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STATISTICALLY CONVERGENT DOUBLE SEQUENCE SPACES IN n-NORMED SPACES

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Abstract: The concept of statistical convergence was introduced by Stinhauss[21] in 1951. M.Gürdal and S.Pehlivan[9] defined statistical convergence in 2-normed spaces in 2009. In this paper, we study statistical convergence of double sequence spaces in n-normed spaces. We show that some properties of statistical convergence of double sequences also hold in n-normed spaces. The results here in proved are analogus to those by Vakeel.A.Khan and Sabiha Tabassum [Appllied Mathematics, Scientific Research Publishing, USA,2(4)(2011):398-402].

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1 Introduction

The notion of statistical convergence was introduced by Fast[1] and Schoenberg[20] independently. Later on it was further investigated by Fridy and Orhan[4]. The idea depends on the notion of density of subset of $\mathbb{N}$. Over the years and under different names statistical convergence has been discussed in the theory of fourier analysis, egrodic theory and number theory. Later on it was further investigated from the sequence space point of view and linked with summability theory by Fridy(1985) and many others. In recent years, generalization of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces.

The concept of 2-normed spaces was initially introduced by Gähler[5,6,7] in the mid of 1960's. Since then, many researchers have studied this concept and obtained various results, see for instance[8].

Let $n \in \mathbb{N}$ and X be a real vector space of dimension d, where $n \leq d$. A real valued function $\|.,...,.\|$ on X^n satisfying the following four conditions:

1. $\|x_1, x_2, ..., x_n\| = 0$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent;
2. $\|x_1, x_2, ..., x_n\|$ is invariant under permutation:
3. $\|\alpha x_1, x_2, ..., x_n\| = \alpha\|x_1, x_2, ..., x_n\|$, for any $\alpha \in R$:
4. $\|x + x', x_2, ..., x_n\| \leq \|x, x_2, ..., x_n\| + \|x', x_2, ..., x_n\|$ is called an $n - norm$ on X, and the pair

$(X, \|.,.,...,.\|)$ is then called an n-normed space.

Example 1 As a standard example of a n-normed space we may take R^n being equipped with the n-norm $\|x_1, x_2, ..., x_n\|_E =$ the volume of the n-dimensional parallelopiped spaned by the vectors $x_1, x_2, ..., x_n$ which may be given explicitly by the formula

$$\|x_1, x_2, ..., x_n\|_E = |\det(x_{ij})|,$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{in}) \in R^n$ for each $i = 1, 2, ..., n$.

Example 2 Let $(X, \|.,.,...,.\|)$ be an n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, ..., a_n\}$ be a linearly independent set in X. Then the following function $\|.,.,...,.\|_\infty$ defined by

$$\|x_1, x_2, ..., x_n\|_\infty = \max\{\|x_1, x_2, ..., x_{n-1}, a_i\| : i = 1, 2, ..., n\}$$

defines an $(n-1)-$norm on X with respect to $\{a_1, a_2, ..., a_n\}$.

Example 3 Let $n \in \mathbb{N}$ and $(X, \langle.,.\rangle)$ be a real inner product space of dimension $d \geq n$. Then the following function $\|.,.,...,.\|_S$ on $X \times ..., \times X$(n factor) defined by

$$\|x_1, x_2, ..., x_n\|_S = [\det(\langle x_i, x_j\rangle)]^{\frac{1}{2}}$$

A sequence (x_j) in an n-normed space $(X, \|.,.,...,.\|)$ is said to be converge to some $L \in X$ in the n-norm if

$$\lim_{j\to\infty} \|x_j - L, z_1, ..., z_{n-1}\| = 0, \quad \text{for every } z_1, ..., z_{n-1} \in X.$$

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A sequence (x_j) in an n-normed space $(X, \|., ., ..., .\|)$ is said to be Cauchy with respect to the n-norm if

$$\lim_{j,k\to\infty} \|x_j - x_k, z_1, ..., z_{n-1}\| = 0, \text{ for every } z_1, ..., z_{n-1} \in X.$$

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the n-norm. Any complete n-normed space is said to be Banach space.

We recall some facts connecting with statistical convergence. If K is subset of positive integers $\mathbb{N}$, then K_n denotes the set $\{k \in K : k \leq n\}$. The natural density of K is given by $\delta(K) = \lim_{n\to\infty} \frac{|K_n|}{n}$, where $|K_n|$ denotes the number of elements in K_n, provided this limit exists. Finite subsets have natural density zero and $\delta(K^c) = 1 - \delta(K)$ where $K^c = \mathbb{N} \setminus K$, that is the complement of K. If $K_1 \subseteq K_2$ and K_1 and K_2 have natural densities then $\delta(K_1) \leq \delta(K_2)$. Moreover, if $\delta(K_1) = \delta(K_2) = 1$, then $\delta(K_1 \cap K_2) = 1$ (see[2]).

A real number sequence $x = (x_j)$ is statistically convergent to L provided that for every $\epsilon > 0$ the set $\{n \in \mathbb{N} : |x_j - L| \geq \epsilon\}$ has natural density zero. The sequence $x = (x_j)$ is statustically Cauchy sequence if for each $\epsilon > 0$ there is positive integer $N = N(\epsilon)$ such that $\delta(\{n \in \mathbb{N} : |x_j - x_N(\epsilon)\}) = 0$.(see[3])
If $x = (x_j)$ is a sequence that satisfies some property P for all n except a set of natural density zero, then we say that (x_j) satisfies some property P for "almost all n".

An *Orlicz Function* is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, nondecreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$.

If convexity of M is replaced by $M(x + y) \leq M(x) + M(y)$, then it is called a *Modulus funtion* (see Maddox [18]). An Orlicz function may be bounded or unbounded. For example, $M(x) = x^p (0 < p \leq 1)$ is unbounded and $M(x) = \frac{x}{x+1}$ is bounded.
Lindesstrauss and Tzafriri [17] used the idea of Orlicz sequence space;

$$l_M := \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}$$

which is Banach space with the norm

$$\|x\|_M = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}.$$

The space l_M is closely related to the space l_p, which is an Orlicz sequence space with $M(x) = x^p$ for $1 \leq p < \infty$.

An Orlicz function M satisfies the $\Delta_2 -$ *condition* ($M \in \Delta_2$ for short) if there exist constant $K \geq 2$ and $u_0 > 0$ such that

$$M(2u) \leq KM(u)$$

whenever $|u| \leq u_0$.

Note that an Orlicz function satisfies the inequality

$$M(\lambda x) \leq \lambda M(x) \text{ for all } \lambda \text{ with } 0 < \lambda < 1.$$

Orlicz function has been studied by V.A.Khan[10,11,12,13] and many others.

Throughout a double sequence $x = (x_{jk})$ is a double infinite array of elements x_{jk} for $j, k \in \mathbb{N}$.
Double sequences have been studied by V.A.Khan[14,15,16], Moricz and Rhoades[19] and many others.
A double sequence $x = (x_{jk})$ called statistically convergent to L if

$$\lim_{m,n\to\infty} \frac{1}{mn} |(j, k) : |x_{jk} - L| \geq \epsilon, j \leq m, k \leq n| = 0$$

where the vertical bars indicate the number of elements in the set.(see[15])
In this case we write $st_2 - \lim x_{jk} = L$.

Remark 1 If x is statistically connvergent to the number l, then l is determined uniquely.

2 If x is bounded convergent double sequence the it is also statistically convergent to the same number. If x is unbounded convergent double sequence, then x is statistically convergent.

3 If x is statistically convergent, then x need not be convergent and it is not necessarily bounded.

Example 4 Let $x = (x_{jk})$ be defined as

$$x_{jk} = \begin{cases} k, j = 1, & \forall\ k, \\ 0, & \text{otherwise} . \end{cases}$$

Then x is statistically convergent to 0 since

$$\lim_{m,n} \frac{1}{mn} |\{(j, k) : |x_{jk} - 0| \geq \epsilon\}| \leq \lim_{m,n} \frac{n}{mn} = 0$$

Example 5 Let $x = (x_{jk})$ be defined as

$$x_{jk} = \begin{cases} j, \text{if j is square, ,} & \forall\ k, \\ 2, & \text{otherwise} . \end{cases}$$

Then x is neither convergent nor bounded but statistically convergent to 2.

2. Main Results.

Recently V.A.Khan and Sabiha Tabassum[16] defined statistical convergence of double sequences in 2-normed spaces. In this section we introduce the notion of statistical covergence in n-normed spaces and give the main results of the paper.

Definition 1 Let (x_{jk}) be a double sequence in n-normed space $(X, \|.,.,...,.\|)$. The sequence (x_{jk}) is said to be statistically convergent to L, if for every $\epsilon > 0$,

$$\lim_{m,n\to\infty} \frac{1}{mn}|(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon, J \leq m, k \leq n| = 0$$

for each nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$. In this case we write

$$st_2 - \lim_{j,k\to\infty} \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|.$$

Definition 2 Let (x_{jk}) be a double sequence in n-normed space $(X, \|.,.,...,.\|)$. The sequence (x_{jk}) is said to be statistically Cauchy sequence in X if for every $\epsilon > 0$ and every nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$, there exists a number $p = p(\epsilon, z_{j_2}, z_{j_3}, ..., z_{j_n})$ and $q = q(\epsilon, z_{j_2}, z_{j_3}, ..., z_{j_n})$ such that

$$\lim_{m,n\to\infty} \frac{1}{mn}\{j,k \in \mathbb{N} \times N : \|x_{jk} - x_{pq}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon \;\; j \leq m, k \leq n| = 0$$

V.A.Khan and Sabiha Tabassum[16] defined a double sequence (x_{jk}) in 2-normed space $(X, \|.,\|)$ to be Cauchy with respect to the 2-norm if

$$\lim_{j,p\to\infty} \|x_{jk} - x_{pq}, z\| = 0 \to \text{ for every } z = (z_{jk}) \in X \text{ and } k, q \in \mathbb{N}.$$

If every Cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the 2-norm. Any complete 2-normed space is said to be 2-Banach space.

Theorem 1 Let (x_{jk}) be a double sequence in n-normed space $(X, \|.,.,...,.\|)$ and $L, L' \in X$. If $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$ and $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L', z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, then $L = L'$.

Proof Assume $L \neq L,$. Then $L - L' \neq 0$, so there exists a $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$, such that $L - L'$ and $z_{j_2}, z_{j_3}, ..., z_{j_n}$ are linearly independent. Therefore

$$\|L - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\| = 2\epsilon, \text{ with } \epsilon > 0.$$

Now

$$2\epsilon = \|(L - x_{jk}) + (x_{jk} - L'), z_{j_2}, z_{j_3}, ..., z_{j_n}\|$$

$$\leq \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| + x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|.$$

So $\{(j,k) : \|x_{jk} - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \epsilon\} \subseteq \{(j,k) : \|x_{jk} - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \epsilon\}$.
But $\delta(\{(j,k) : \|x_{jk} - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \epsilon\}) = 0$.
Contradicting the fact that $x_{jk} \to L'(stat)$.

Theorem 2 Let the double sequence (x_{jk}) and (y_{jk}) in n-normed space $(X, \|.,.,...,.\|)$. If (y_{jk}) is a convergent sequence such that $x_{jk} = y_{jk}$ almost all n, then (x_{jk}) is statistically convergent.

Proof Suppose $\delta(\{(j,k) \in N \times N : x_{jk} \neq y_{jk}\|\}) = 0$ and $\lim_{j,k\to\infty} \|y_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z\|$. Then for every $\epsilon > 0$ and $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$.

$$\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\} \subseteq \{(j,k) \in N \times N : x_{jk} \neq y_{jk}\}.$$

Therefore
$\delta(\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\})$

$$\leq \delta(\{(j,k) \in N\times N : \|y_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}) + \delta(\{(j,k) \in N \times N : x_{jk} \neq y_{jk}\}). \tag{1}$$

Since $\lim_{n\to\infty} \|y_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$ for every $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$, the set $\{(j,k) \in N \times N : \|y_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}$ contains finite number of integers. Hence, $\delta(\{(j,k) \in N \times N : \|y_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}) = 0$.
Using enequality (1), we get

$$\delta(\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}) = 0$$

for every $\epsilon > 0$ and $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$.
Consequently, $st_2 - \lim \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$.

Theorem 3 Let the double sequence (x_{jk}) and (y_{jk}) in 2-normed space $(X, \|.,.,...,.\|)$ and
$L, L' \in X$ and $a \in \mathbb{R}$. If $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$ and
$st_2 - \lim \|y_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L', z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, for every nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$, then

(i) $st_2 - \lim \|x_{jk} + y_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L + L', z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, for each nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$ and

(ii) $st_2 - \lim \|ax_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|aL, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, for each nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$.

Proof(i) Assume that $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$,
and
$st_2 - \lim \|y_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L', z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, for every nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$. Then $\delta(K_1) = 0$ and $\delta(K_2) = 0$ where

$$K_1 = K_1(\epsilon) := \left\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \frac{\epsilon}{2}\right\}$$

$$K_2 = K_2(\epsilon) := \left\{(j,k) \in N \times N : \|y_{jk} - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \frac{\epsilon}{2}\right\}$$

for every $\epsilon > 0$ and $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$. Let

$$K = K(\epsilon) := \{(j,k) \in N \times N : \|x_{jk} + y_{jk} - (L + L'), z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}.$$

To prove that $\delta(K) = 0$, it is sufficient to prove that $K \subset K_1 \cup K_2$. Suppose $j_0, k_0 \in K$. Then

$$\{\|x_{j_0 k_o} + y_{j_0 k_0} - (L + L'), z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\} \qquad (2).$$

Suppose to the contrary that $j_0, k_0 \notin K_1 \cup K_2$. Then $j_0, k_0 \notin K_1$ and $j_0, k_0 \notin K_2$. If $j_0, k_0 \notin K_1$ and $j_0, k_0 \notin K_2$ then $\|x_{j_0 k_0} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \frac{\epsilon}{2}$ and $\|x_{j_0 k_0} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \frac{\epsilon}{2}$. Then, we get

$$\|x_{j_0 k_o} + y_{j_0 k_0} - (L + L'), z_{j_2}, z_{j_3}, ..., z_{j_n}\| \leq \|x_{j_0 k_o} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| + \|y_{j_0 k_0} - L', z_{j_2}, z_{j_3}, ..., z_{j_n}\|$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

which contradicts (2). Hence $j_0, k_0 \in K_1 \cup K_2$, that is, $K \subset K_1 \cup K_2$.

(ii) Let $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|, a \in \mathbb{R}$ and $a \neq 0$. Then

$$\left(\left\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \frac{\epsilon}{|a|}\right\}\right) = 0.$$

Then we have

$$\{(j,k) \in N \times N : \|ax_{jk} - aL, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\} = \{(j,k) \in N \times N : |a| \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \epsilon\}$$

$$= \left\{(j,k) \in N \times N : \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \geq \frac{\epsilon}{|a|}\right\}.$$

Hence, the right handside of above equality equals 0. Hence, $st_2 - \lim \|ax_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|aL, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$, for every nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$.

From Theorem 1 of Fridy[3] we have

Theorem 4 Let (x_{jk}) be statistically Cauchy sequence in a finite dimensional n-normed space $(X, \|., ., ..., .\|)$. Then there exists a convergent double sequence (y_{jk}) for all in $(X, \|., ., ..., .\|)$ $j, k \in N$ such that $x_{jk} = y_{jk}$ for almost all n.

Proof See proof of Theorem 2.9[9]

Theorem 5 Let (x_{jk}) be a double sequence in n-normed space $(X, \|., ., ..., .\|)$. The double sequence (x_{jk}) is statistically convergent if and only if (x_{jk}) is a statistically Cauchy sequence.

Proof Assume that $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$ for every nonzero $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$ and $\epsilon > 0$.
Then, for every $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$,

$$\|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \frac{\epsilon}{2} \quad \text{almost all n},$$

and if $p = p(\epsilon, z_{j_2}, z_{j_3}, ..., z_{j_n})$ and $q = q(\epsilon, z_{j_2}, z_{j_3}, ..., z_{j_n})$ is chosen so that

$$\|x_{pq} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| < \frac{\epsilon}{2},$$

then, we have

$$\|x_{jk} - x_{pq}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| \leq \|x_{jk} - L, z_{j_2}, z_{j_3}, ..., z_{j_n}\| + \|L - x_{pq}, z_{j_2}, z_{j_3}$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} \quad \text{almost all n.}$$
$$= \epsilon \quad \text{almost all n} \; .$$

Hence, (x_{jk}) is statistically Cauchy sequence.

Conversely, assume that x_{jk} is a statistically Cauchy sequence. By Theorem 4, we have $st_2 - \lim \|x_{jk}, z_{j_2}, z_{j_3}, ..., z_{j_n}\| = \|L, z_{j_2}, z_{j_3}, ..., z_{j_n}\|$ for each $z_{j_2}, z_{j_3}, ..., z_{j_n} \in X$.

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