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Is the Equity Profile in India Flat: Statistical Evidence

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ABSTRACT

The paper econometrically estimate investors' optimal portfolios are independent of their investment horizon. When ex ante diversification is investigated there appears to be no evidence of increased demand for equity over a longer investment horizons in India. That is, in India we obtain a flat equity profile over the investment horizons. Therefore, the mean-aversion in fixed-income explains the time diversification effect. The results also indicate that cross- correlation amongst asset returns do not seem to play any role in time diversification either.

Keywords: *Stochastic dominance, Vector Autoregressive (VAR) representation, Sharpe ratio, Akaike Information Criterion (AIC).*

1.   INTRODUCTION

Over the last few years the growing acceptance of life cycle investment products, such as target retirement mutual funds, has renewed interest in time diversification. The objective in this paper is not to prove or disprove time diversification, but to evaluate whether the concept must be valid for a 'horizon-based' asset-allocation framework to be viable and appropriate[1]. Experts like Kritzman (1994) define time diversification as the notion that above-average returns of assets tend to offset below-average returns over a longer time horizon. However, he points out that while the annualized dispersion of returns moderates toward the expected mean, the dispersion of terminal wealth increases as investment horizon increases. He suggests that although the probability of losing money in stocks is lower over longer investment horizons than shorter ones, the size of the potential loss increases. This view supports the commonly held notions that younger investors should favor a portfolio heavily weighted in stocks to capitalize on the equity risk premium relative to bonds and Treasury bills and that, over long enough horizons, this equity risk premium was reliable.

Samuelson [17] rejected the premise that the risk of stocks decreased over longer time horizons.

According to Samuelson, the investment horizon can have no effect on portfolio composition. Relying on utility theory, he said that investors want to maximize the utility of wealth, rather than expected return or terminal wealth. That is, investors should be interested in what happens to their wealth over time, not just at a point in time (such as at retirement).

The practice of rationalizing high equity allocations for investors with longer investment horizons seems to have been common enough in the financial community to be considered "oversold" in Samuelson's opinion. Many joined the debate in different ways on either side, adding additional layers to this increasingly complex topic. Bodie [2] used option pricing theory to illustrate how the cost of insuring against a stock return below the risk-free rate increased, rather than decreased, with longer contracts. Since higher option premiums suggested higher perceived risk for longer contracts, he concluded that time diversification was not evident.

Reichenstein and Dorsett [14] modeled two sets of return projections, one based on the "random walk" assumption that is common among detractors of time diversification and another based on mean reversion, common among its supporters. Both models supported the notions that it is reasonable for investors with longer investment horizons to have larger allocations to risky assets (such as stocks) and that a portfolio's relative risk depends upon the length of the holding period.

Fisher and Statman [7] evaluated not only the time diversification issue but the assumptions often used to rebut the concept, for example, the assumption that stock market returns follow a random walk pattern, or that investors' future wealth depends only on their investment portfolios. Though a debatable topic itself the mean reversion hypothesis would suggest that a longer investment horizon would better enable investors to weather adverse market outcomes if experienced at the outset of the investment strategy. Samuelson's utility argument is based on the standard finance assumption

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[1] The aim is not to provide, as in stochastic dominance theory of investment, alternative measures of investment superiority or to evaluate whether an asset A is stochastically superior to asset B. Asset A stochastically dominate asset B in the sense of what is known as first degree stochastic dominance (FDSD) when the probability that the return on A exceeds any value y is greater than the probability that the return on B exceeds y, for all possible values of x Asset A stochastically dominates asset B in the sense of second degree stochastic dominance (SDSD) when the following condition holds:

that investors are always risk averse and that therefore risk tolerance does not vary with wealth. However, he also notes that this may not always be the case and concedes that people may be less risk-tolerant in absolute terms when they face poverty than when they are affluent.

Kahneman and Tversky [11] also suggest that investors do not display constant risk aversion, but rather, behave as if they are risk averse in particular settings but not in others. Irrationality, according to Kritzman [12] may also be one reason t an investor might adopt a time-based investment strategy but behaviorists might note that investors are neither irrational nor rational, but 'normal'. In other words, investors commonly suffer from the behavioral bias of 'recency' embracing recent performance in their return expectations... This is what we would like to investigate and do it empirically. But, unlike previous studies, we will do it following a different path because allocation based on asset-class return expectations that seemed so reasonable at the onset of an investment may seem less palatable when the market turns down.

The vast majority of the previous studies are *ex post* in nature use past returns so, what we will do here is to test time diversification in the context of ex *ante* investment behavior. In large part, it is this disconnect between the expected lower risk of an investment in stocks over the long run and the expected higher risk of such an investment in the short run that creates doubt and can foster poor decision-making under stress.

Let us therefore, submit two alternative definitions of time diversification. Under the first definition at each point in time investors form risk-adjusted conditional expectations of future returns. The asset with the highest ex ante Sharpe ration is then selected. This decision process is recursively applied through the data for investors with various investment horizons and the total number of equity positions taken is calculated. A significant increase in the number of equity positions as the investment horizon is increased is taken as evidence in favor of *ex ante* time diversification.

Under the second definition, we form optimal portfolios within a mean-variance framework using the estimated conditional expectations of future returns. If the equity weight significantly increases as the investment horizon is increased then this is also taken as evidence of *ex ante* time diversification. Actually what we do here is offer a rationale for the observed effects that is consistent with the empirical facts concerning the time series behavior of asset returns. Before examining the strategies being adopted by investors, we will first describe the method which is the method of VAR representation through which return expectations are generated.

2. VAR REPRESENTATION

Given the limited availability of data over the sample period, real asset returns are assumed to be determined by their own pas values and past values of competing assets. Therefore, real returns are modeled following Vector Auto Regressive (VAR) representation:

$$
\begin{bmatrix} r_t^e \\ r_t^g \\ r_t^b \end{bmatrix} = \begin{bmatrix} A(L) & B(L) & C(L) \\ D(L) & E(L) & F(L) \\ G(L) & H(L) & I(L) \end{bmatrix} =
$$

$$
\begin{bmatrix} r_{t-1}^e \\ r_{t-1}^g \\ r_{t-1}^b \end{bmatrix} + \begin{bmatrix} v_{t,1} \\ v_{t,2} \\ v_{t,3} \end{bmatrix} \qquad (1)
$$

where r_t^e, r_t^g r_t^b denote the one – period real returns to equity, bonds, and bills, respectively and $A(L)$, $B(L)$, etc are lag polynomials each of order p. This VAR can be more compactly written as a

$$
r_t = \gamma\, r_{t-1} + v_t \qquad (2)
$$

where r_t is a $3p$ x 1 vector of returns, γ is a $3p$ x $3p$ matrix of coefficients, $v_t \sim \mathrm{IN}[0, \Omega]$ with expectation $\mathrm{E}[v_t] = 0$ and variance matrix $V[v_t] = \Omega$. Assuming the forecasting model coincides with the DGP, Clements and Hendry [4] and Bandi and Russell [1] show that the forecasts of the *N*- horizon ahead returns are given by

$$
\bar{r}_N \equiv E_t\, r_{t+1},_N = \sum_{j=I}^{N} E_t r_{t+j} = \sum_{j=1}^{N} \gamma^j r_t \qquad (3)
$$

With the associated forecast error variance-covariance matrix is:

$$
\Sigma_N = V_t\,[\ r_{t-1,N} - E_t r_{t+1,N}] = \sum_{j=1}^{N} \Phi_j \Omega\, \Phi_j' \quad \text{where}
$$

$$
\Phi_j = \sum_{i=0}^{N-j} \gamma^i \qquad (4)
$$

The expression given in (3) and (4) are used to construct expected returns for an investment strategy and the forecast error variance-covariance matrix[2].

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[2] This VAR representation is time-consistent in that only information that is publicly available at time *t* is used to make investment decisions at time *t* +1. However, it can be argued that investors were not quite knowledgeable in applying statistical methodologies in their decision making. The purpose of our work is to find investor's decisions in real time; it does not try to replicate the investment decisions in real time. We attempt to capture, in the spirit of Strong and Taylor [15], the

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3. INVESTMENT STRATEGIES

Let us assume that investors will adopt in the spirit of Tokat and Stochton [16] one of two possible strategies. The first strategy is based on market betting while the second strategy involves optimal portfolio construction in a mean variance framework.

Strategy 1. The first strategy used by investors is based on taking a position in one of the three assets, namely, equity, bonds, or bills. The decision is based on the conditional expectation of the asset's return over the appropriate investment horizon and the reliability of the expectation. These two characteristics are combined in the following *ex ant e* Sharpe ratio.[3]

$$\bar{r}_{t+1,N} = \frac{E_t r_{t+1,N} - r_f}{\sigma_{t+1,N}} \tag{5}$$

where $\bar{r}_{t+1,N}$ is the risk-adjusted expected return to an asset from $t+1$ to $t+N$, $E_t r_{t+1,N}$ is the conditional expectation of the return to an asset from $t+1$ to $t+N$, $\sigma_{t+1,N}$ is the standard deviation of the forecast error from $t+1$ to $t+N$, r_f is the risk free rate. We assume that the asset with the highest $\bar{r}_{t+1,N}$ is

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important features of two different investment strategies used by rational investors.

$$\int [\, F_A(X) - F_B(X)\,] \le 0 \quad \forall z$$

Denoted A SDSD B. See Hipp [9], Das [5]

[3]The Sharpe ratio, S is used to characterize how well the return of an asset compensates the investor for the risk taken, the higher the Sharpe ratio number, the better. The Sharpe ratio, in fact, increases with the investment horizon $\hat{T}$, $S = \sqrt{\dfrac{T}{\hat{T}}}$ T is the time interval. The Sharpe ratio

has as its principal advantage that it is directly computable from any observed series of returns without need for additional information surrounding the source of profitability. Other ratios such as the 'bias ratio' have recently been introduced into the literature to handle cases where the observed volatility may be an especially poor proxy for the risk inherent in a time-series of observed returns. But While most of the ratios , for example, the 'Treynor ratio' works only with systematic risk of a portfolio, the Sharpe ratio observes both systematic and idiosyncratic risks. See Bouchaud and Potters [3]

held by the investor at t. As real returns are used the risk-free rate is set equal to the return on a Perfect inflation hedge asset. In the absence of any inflation premia or risk premia this rate is set equal to zero.

Strategy 2. An alternative investment strategy is also considered whereby investors take positions (short or long) in each of the available assets (equity, bonds, and/or bills) such that the variance of particular return is minimized for a given level of desired expected return. These investors maximize their expected utility by holding such portfolios. The investor's problem then is:

$$\text{Min } \frac{1}{2} w_N' \, \Sigma_N \, w_N$$

Subject to
$$I' \, w_N = I \ I$$
$$\bar{r}' \, w_N = \mu \tag{6}$$

where w_N is a vector of N-period horizon portfolio weights and μ is the desired expected portfolio return. The optimal weights are given by

$$w_N{}^* = \lambda \, \Sigma_N^{-1} I + \gamma \, \Sigma_N^{-1} \bar{r}_N' \tag{7}$$

where $\lambda = (C - \mu B)/\Delta$,
$\gamma = (\mu A - B)/\Delta$,
$A = I \Sigma_N^{-1} I$, $B = I' \Sigma_N^{-1} \bar{r}_N$, $C = \bar{r}_N \Sigma_N^{-1} \bar{r}_N$,
and $\Delta \equiv AC - B^2$.

To ensure that the desired portfolio return is greater than the expected return on the global minimum variance portfolio, μ is set equal to the return on the global minimum variance portfolio $(= B/A)$ plus some annualized percentage 'excess return' denoted δ.

This necessarily results in a positive relationship between δ and the standard deviation of portfolio returns and thus enables inferences to be drawn about the relation between the asset weight and the degree of risk aversion. Investors here are assumed to base their investment decisions on real return. The real return is defined as the nominal return observed at time t minus the inflation rate observed at time $t - 2$ because the real return cannot be observed at time t because the appearance of inflation data are approximately delayed by two months.

4. DATA AND METHODOLOGY

The data comprise monthly return series for equity, bonds, and Treasury bills in India. Data come from Securities and Exchange Bond of India (Annual Reports) and Handbook of Statistics – the Indian Security Market (1995- 2007), and The Reserve Bank of India Bulletin, providing returns data for equity, short and long term government bonds, corporate bonds for the years under investigation. All returns are continuously compounded holding period returns. The holding period varies between one month and ten years: for the 1995-2007 data, the holding periods are given in months. Here we use $N = \{1, 12, 60\}$; the longer data runs add $N = 120$. For all months and periods the returns are non-overlapping.

The VAR methodology represented above, a time-consistent methodology is a convenient way of incorporating this basic feature. The VAR is estimated recursively using India's monthly data. An initial learning period of fifteen years is used for the first estimate of the expected returns. The estimation process is then repeated using one more month of return data. The order of the VAR is determined by the Akaike Information Criterion (AIC) as this produces VAR specifications that yield forecasts, as shown in Johansen [10] that are superior to forecasts produced by VAR specifications with alternative information criterion. After the VAR is estimated the N- horizon expected returns and N- horizon forecasts variances are calculated for $\{N = 1, 12, 60, 120\}$ and $\delta = \{0\%, 1\%\}$. The information is then used by investors adopting the above strategies. If strategies 1 is used then the investor purchases one of the three assets while investors who use strategy 2 will place a proportion of their wealth in each of the three assets. The process is repeated until the entire sample period is used. When strategy 1 is assumed this means that a series of one's (equity position taken) and zeros (no equity position) is obtained for each investment horizon while a series of equity weights is obtained for each investment horizon when strategy 2 is adopted. In both cases, these series denoted x_t (short horizon) and x_t' (long horizon) are averaged to give either a proportion of time that equity positions are taken (strategy1) or a mean equity weight (strategy2).

For a null hypothesis of *ex ante* time diversification is tested by comparing the equity proportions (mean weight) at the one month investment horizon, dented x_t' with the equity proportions (or mean weight) at the longer investment horizons (one year, five years, and ten years), and denoted $\overline{x}_t$. If normality assumption is used then a sample t- test of the difference of two means could be used. However, in the current application this assumption cannot be made[4]. Moreover, the investment horizons and the proportions (or mean weights) obtained when using the short investment horizons make use of the overlapping data. To incorporate this time-dependency in the data, we make use of the studentized bootstrap method with block *re sampling* using the proportions (or mean weights) calculated using short and long investment horizons the following test statistic is calculated[5].

$$Z_0 = \frac{\overline{x}_t - \overline{x}_t'}{\sqrt{\hat{\sigma}_x^2 / T + \hat{\sigma}_{x'}^2 / T}} \qquad (8)$$

where $\hat{\sigma}_x^2$ and $\hat{\sigma}_{x'}^2$ denote the sample variances of x_t and x_t' calculated over the long and short Investment horizons. Respectively. The distribution of this test statistic is calculated using block bootstrap *re sampling*. In particular R values of

$$Z^* = \frac{\overline{x}_t^* - \overline{x}_t'^* - (\overline{x}_t - \overline{x}_t')}{\sqrt{\hat{\sigma}_{x^*}^2 / T + \hat{\sigma}_{x'}^2 / T}} \qquad (9)$$

are generated where statistics denoted by * indicate that they are based on the shuffled blocks of x_t land x_t'. Finally Z_0 statistic is compared with the R separate Z^* statistics and (two sided) p values are calculated. In the particular application *we* set $R = 100$ i.e., the p- value is calculated using a 100 repetition bootstrap technique[6,7]

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[4] To compare the goodness of fit we use the Kolmogorov distance $D = sup_x [F(x) - \overline{F}(x)]$ and its standardized counterpart, $\sqrt{T} D$ where T is the sample size and F and $\overline{F}$ are the empirical and fitted cumulative density functions, respectively. Of the different distributions we considered, for example, Laplace, generalized exponential of the second kind, Student's t did not produce the lowest Kolmogov D value, see Marsaglia et al [13]

[5] The usual to sample t – statistic

$$Z = \frac{\overline{x}_t - \overline{x}_t' - (\mu_x - \mu_{x'})}{\sqrt{\sigma_x^2 / T + \sigma_{x'}^2 / T}}$$

[6] This technique is appropriate when using auto correlated time series and/or series that have time-varying conditional Variances. To maintain the time-dependency in the data, blocks of data are selected at random and used to construct p Values. Five overlapping blocks are selected with each replication of the data. For details of this technique, see Davison and Hinkkley [6].

[7] See Gongalves and Meddahi [8]

5. ESTIMATION

The total number of equity positions and mean equity weights are presented in Table 1.These are calculated using the entire sample period and over various sub-periods. The results of the horizon invariance test are presented in the final three columns of Table 1 (Appendix). There is no evidence of *ex ante* (strategy 1) time diversification to the Indian equity markets. For instance, 135 equity positions are taken by investors with a one-month investment horizon between 1955 and 1998. This compares to 129 equity propositions taken by investors with a ten-year investment horizon over the same period. Moreover, the *p-* values associated with this difference equal 1. In addition equity is not the most popular asset across all investment horizons.

When investors are assumed to follow strategy 2, the results for Panel B reveal evidence of time diversification effects only during the first sub-period 1990-1998. In particular, when $\delta = 1\%$ .the p- value associated with the horizon invariance test all indicates significant time diversification effects. However, over the whole period, no time diversification effects are seen though the equity weights does increase from 2% when $N = 1$ to 32 % when $N = 120$ when $\delta = 1\%$. The contribution of mean-aversion in fixed-income assets to this finding can be explained by the fact that the forecast of fixed-income returns and the associated variances vary over N *in* accordance with the assumption of a random process. The results do indicate that the restriction does indeed lead to a flat equity profile under the investment horizon.[8]

Therefore, the mean-aversion in fixed income contributes to the time diversification effect. Manipulation of the weight formula in (7) also enables us to consider the role of predicted cross-correlations amongst asset returns. The restriction of zero cross correlations is imposed by setting the off-diagonal elements in $\sum N$ to zero. The results indicate that these cross correlations do not contribute to the time diversification effect.

6. CONCLUSION AND A FEW REMARKS

We investigated on the basis of Indian data if it is reasonable and appropriate for investors with longer investment horizons to allocate a large portion of their portfolios to risky assets, particularly equities. When *ex ante* diversification is investigated we find no evidence for increased demand for equity over longer investment horizons in the context of the Indian markets. In Indian market the time diversification is the result of mean-aversion in fixed income assets and predicted cross-correlation amongst the asset returns. We expected for most investment strategies based on age or time that the longer the time horizon, the larger will be the relative weight of equities in the portfolio. Although in the Indian market some horizon-based funds maintain a more or less static allocation, others, such as target retirement funds, do not seem to moderate the equity allocation in a predictable manner as time passes and the target horizon approaches. Indeed the investment horizon is considered by leading investment and financial planning professional associations in India as not being a key factor in developing investment policy statements and asset allocations. The point is: not every investor is equally prepared—either emotionally or financially— to contend with the uncertainty that comes with seeking returns above the risk-free rate.

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[8] In particular, both the forecast return and variances are assumed to increase in direct proportion to *N*. this restriction is achieved by direct manipulation of the weight formula given in (7) above.

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APPENDIX

Table 1

Ex Ante Time Diversification

			Equity Proportion weights				Horizon invariance		
Period	Filter	δ	N=1	N=12	N=60	N=120	1 v 12	1 v 60	1v120
Panel A Indian Data [Market-timing]									
1990-1998	N0		80	70	68	68	0.70	0.80	0.70
1998-2010	N0		55	57	60	61	0.80	0.75	0.90
1990-2010	N0		135	127	128	129	0.97	0.87	1.00
Panel B Indian Data [Mean Variance Portfolio]									

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Period	Filter	δ								
1990-1998	N0	0%	0.01	0.01	0.01	0.01		0.34	0.44	0.52
	N0	1%	0.03	0.54	0.98	1.32		0.00	0.00	0.00
1998-2010	N0	0%	0.01	0.03	0.03	0.42		0.55	0.55	0.55
	N0	1%	0.01	0.01	0.02	0.48		0.60	0.61	0.63
1990-2010	N0	0%	0.01	0.03	0.03	0.24	0.21	0.20	0.21	0.23
	N0	1%	0.01	0.04	0/03	0.49		0.23	0.23	0.18
	CC	1%	0.02	0.04	0.04	0.23		0.76	031	0.33
	FI	1%	0.01	0.05	0.12	0.32		0.71	0.68	0.58
	FI = CC	1%	0.01	0.04	0.05	0.41		0.06	0.08	0.06

Notes:

This table prepares the number of times equity is selected when the asset with the highest risk-adjusted Expected returns is held by investments strategies, (strategy 1-market timing and by the mean weights associated with equity in mean-variance framework (strategy 2 – mean variance portfolio. In both cases, four different investment horizons (N) are considered and there are three assets (equity, bonds, and bills) available. Expected returns are generated by a Vector Autoregressive (VAR) model with lag length selected by AIC and where a twenty year planning period has been assumed. The p- values associated with the horizon invariance test are given in the last three columns of the table. Panel A reports these for 1955 to 1975 Indian data when strategy 1 is adopted. Panel B for 1955 to 1998 when strategy 2 is adopted. The annualized excess mean return over the global minimum variance portfolio is given by δ. Filter refers to whether fixed-income asset returns, denoted (FT) have been filtered to remove the effects of mean-aversion, whether predicted asset return cross-correlations have been set to zero, denoted (CC) 0r, whether both of these filters have been applied, denoted (FI + CC).