [34], Rao et al. [35], Singh et al. [36], Bali and Pradhan [37] have studied bulk viscous fluid cosmological models in different physical context.

With these motivations and following the technique of Pradhan and Amirhashchi [38], in this paper, we have investigated a new spatially homogeneous and anisotropic Bianchi type-I cosmological model by considering a dissipative fluid. The Einstein's field equations are solved explicitly. The out line of the paper is as follows: In Sect. 2, the metric and the field equations are described. Section 3

deals with the solutions of the field equations by considering two different values of the scale factors. This also contains the thermodynamic equation and its aspects of the models. Finally, conclusions are summarized in the last Sect. 4.

II. THE METRIC AND FIELD EQUATIONS

We consider totally anisotropic Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where the metric potentials A, B and C are functions of cosmic time t alone. This ensures that the model is spatially homogeneous.

We define the following parameters to be used in solving Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-I model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \quad (2)$$

A volume scale factor V is given by

$$V = a^3 = ABC. \quad (3)$$

In analogy with FRW universe, we also define the generalized Hubble parameter H and deceleration parameter q as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad (4)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}, \quad (5)$$

where an over dot denotes derivative with respect to the cosmic time t.

Also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (6)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively.

The Einstein's field equations (in gravitational unit $8\pi G = c = 1$) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}, \quad (7)$$

where T_{ij} is the stress energy tensor of matter which, in case of viscous fluid, has the form [39]

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \eta\mu_{ij}, \quad (8)$$

with

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;i} = p - (3\xi - 2\eta)H \quad (9)$$

and

$$\mu_{ij} = u_{i;j} + u_{j;i} + u_i u^\alpha u_{j;\alpha} + u_j u^\alpha u_{i;\alpha}. \quad (10)$$

In the above equations, ξ and η stand for the bulk and shear viscosity coefficients respectively; ρ is the matter density; p is the isotropic pressure and u^i is the four-velocity vector

satisfying $u^i u_i = -1$.

In a co-moving coordinate system, where $u^i = \delta^i_0$, the field equations (7), for the anisotropic Bianchi type-I space-time (1) and viscous fluid distribution (8), yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + 2\eta\frac{\dot{A}}{A}, \quad (11)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + 2\eta\frac{\dot{B}}{B}, \quad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + 2\eta\frac{\dot{C}}{C}, \quad (13)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho. \quad (14)$$

The usual definitions of the dynamical scalars such as the expansion scalar (θ) and the shear scalar (σ) are considered to be

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} \quad (15)$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \quad (16)$$

where

$$\sigma_{ij} = u_{i;j} + \frac{1}{2}(u_{i;k}u^k u_j + u_{j;k}u^k u_i) + \frac{1}{3}\theta(g_{ij} + u_i u_j). \quad (17)$$

The anisotropy parameter (A_m) is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \quad (18)$$

The energy conservation equation $T^{ij}_{;j} = 0$, leads to

$$\dot{\rho} = -(p + \rho)\theta + \xi\theta^2 + 4\eta\sigma^2. \quad (19)$$

It follows from (19) that for contraction, that is, $\theta < 0$, we have $\dot{\rho} > 0$ so that the matter density increases or decreases depending on whether the viscous heating is greater or less than the cooling due to expansion.

III. SOLUTION OF FIELD EQUATIONS

We have a system of four independent equations $(11) - (14)$ and seven unknown variables, namely A, B, C, p, ρ, ξ and η. So for complete determinacy of the system, we need three appropriate relations among these variables that we shall consider in the following section and solve the field equations.

We follow the approach of Saha [19] to solve the field equations $(11)-(14)$. Subtracting (11) from (12), (11) from (13), (12) from (13) and taking second integral of each, we get the following three relations

$$\frac{A}{B} = d_1 \exp\left(x_1 \int a^{-3}e^{-2\int \eta dt}dt\right), \quad (20)$$

$$\frac{A}{C} = d_2 \exp\left(x_2 \int a^{-3}e^{-2\int \eta dt}dt\right), \quad (21)$$

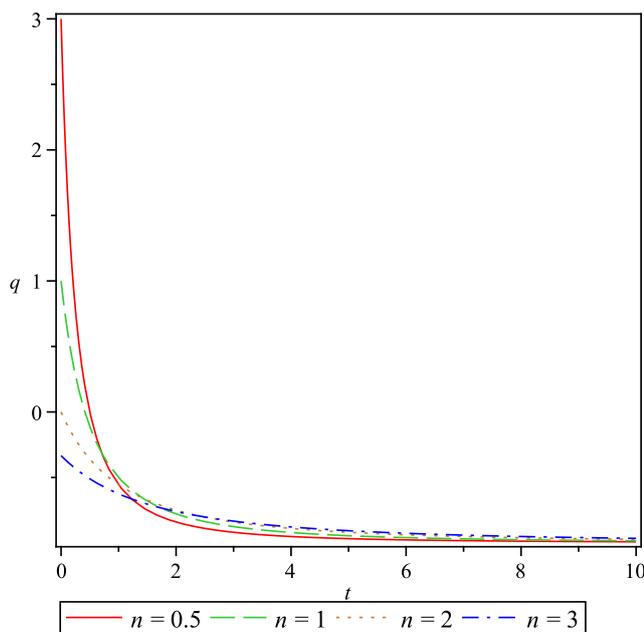$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3}e^{-2\int \eta dt}dt\right), \quad (22)$$

Figure 1. The plot of deceleration parameter q versus t

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

From $(20)-(22)$, the metric functions can be explicitly written as

$$A(t) = a_1 a \exp\left(b_1 \int a^{-3} e^{-2\int \eta dt} dt\right), \quad (23)$$

$$B(t) = a_2 a \exp\left(b_2 \int a^{-3} e^{-2\int \eta dt} dt\right), \quad (24)$$

$$C(t) = a_3 a \exp\left(b_3 \int a^{-3} e^{-2\int \eta dt} dt\right), \quad (25)$$

where

$$a_1 = \sqrt[3]{d_1 d_2}, \ a_2 = \sqrt[3]{d_1^{-1} d_3}, \ a_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$b_1 = \frac{x_1 + x_2}{3}, \ b_2 = \frac{x_3 - x_1}{3}, \ b_3 = \frac{-(x_2 + x_3)}{3}.$$

These constants satisfy the following two relations

$$a_1 a_2 a_3 = 1, b_1 + b_2 + b_3 = 0. \quad (26)$$

Thus the metric functions are found explicitly in terms of the average scale factor a.

For any physically relevant model, the Hubble parameter and deceleration parameter (DP) are the most important observational quantities. The first quantity sets the present time scale of the expansion while the second one reveals that the present state of evolution of universe is speeding up instead of slowing down as expected before the type Ia supernovae observations [40], [41]. Therefore, following Pradhan et al. [38], we consider the DP as a variable for which we consider the variation of scale factor a with cosmic time t by the relation

$$a(t) = \sqrt{t^n e^t}, \quad (27)$$

n is an arbitrary constant and the constant of integration is absorbed in t without any loss of generality.

From (5) and (27), we get the time varying DP as

$$q(t) = \frac{2n}{(n+t)^2} - 1. \quad (28)$$

From Eq. (28), we observe that $q > 0$ for $t < \sqrt{2n} - n$ and $q < 0$ for $t > \sqrt{2n} - n$. It is observed that for $0 < n < 2$, our model is evolving from deceleration phase to acceleration phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observation. Figure 1 graphs the deceleration parameter (q) versus time which gives the behaviour of q from decelerating to accelerating phase for different values of n. Thus our derived model has accelerated expansion at present epoch which is consistent with recent observations of Type Ia supernova (Riess et al. [40]; Perlmutter et al. [41]) and CMB anisotropies (Bennett et al. [42]; de Bernardis et al. [43]; Hanany et al. [44]).

Next, we assume that the coefficient of shear viscosity (η) is proportional to the expansion scalar (θ) i.e. $\eta \propto \theta$, which leads to

$$\eta = \eta_0 \theta, \quad (29)$$

where η_0 is proportionality constant. Such relation has already been proposed in the physical literature as a physically plausible relation [9].

Finally to conveniently specify the source, we assume the perfect gas equation of state, which may be written as

$$p = \gamma \rho, 0 \leq \gamma \leq 1. \quad (30)$$

Using Eqs. (15), (27) and (29) into $(23)-(25)$, we get the following expressions for the scale factors

$$A = a_1 (t^n e^t)^{\frac{1}{2}} \exp\left[b_1 \int \left\{(t^n e^t)^{-\frac{3}{2}(1+2\eta_0)}\right\} dt\right], \quad (31)$$

$$B = a_2 (t^n e^t)^{\frac{1}{2}} \exp\left[b_2 \int \left\{(t^n e^t)^{-\frac{3}{2}(1+2\eta_0)}\right\} dt\right], \quad (32)$$

$$C = a_3 (t^n e^t)^{\frac{1}{2}} \exp\left[b_3 \int \left\{(t^n e^t)^{-\frac{3}{2}(1+2\eta_0)}\right\} dt\right], \quad (33)$$

The physical parameters such as directional Hubble factors (H_i), Hubble parameter (H), expansion scalar (θ), spatial volume (V), anisotropy parameter (A_m) and shear scalar (σ) are given by

$$H_i = \frac{1}{2}\left(\frac{n}{t} + 1\right) + b_i (t^n e^t)^{-\frac{3}{2}(1+2\eta_0)} \quad (34)$$

$$H = \frac{1}{2}\left(\frac{n}{t} + 1\right), \quad (35)$$

$$\theta = \frac{3}{2}\left(\frac{n}{t} + 1\right), \quad (36)$$

$$V = (t^n e^t)^{\frac{3}{2}}, \quad (37)$$

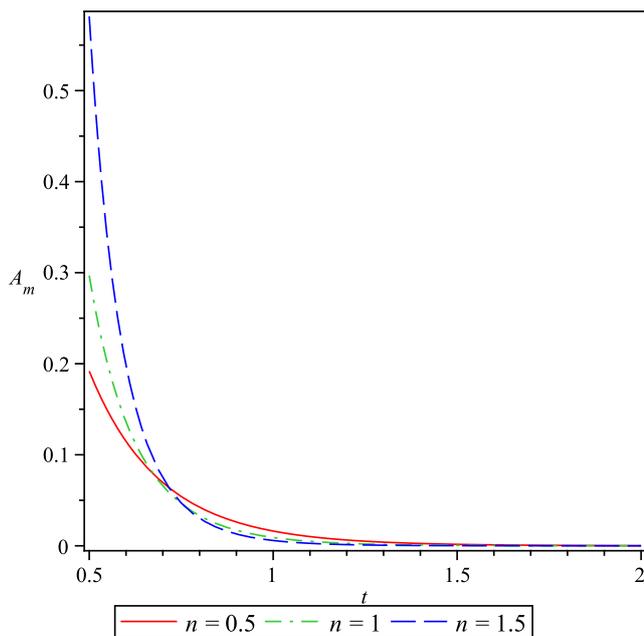$$A_m = \frac{1}{3}\beta_1 \frac{t^2 (t^n e^t)^{-3(1+2\eta_0)}}{(n+t)^2}, \quad (38)$$

Figure 2. The plot of anisotropy parameter A_m versus t for $\beta_1 = 1$, $\eta_0 = 0.1$
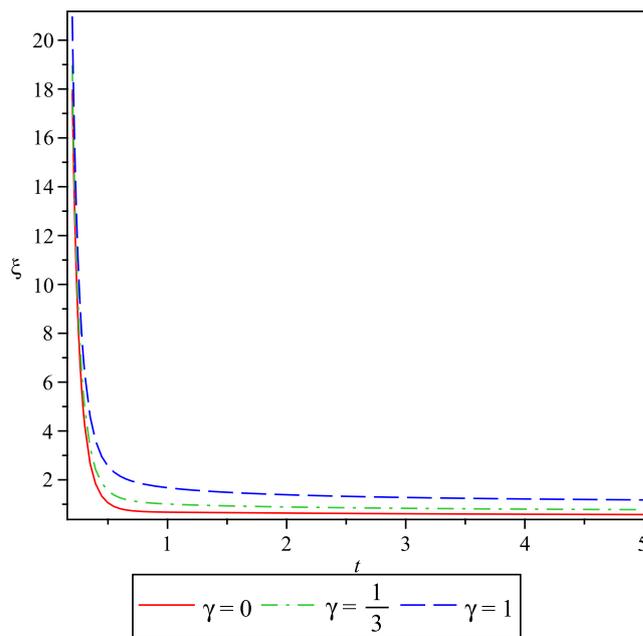

Figure 4. The plot of bulk viscosity coefficient ξ versus t for $n = \beta_4 = 1$, $\eta_0 = 0.1$

where

$$\beta_2 = b_1 b_2 + b_2 b_3 + b_1 b_3,$$

$$\beta_3 = b_2^2 + b_3^2 + b_2 b_3,$$

The expressions for isotropic pressure, bulk viscosity and shear viscosity are given by

$$p = \frac{3\gamma}{4}\left(\frac{n}{t}+1\right)^2 + \beta_2 \gamma (t^n e^t)^{-3(1+2\eta_0)}, \qquad (42)$$

$$\xi = \frac{1}{2}(1+\gamma)\left(\frac{n}{t}+1\right) - \frac{2n}{3t(n+t)} + \frac{2}{3}(\beta_2\gamma+\beta_3)\left(\frac{t}{n+t}\right)^{-3(1+2\eta_0)}, \qquad (43)$$

$$\eta = \frac{3\eta_0}{2}\left(\frac{n}{t}+1\right). \qquad (44)$$

It is already observed that the above set of solutions satisfy the energy conservation equation (19) identically. Therefore, the above solutions are exact solutions of Einstein's field equations (11)−(14). From Eqs. (36) and (37), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. From Eqs. (31)−(33), we see that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity [45]. All the physical quantities pressure (p), energy density (ρ), bulk viscosity (ξ), shear viscosity (η), Hubble factor (H) and shear scalar (σ) diverge at $t = 0$. As $t \to \infty$, scale factors and volume becomes infinite whereas p, ρ approach to zero.

Figure 2 depicts the variation of anisotropic parameter A_m versus time t. From the figure, we observe that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in our derived model at present epoch.
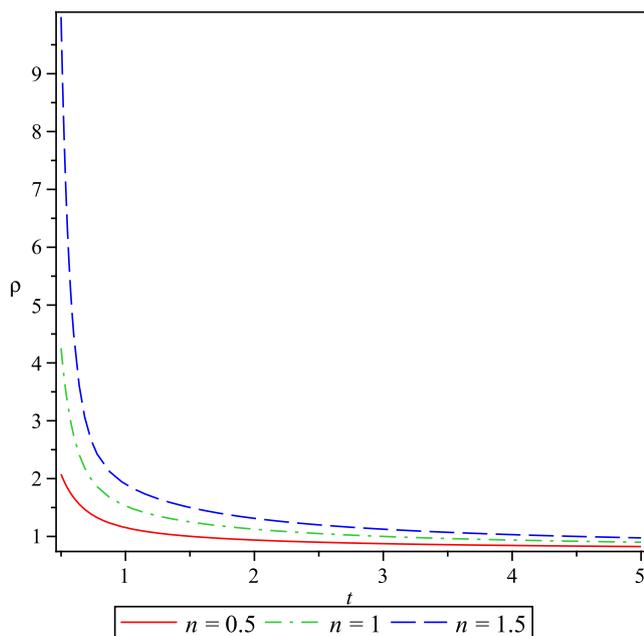

Figure 3. The plot of energy density ρ versus t for $\beta_2 = 1$, $\eta_0 = 0.1$

$$\sigma^2 = \frac{1}{2}\beta_1 (t^n e^t)^{-3(1+2\eta_0)}, \qquad (39)$$

where

$$\beta_1 = b_1^2 + b_2^2 + b_3^2.$$

The energy density and effective pressure of the model read as

$$\rho = \frac{3}{4}\left(\frac{n}{t}+1\right)^2 + \beta_2 (t^n e^t)^{-3(1+2\eta_0)}, \qquad (40)$$

$$\bar{p} = \frac{n}{t^2} - \frac{3}{4}(1-2\eta_0)\left(\frac{n}{t}+1\right)^2 - \beta_3 (t^n e^t)^{-3(1+2\eta_0)}, \quad (41)$$

From Eqs. (40) and (42) we observed that the energy density ρ and isotropic pressure p are always positive

and decreasing functions of time. Figure 3 describes the variation of energy density (ρ) with cosmic time t. From this figure we observe that ρ is a positive decreasing function of time.

Figure 4 plots the variation of bulk viscous coefficient ξ versus time t. From this figure we observe that ξ is a positive decreasing function of time and it approaches to a constant quantity which is near to zero. This is in good agreement with physical behaviour of ξ.

From Eqs. (36) and (39), we get

$$\frac{\sigma^2}{\theta^2} = \frac{2}{9}\beta_1 \left(\frac{t}{n+t}\right)^2 (t^n e^t)^{-3(1+2\eta_0)}. \qquad (45)$$

The above equation shows that the ratio of shear and expansion scalars is a decreasing function of time and decays to zero as $t \to \infty$. Hence the model approaches to isotropy for large value of time i.e. at present epoch. Also we observe that the rate of decay falls in the absence of shear viscosity.

From (36) and (40), one can get the following relation

$$\frac{\rho}{\theta^2} = \frac{1}{3} + \frac{4}{9}\beta_2 \left(\frac{t}{n+t}\right)^2 (t^n e^t)^{-3(1+2\eta_0)}. \qquad (46)$$

The above relation shows that $\frac{\rho}{\theta^2}$ is maximum at $t = \infty$ and the maximum value is $\frac{1}{3}$. It is important to note here that $\lim_{t \to 0} \left(\frac{\rho}{\theta^2}\right)$ spread out to be constant. Therefore the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [46].

It is worth mentioned here that the anisotropy parameter decreases faster with time due to the presence of viscosity [see, Fig. 2, and Eq. (38)]. So it is concluded that viscosity played an important role in the process of isotropization of the large scale structure of the universe. We also conclude that the model represents shearing, non-rotating and expanding universe, which starts with a big bang and approaches to isotropy at present epoch.

IV. THERMODYNAMIC EQUATIONS

The energy in a comoving volume is $U = \rho V$. The equation for production of entropy S in a comoving volume due to the dissipative effects in a fluid with temperature T is given by

$$T\dot{S} = \dot{U} + p\dot{V} = 3V(3\xi + 2\eta A_m)H^2. \qquad (47)$$

In a cosmic fluid where the energy density and pressure of the cosmic fluid are functions of temperature only, $\rho = \rho(T)$, $p = p(T)$ and where the cosmic fluid has no net charge, we obtain easily (Grøn [47])

$$S = \frac{V}{T}(\rho + p). \qquad (48)$$

From (47) and (48), we get the following expression for the entropy production rate in viscous Bianchi type-I universe

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta A_m)H^2}{\rho + p}. \qquad (49)$$

For a fluid obeying the equation of state (30), (48) and (49) become

$$S = \frac{V}{T}(1 + \gamma)\rho, \qquad (50)$$

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta A_m)H^2}{(1 + \gamma)\rho}. \qquad (51)$$

Equation (51) can be rewritten as

$$\frac{\dot{S}}{S} = \frac{\xi + 4\eta(\sigma^2/\theta^2)}{(1 + \gamma)(\rho/\theta^2)}, \qquad (52)$$

which gives the rate of change of entropy with time.

Let the entropy density be s so that

$$s = \frac{S}{V} = \frac{(1 + \gamma)\rho}{T}. \qquad (53)$$

It defines the entropy density in terms of the temperature.

The first law of thermodynamics may be written as

$$d(\rho V) + \gamma\rho dV = (1 + \gamma)Td\left(\frac{\rho V}{T}\right), \qquad (54)$$

which on integration, yields

$$T \sim \rho^{\left(\frac{\gamma}{1+\gamma}\right)}. \qquad (55)$$

From (53) and (55), one can get

$$s \sim \rho^{\left(\frac{1}{1+\gamma}\right)}. \qquad (56)$$

The entropy in a comoving volume then varies according to

$$S \sim sV. \qquad (57)$$

These equations are not valid for a vacuum fluid with $\gamma = -1$. For a Zel'dovich fluid ($\gamma = 1$), we get

$$T \sim \rho^{\frac{1}{2}} \text{ and } s \sim \rho^{\frac{1}{2}}, \qquad (58)$$

so that the entropy density is proportional to the temperature.

Using (37) and (40), we find the respective temperature (T), entropy density (s) and total entropy (S) from (55)−(57), as

$$T = T_0 \left[\frac{3}{4}\left(\frac{n}{t} + 1\right)^2 + \beta_2(t^n e^t)^{-3(1+2\eta_0)}\right]^{\frac{\gamma}{(1+\gamma)}}, \qquad (59)$$

$$s = s_0 \left[\frac{3}{4}\left(\frac{n}{t} + 1\right)^2 + \beta_2(t^n e^t)^{-3(1+2\eta_0)}\right]^{\frac{\gamma}{(1+\gamma)}}, \qquad (60)$$

$$S = S_0 \left[\frac{3}{4}\left(\frac{n}{t} + 1\right)^2 + \beta_2(t^n e^t)^{-3(1+2\eta_0)}\right]^{\frac{\gamma}{(1+\gamma)}}, \qquad (61)$$

where T_0, s_0 and S_0 are positive constants.

From Eq. (61) we observe that the total entropy S increases with time.

The rate of change of entropy is obtained as

$$\frac{\dot{S}}{S} = \frac{3}{(1+\gamma)\left[\frac{3}{4}\left(\frac{n}{t} + 1\right)^2 + \beta_2(t^n e^t)^{-3(1+2\eta_0)}\right]} \times$$

$$\left[\frac{3}{8}(1+\gamma)\left(\frac{n}{t}+1\right)^3 - \frac{3n(n+t)}{2t^3} + \right.$$

$$\left. \frac{3}{2}(2\eta_0\beta_1 + \gamma\beta_2 + \beta_3)\left(\frac{n}{t}+1\right)(t^n e^t)^{-3(1+2\eta_0)} \right]. \quad (62)$$

From above equation we observe that the relation $\frac{\dot{S}}{S} > 0$. This also implies that the total entropy increases with time in Bianchi type-I model presented in this paper.

V. Concluding Remarks

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-I space-time with bulk and shear viscosity in the context of general relativity. The Einstein's field equations have been solved exactly by considering a scale factor $a(t) = \sqrt{t^n e^t}$ which yields a time dependent deceleration parameter (DP). In literature it is a plebeian practice to consider a constant deceleration parameter. Now for a Universe which was decelerating in past and accelerating at present epoch, the DP must show signature flipping. Therefore our consideration of DP to be variable is physically justified and consistent with recent observations.

For different choice of n, we can generate a class of viscous fluid models in Bianchi type-T space-time. For $n = 1$, Eq. (27) reduces to $a(t) = \sqrt{t e^t}$ and it yields $q(t) = \frac{2}{(1+t)^2} - 1$. It is observed from Figures 1 to 4 that all the physical parameters are also in good harmony with current observations. All the values of physical parameters can be derived from their respective expressions by putting $n = 1$.

The derived models represent expanding, shearing and non-rotating universe, which approach to isotropy for large value of t. This is consistent with the behaviour of the present universe as already discussed in introduction.

Due to dissipative processes, the mean anisotropy and the shear of the cosmological model of the universe for Bianchi type-I space-time tend to zero very rapidly in both cases. It has also been observed that shear coefficient (η) plays more important role than bulk coefficient (ξ) in the isotropization process of the universe. Therefore it may be possible that the isotropy observed in the present universe, is a consequence of the viscous effects in the cosmic fluid right from the beginning of the evolution of the universe.

The basic equations of thermodynamics for the present models of the universe in Bianchi type-I space-time have been derived. We have observed that the total entropy of the system increases with time. Since we know that from the state of thermodynamic equilibrium, the law deduced the principle of increase of entropy and explains the phenomenon of irreversibility in nature. Hence our results derived in the present paper are in good agreement with second law of thermodynamics.

In absence of viscosity i.e. if we set $\xi \to 0$ and $\eta \to 0$ ($\eta_0 = 0$), we can obtain the solutions of Einstein's field equations (7) for perfect fluid distribution in Bianchi type-I space-time. It is a straight forward calculations from our results described in Sect. 3, we do not need to mention here.

Finally, the exact solutions presented in this paper are new and may be useful for better understanding of the evolution of the universe in Bianchi type-I space-time with viscous effects as well as for perfect fluid distribution in general theory of gravitation.

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