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A Fuzzy Residual Network Approach to Minimum Cost Flow Problem with Fuzzy Parameters

M. Bagherian
Department of Applied Mathematics, Faculty of Mathematical Science, University of Guilan, Rasht, Iran.
mbagherian@guilan.ac.ir

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ABSTRACT

The Minimum Cost Flow (MCF) problem has been defined as to determine a least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes. In this paper the minimum cost flow problem with fuzzy parameters is considered. Based on integer solution property of the problem, the Yager ranking indices can be calculated for fuzzy arc costs to change the fuzzy arc costs to crisp ones. Consequently the problem can be converted to a MCF problem with fuzzy arc capacities and fuzzy supply\demands. Then by defining the fuzzy residual network, the MCF algorithms are developed to solve this problem efficiently. In fact the aim of this paper is to consider the theoretical aspects of the problem, not numerical results of the presented algorithm. So the computational complexity of the proposed method is discussed.

Keywords: *Minimum cost flow problem; Fuzzy interval valued data; Complexity;*

1. INTRODUCTION

MCF problem represents a general form of network flow problems. It is also used to solve several real-world applicational problems and can be solved very efficiently by strongly polynomial algorithms [1]. MCF can be defined as follows. Let G=(N,A) be a directed network with node set N, arc set A, a cost c_{ij} and a capacity u_{ij} associated with each arc $(i,j)\in A$. We associate with each node $i\in N$ a number b(i) which indicates its supply or demand depending on whether b(i)>0 or b(i)<0. Node i is called a transshipment node if b(i)=0.The MCF problem can be stated as:

$$Min. \quad \sum_{(i,j)\in A} c_{ij}x_{ij} \qquad (1-1a)$$

$$s.t: \quad \sum_{j\in N} x_{ij} - \sum_{j\in N} x_{ji} = b(i)\,,\ \forall i\in N \qquad (1-1b)\,.$$

$$0\le x_{ij}\le u_{ij}\,,\ \forall (i,j)\in A \qquad (1-1c)$$

The objective is to minimize the total cost of shipment. Constraints (1-1b) represent the conservation of flows, and constraints (1-1c) represent the capacity constraints.

In general, the MCF problem also has some additional assumptions such as: (a) supplies, demands, and capacities must be integers; (b) the network is directed; (c) the supply\demands satisfy the condition $\sum_{i\in N} b(i)=0$; and (d) the MCF problem has a feasible solution.

In actual practice including internet and other communication networks, the costs, capacities and supply\demands of the network are generally vague or uncertain. This vagueness in parameters can arise in a number of ways. There are errors in determining the quantities, also in some situations variability and uncertainty within the network are such that the parameters are only known to lie within specified ranges, or may vary in time within these ranges. There are articles discussing the network flow problems where arc capacities are random variables. [2, 3, 4]. However probability distributions require either a priori predictable regularity or a posteriori frequency determination to construct, Moreover the premise that imprecision can be equated with randomness is questionable [5]. As an alternative, uncertain values can be represented by membership functions of the fuzzy set theory [6, 7]. In the literature, there are articles address fuzzy network flow problems. Dubois [8], Klein [9], and Yager [10] have studied the fuzzy shortest path problem. Okada and Soper [11] developed an algorithm based on the labeling method for a multicriteria shortest path to solve a shortest path problem. A number of no dominated paths are obtained and offered to the decision maker. Hernandes et al [12] proposed an algorithm for the shortest problem which can be applied in graphs with negative parameters and it can detect whether there are negative circuits. Tai and Kao [13] have considered network flow problem with only fuzzy arc costs. They solved this problem by converting it to a crisp one. Chanas and Kolodziejczyk [14] proved an equivalent theorem to the Ford and Fulkerson one concerning the classic task of maximum flow and proposed an algorithm for searching maximum flow in a network with fuzzy arc capacities. Diamond [15] proved a fuzzy maximum flow-minimum cut theorem and proposed an algorithm to solve maximum flow problem in a network with fuzzy number arc capacities. Shih and Lee [16] solved a multi-level minimum cost flow

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problem with fuzzy arc costs and fuzzy arc capacities via a modified fuzzy linear programming technique by Negi and Lee [17]. They converted the problem into a mixed integer linear programming problem. Their procedure is based on linear programming formulation of the problem, not on the network structure of it while there exist minimum cost flow algorithms based on the network structure of the problem which are much more efficient than the linear programming based ones [1]. Ghatee and Hashemi [18] converted the fully fuzzified MCFP into three crisp problems solvable in polynomial time using a ranking function which is a total order. They proposed ranking function-based solutions using Hukohara's difference for mathematical modeling of flows, and by combining the two procedures; they used existing combinatorial algorithms to solve the fully fuzzified minimal cost flow problem. Ghatee et al [19] also proposed the Preemptive priority-based algorithms for the minimal cost flow problem (MCFP) with fuzzy link costs, say fuzzy MCFP. With respect to the most possible case, the worst case and the best case, they converted the fuzzy MCFP into a 3-objective MCFP. Applying a lexicographical ordering on the objective functions of derived problem, they provided two algorithms to find the preemptive priority-based solution(s), namely p-successive shortest path algorithm and p-network simplex algorithm. In their proposed techniques 3 problems should be solved. In [20, 21, 22] Ghatee and Hashemi deal with fuzzy quantities and relations in multi-objective minimum cost flow problem. When t-norms and t-conorms are available, they used the goal programming to minimize the deviation among the multiple costs of fuzzy flows and the given targets while the fuzzy supplies and demands are satisfied. They used a scheme based on genetic algorithm together with fuzzy minimum cost flow problem. In [23, 24] Ghatee ET. Al. deal with imprecise network problems and in [25] they investigate the fuzzy generalized minimal cost flow problem. In none of the above mentioned papers the fuzzy residual network has been used. In this paper we consider MCF problem with fuzzy arc costs, fuzzy arc capacities and fuzzy supply\demands which is more general than the model of Shih and Lee [16] where supply/demands are not considered as fuzzy numbers. Our procedure is more general than the model of Tai and Kao [13] too, where only arc costs are considered fuzzy. Then by defining the fuzzy residual network, we adapt the MCF algorithms to solve the MCF problem with fuzzy parameters more efficiently than Ghatee and Hashemi [22] where it is necessary to solve 3 problems using combinatorial procedures which are not as efficient as residual network based minimum cost flow algorithms [1] in terms of complexity. The rest of the paper is organized as follows. In section 2, the fuzzy MCF problem is formulated. In section 3 the transformation of the fuzzy arc costs to crisp ones is presented. In section 4 some useful definitions and results about fuzzy numbers are discussed. In section 5 the fuzzy residual network which is the main contribution of this work is introduced and it is used to develop fuzzy MCF algorithms for solving the MCF problem with fuzzy arc capacities and fuzzy supply\demands. A numerical example is presented to show the performance of the algorithm. The last section ends the paper with conclusion and some suggestions for further research.

2. FUZZY MINIMUM COST FLOW PROBLEM

In many actual situations, the arc capacities, arc costs and supply\demands of a flow network are vague and must be considered as fuzzy numbers, thus the fuzzy MCF problem can be formulated as:

$$Min. \quad \sum_{(i,j)\in A} \tilde{c}_{ij} x_{ij} \qquad (2-1a)$$

$$s.t: \quad \sum_{j\in N} x_{ij} - \sum_{j\in N} x_{ji} = \tilde{b}(i)\,,\ \forall i \in N \qquad (2-1b).$$

$$0 \le x_{ij} \le \tilde{u}_{ij}\,,\ \forall (i,j)\in A \qquad (2-1c)$$

Where $\tilde{c}_{ij}$, $\tilde{u}_{ij}$, and $\tilde{b}_i$ represent the fuzzy cost, fuzzy capacity of each arc $(i, j)\in A$, and fuzzy supply\demands of each node $i \in N$, respectively.

3. TRANSFORMATION OF FUZZY ARC COSTS TO CRISP ONES

Since the objective function of the problem (2-1) is a fuzzy number it cannot be minimized directly. To tackle this problem, one way is transforming the problem into multiple objectives. Buckley and Feuring [26] transformed the objective function of a fuzzy linear programming problem to a multiple objective programming problem which leads to 3 problems to be solved. As an alternative procedure the ranking methods can be used to convert the fuzzy numbers to crisp ones, which leads to only one problem to be solved. Many ranking methods concerning the dominance of fuzzy numbers have been proposed and discussed [27, 28-30, 31, 32]. We transform the fuzzy arc costs to crisp ones via the ranking method of [13] used by Tai and Kao. Given a convex fuzzy number $\tilde{c}$, the classical definition in area compensation is defined by:

$$I(\tilde{c}) = \int_0^1 .5(c_\alpha^l + c_\alpha^u)\,d\alpha\ .$$

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Where (c_α^l, c_α^u) is the α-level cut of $\tilde{c}$. This index is exactly the ranking index developed by Yager [32]. This ranking index has some useful properties.

3.1 Properties of the Yager's ranking index

a. Yager method posses the properties of compensation, linearity and additivity.

b. $I(\tilde{c})$ is the center of the mean value of the fuzzy number, $\tilde{c}$ (see [31]).

c. If $I(\tilde{A}) \le I(\tilde{B})$, then $\tilde{A}$ is dominated by $\tilde{B}$. i.e. $\tilde{A} \le \tilde{B}$, moreover $\min\{\tilde{A}, \tilde{B}\} = \tilde{A}$.

d. If $\tilde{R} = a\tilde{S} + b\tilde{T}$ and $\tilde{W} = c\tilde{U} - d\tilde{V}$, where a, b, c ,and d are constants, then $I(\tilde{R}) = aI(\tilde{S}) + bI(\tilde{T})$ and $I(\tilde{W}) = cI(\tilde{U}) - dI(\tilde{V})$.

To verifying these properties the interested reader is referred to [32]. Based on these properties the fuzzy arc costs of a fuzzy MCF problem can be transformed to crisp ones, by considering $I(\tilde{c}_{ij})$ as the cost of arc $(i, j) \in A$.. After this transformation we have a MCF problem with fuzzy arc capacities and fuzzy supply\demands.

4. FUZZY ARC CAPACITIES AND FUZZY SUPPLY\DEMANDS

In the following subsections we show that a fuzzy number is characterized by its level sets which are compact intervals. This means that any algorithm which is proposed to solve the MCF problem with interval valued parameters can be used to solve a MCF with fuzzy number parameters. So in this paper, we consider the interval valued numbers to represent the fuzzy parameters. Thus given a network G=(N,A), suppose that the capacity function u, class of flows f ,and the supply\demands are known only to fall within specific ranges expressed as compact intervals, that is for each arc $e \in A$, $u(e) = [l_u(e), r_u(e)]$ and $f(e) = [l_f(e), r_f(e)]$ are the fuzzy capacity and fuzzy flow of arc $e \in A$ respectively, and $b(i) = [l_b(i), r_b(i)]$ is the fuzzy supply\demand of node $i \in N$. Since we should work with interval valued data, some useful definitions and descriptions of the interval arithmetic are presented in the next subsection.

4.1 Interval Arithmetic

Let F denote the class of nonempty compact intervals $A = [l_A, r_A]$ on $[0, \infty)$. If $l_A = r_A = a$, the interval A is defined by the real number a. The addition in F is defined as: $A + B = [l_A + l_B, r_A + r_B]$, $A, B \in F$. There is no additive inverse in F, but there is a cancellation law: $A + B = A + C \;\Rightarrow\; B = C$. If $r_A - l_A \le r_B - l_B$ and $l_A \le l_B$ then the equation $A + X = B$ has a unique solution $X \in F$ which is represented by $X = B_{\Theta_H} A$ which is known as Hukuhara difference [35].

Let $A = [l_A, r_A]$ and $B = [l_B, r_B]$ be two compact intervals. Then $A \le B$ if $l_A \le l_B$ and $r_A \le r_B$. The infimum $A \wedge B$ and the supremum $A \vee B$ are defined as:

$$A \wedge B = [\min\{l_A, l_B\}, \min\{r_A, r_B\}],$$
$$A \vee B = [\max\{l_A, l_B\}, \max\{r_A, r_B\}].$$

If $A, B, C \in F$, then $A + B \le C \;\Rightarrow\; A \le C$.

The interested reader is referred to [8, 31, 33, 34] for more complete description of interval arithmetic.

4.2 Fuzzy Flows

An interval valued function f on A is said to be a flow in an interval valued capacity network G = (N , A), if it satisfies the following condition:

$0 \le f(e) \le u(e)$, for each arc $e \in A$, that is $0 \le l_f(e) \le l_u(e)$, and $0 \le r_f(e) \le r_u(e)$; Consequently l_f and r_f are themselves flows corresponding to the capacities l_u and r_u respectively.

4.3 Representing the Fuzzy Numbers as Compact Intervals

A fuzzy number is defined as follows: U is a fuzzy number if all the level sets $[U]^\beta$, $0 \le \beta \le 1$, are compact intervals and there is at least one $z \in \Re$ such that U(z)=1. The infimum and supremum of two fuzzy numbers U and W is defined by using the extension principle as follows:

$$(U \wedge W)_{(t)} = \sup_{\min(r,s)=t} \min\{U(r), W(s)\},$$

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$$(U \vee W)_{(t)} = \sup_{\max(r,s)=t} \min\{U(r), W(s)\}.$$

The important feature is the relation between the operations on compact intervals and those on fuzzy numbers: For $0 \le \beta \le 1$: $[U \wedge W]^{\beta} = [U]^{\beta} \wedge [W]^{\beta}$ and $[U \vee W]^{\beta} = [U]^{\beta} \vee [W]^{\beta}$.

Moreover partial orders can be defined as follows: $U \le W$ if $[U]^{\beta} \le [W]^{\beta}$, for all $0 \le \beta \le 1$.

Addition of fuzzy numbers can be defined through the extension principle in terms of the level sets: $[U+W]^{\beta} = [U]^{\beta} + [W]^{\beta} = [l_U(\beta) + l_W(\beta), r_U(\beta) + r_W(\beta)]$, for $0 \le \beta \le 1$.

4.4 Stacking Theorem (Negoita and Ralescu [36])

Let $\{I_{\beta} : 0 \le \beta \le 1\}$ be a family of compact intervals satisfying:

a. $I_{\beta} \in F$, for all $0 \le \beta \le 1$;

b. $I_{\beta} \subseteq I_{\alpha}$, for all $0 \le \alpha \le \beta \le 1$;

c. $I_{\beta} = \bigcap_{i=1}^{\infty} I_{\beta_i}$, for any non decreasing sequence $\beta_i \to \beta$ in [0,1].

Then there is a fuzzy number U such that $[U]^{\beta} = I_{\beta}$. conversely the level sets of U satisfy these conditions.

Since a fuzzy number is characterized by its level sets which are compact intervals, any algorithm which is proposed to solve the MCF problem with interval valued capacities and supply\demands can be used to solve a MCF with fuzzy number arc capacities and supply\demands.

5. THE FUZZY RESIDUAL NETWORK

All the MCF algorithms which are based on the network structure of the problem rely on the concept of the residual network. For the crisp structure, the residual network G(x) corresponding to a flow x is defined as follows: Each arc $(i, j) \in A$ is replaced with two arcs (i,j) and (j,i). The arc (i,j) has cost c_{ij} and residual capacity $r_{ij} = u_{ij} - x_{ij}$, and arc (j,i) has cost $-c_{ij}$ and residual capacity $r_{ji} = x_{ij}$. The residual network consists only of arcs with positive residual capacity.

For a MCF problem with fuzzy parameters, we define two fuzzy residual networks $G_l(f)$ and $G_r(f)$ for G=(N,A) in which the arc capacities and supply\demands are fuzzy numbers and so is the flow f. i.e.

$[u(e)]^{\beta} = [l_u(e,\beta), r_u(e,\beta)]$,

$[f(e)]^{\beta} = [l_f(e,\beta), r_f(e,\beta)]$,and

$[b(i)]^{\beta} = [l_b(i,\beta), r_b(i,\beta)]$ as follows:

If $r_f(e,\beta) < r_u(e,\beta)$, for arc $e \in A$, add arc e to $G_r(f,\beta)$ with cost $I(\tilde{c}_e)$ and residual capacity $v_r(e,\beta) = r_u(e,\beta) - r_f(e,\beta)$; and if $r_f(e,\beta) > 0$, then add arc –e to $G_r(f,\beta)$ with cost $-I(\tilde{c}_e)$ and residual capacity $v_r(-e,\beta) = r_f(e,\beta)$.

If $l_f(e,\beta) < l_u(e,\beta)$, for arc $e \in A$, add arc e to $G_l(f,\beta)$ with cost $I(\tilde{c}_e)$ and capacity $v_l(e,\beta) = l_u(e,\beta) - l_f(e,\beta)$; and if $l_f(e,\beta) > 0$, then add arc –e to $G_l(f,\beta)$ with cost $-I(\tilde{c}_e)$ and capacity $v_l(-e,\beta) = l_f(e,\beta)$.

We should assume that $\sum_i l_b(i,\beta) = 0$, and $\sum_i r_b(i,\beta) = 0$, for all $0 \le \beta \le 1$.

Now the fuzzy minimum cost flow algorithm can be developed.

5.1 Fuzzy Minimum Cost Flow Algorithm

We adopt the successive shortest path algorithm to work with fuzzy arc capacities, fuzzy arc costs, and fuzzy supply\demands. Using similar modification any other minimum cost flow algorithm such as primal-dual, out-of-kilter, and relaxation can be developed to solve the problem.

Algorithm Fuzzy successive shortest path
begin

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set $\pi(i) = \pi_l(i) = \pi_r(i) = 0$,
$l_e(i,\beta) = l_b(i,\beta)$ and $r_e(i,\beta) = r_b(i,\beta)$ for all $i \in N$;
Set $l_f((i,j),\beta) = 0$, $r_f((i,j),\beta) = 0$ and $c^{\pi}_{l,ij} = c^{\pi}_{r,ij} = I(\tilde{c}(i,j)) - \pi(i) + \pi(j)$ for all arc $(i,j) \in A$;
Initialize the sets $E_l = \{i \in N : l_e(i,\beta) > 0\}$,

$E_r = \{i \in N : r_e(i,\beta) > 0\}$,
$D_l = \{i \in N : l_e(i,\beta) < 0\}$
and $D_r = \{i \in N : r_e(i,\beta) < 0\}$
while $E_l \neq \emptyset$ or $E_r \neq \emptyset$ do
Begin select a node $k_l \in E_l$, a node $k_r \in E_r$, a node $t_l \in D_l$ and a node $t_r \in D_r$; determine shortest path distances $d_l(i)$ and $d_r(i)$ from nodes k_l and k_r to all other nodes in $G_l(f,\beta)$ and $G_r(f,\beta)$ with respect to the reduced costs c^{π}_{ij} respectively;

let p_l and p_r denote the shortest paths from nodes k_l and k_r to nodes $t_l \in D_l$ and $t_r \in D_r$ in $G_l(f,\beta)$ and $G_r(f,\beta)$ respectively;update $\pi_l = \pi_l - d_l$,and $\pi_r = \pi_r - d_r$

Let
$\delta_l(\beta) = \min\{l_b(k_l,\beta), -l_b(t_l,\beta), v_l((i,j),\beta) : (i,j) \in p_l\}$
and
$\delta_r(\beta) = \min\{r_b(k_r,\beta), -r_b(t_r,\beta), v_r((i,j),\beta) : (i,j) \in p_r\}$
;augment $\delta_l(\beta)$ units of flow along the path p_l and $\delta_r(\beta)$ units of flow along the path p_r ;update $l_f((i,j),\beta)$, $r_f((i,j),\beta)$, $l_b(i,\beta)$, $r_b(i,\beta)$, $G_r(f,\beta)$, and $G_l(f,\beta)$, E_l, E_r, D_l, D_r, $c^{\pi}_{l,ij}$,and $c^{\pi}_{r,ij}$; end; end.

5.2 Complexity Discussion

As it is seen, the modifications added to successive shortest path algorithm, does not affect the complexity of it. So the proposed approach can solve the fuzzy MCF problem in O(nB S(n, m, nC)) where S(n, m, C) denotes the time taken to solve a shortest path problem with nonnegative arc lengths and B is an upper bound on the largest supply of any node, and the costs in the residual network are bounded by nC. The algorithm requires pseudo polynomial time to solve the MCF problem with fuzzy parameters since it is polynomial in n, m and the largest supply B. The double scaling algorithm [1] solves the capacitated MCF problem with fuzzy parameters in O (nm log B log(nC)) time. When the double scaling algorithm implemented using a dynamic tree data structure, one of the best polynomial time algorithms for solving the MCF problem with fuzzy parameters is obtained.

5.3 Example: Consider the network of Figure 1.

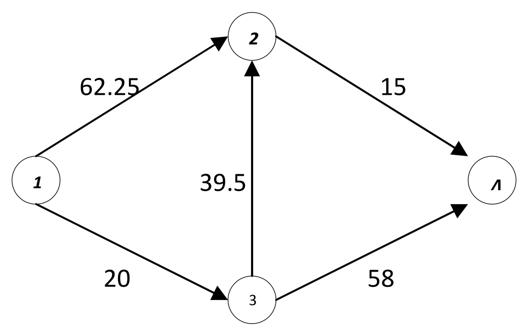

Fig 1

Where $\tilde{c}_{13} = (52,62,65,70)$, $\tilde{c}_{34} = (10,13,17,20)$, $\tilde{c}_{12} = (10,20,30)$, $\tilde{c}_{24} = (52,55,60,65)$,and $\tilde{c}_{23} = (35,38,40,45)$, thus $I(\tilde{c}_{13}) = 62.25$, $I(\tilde{c}_{34}) = 15$, $I(\tilde{c}_{12}) = 20$, $I(\tilde{c}_{24}) = 58$, $I(\tilde{c}_{23}) = 39.5$ which are shown near each arc. The fuzzy supply\demands are $b(1) = [2+\beta, 4-\beta]$, $b(2) = 0$, $b(3) = 0$,and $b(4) = [-2-\beta, -4+\beta]$ and the fuzzy arc capacities are as follows: $u_{13}(\beta) = [1+\beta, 3-\beta]$, $u_{12}(\beta) = [1+\frac{\beta}{2}, 2-\frac{\beta}{2}]$, $u_{23}(\beta) = [1+2\beta, 5-2\beta]$,

$u_{34}(\beta) = [2+2.5\beta, 7-2.5-\beta]$,
and $u_{24}(\beta) = [1+2.5\beta, 6-2.5\beta]$.

Iteration 1:

$l_f(e,\beta) = 0$, $r_f(e,\beta) = 0$ $c^{\pi}_{l,ij} = c^{\pi}_{r,ij} = I(\tilde{c}_{ij})$, for all arcs.

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We find $p_l = p_r = 1 \to 2 \to 3 \to 4$ as the shortest paths from node 1 to node 4.

$$\delta_l(\beta) = \min\{2+\beta, 1+\frac{\beta}{2}, 1+2\beta, 2+2.5\beta\} = 1+\frac{\beta}{2};$$

and

$$\delta_r(\beta) = \min\{4-\beta, 2-\frac{\beta}{2}, 5-2\beta, 7-2.5\beta\} = 2-\frac{\beta}{2}$$

;

By augmenting $\delta_l(\beta)$ units of flow along p_l and $\delta_r(\beta)$ units of flow along p_r, we get: $l_f((1,2),\beta) = l_f((2,3),\beta) = l_f((3,4),\beta) = 1+\frac{\beta}{2}$ and

$$r_f((1,2),\beta) = r_f((2,3),\beta) = r_f((3,4),\beta) = 2-\frac{\beta}{2},$$

and

Now we update $\pi_r(1) = \pi_l(1) = 0$, $\pi_r(2) = \pi_l(2) = -20$, $\pi_r(3) = \pi_l(3) = -59.5$, and $\pi_r(4) = \pi_l(4) = -74.5$. $b(1) = [1+\frac{\beta}{2}, 2-\frac{\beta}{2}]$, $b(4) = [-1-\frac{\beta}{2}, -2+\frac{\beta}{2}]$

Iteration 2:

$$\pi_l(1,\beta) = \pi_r(1,\beta) = 0,$$

$$\pi_l(2,\beta) = \pi_l(3,\beta) = \pi_r(3,\beta) = \pi_r(2,\beta) = 131.75,$$

and $\pi_l(4,\beta) = \pi_r(4,\beta) = 131.75$

We find $p_l = p_r = 1 \to 3 \to 4$ as the shortest paths from node 1 to node 4.

$$\delta_l(\beta) = \min\{1+\frac{\beta}{2}, 1+2\beta, 1+\beta\} = 1+\frac{\beta}{2},$$

and

$$\delta_r(\beta) = \min\{2-\frac{\beta}{2}, 3-\beta, 5-2\beta\} = 2-\frac{\beta}{2}.$$

By augmenting $\delta_l(\beta)$ units of flow along p_l and $\delta_r(\beta)$ units of flow along p_r, we get:

$$l_f((1,2),\beta) = l_f((2,3),\beta) = l_f((1,3),\beta) = 1+\frac{\beta}{2},$$

$l_f((3,4),\beta) = 2+\beta$ and

$r_f((1,2),\beta) = r_f((2,3),\beta) = r_f((1,3),\beta) = 2-\frac{\beta}{2}$,and

$$r_f((3,4),\beta) = 4-\beta$$

Thus $b(1,\beta) = b(4,\beta) = 0$ and the algorithm terminates.

6. CONCLUSION

In this paper the MCF problem with fuzzy parameters has been considered. The problem is modeled as a MCF problem with fuzzy interval valued parameters. Then by defining the fuzzy residual network, the well-known MCF algorithms were developed to solve the problem. Since these algorithms are based on the network structure of the problem, they are much more efficient than the previous works which are based on linear programming procedures like network simplex algorithm. In our model arc capacities, arc costs and supply\demands were considered fuzzy numbers. A challenging research topic is the case of fuzzy constraints.

REFERENCES

[1] R.K. Ahuja, T.L. Magnanti, and J.B. Orlin. Network flows: Theory, Algorithms, and Applications. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[2] G. D. Klockner and G. L. Nemhauser, A dynamic network flow problem with uncertain arc capacities: Formulation and problem structure, Operations Research, vol.48, no.2, 2000, pp. 233-242.

[3] G. H. Polychronopoulos and J. N. Tsitsiklis, Stochastic shortest path problems with resources, Networks, vol. 27, 1996, pp. 131-143.

[4] W. B. Powell and R. K. –M. Cheung, A network resource decomposition method with random arc capacities, Networks, vol. 24, 1994, pp. 369-384.

[5] R. E. Bellman and L. A Zaheh, Decision-making in a fuzzy environment, Management Science, vol. 17, 1970, pp. 141-164.

[6] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, vol. 1, 1978, pp. 3-28.

ARPN Journal of Science and Technology

http://www.ejournalofscience.org

[7] H. J. Zimmermann, Fuzzy Set Theory and Its Applications. Norwell, MA: Kluwer, 1996.

[8] D. Dubois and H. Prade, Fuzzy Sets and Systems: Theory and applications. New York: Academic, 1980.

[9] C. M. Klein, Fuzzy shortest paths, Fuzzy Sets and Systems, vol. 39, 1991, pp. 27-41.

[10] R. Yager, Path of least resistance on possibilistic production systems, Fuzzy Sets and Systems, vol. 19, 1986, pp. 121-132.

[11] S. Okada and T. Soper, A shortest path problem on a network with fuzzy arc lengths, Fuzzy Sets and Systems, vol. 109, 2000, pp. 129-141.

[12] F. Hernandes, M T. Lamata, J. L. Verdegay and A. Yamakami, The shortest path problem on networks with fuzzy parameters, Fuzzy Sets and Systems Volume 158, Issue 14, 16 July 2007, Pages 1561-1570.

[13] S. Tai and C. Kao, Network flow problems with fuzzy arc lengths, IEEE Transactions on Systems, Man and Cybernetics, B. vol. 34, 2004, pp.765-769.

[14] S. Chanas and W. Kolodziejczyk, Maximum flow in a network with fuzzy arc capacities, Fuzzy Sets and Systems, vol. 8, 1982, pp. 165-173.

[15] P. Diamond, A fuzzy max-flow min –cut theorem, Fuzzy Sets and Systems, vol. 119, 2001, pp. 139-148.

[16] H. S. Shih and E. S. Lee, Fuzzy multi-level minimum cost flow problems, Fuzzy Sets and Systems, vol. 107, 1999, pp. 159-176.

[17] D. S. Negi and E. S. Lee, Possibility programming by the comparison of fuzzy numbers, Computers and Mathematics with Applications, vol. 25, 1993, pp. 43-50.

[18] M. Ghatee and S. M. Hashemi, Ranking function-based solutions of fully fuzzified minimal cost flow problem, Information Sciences: an International Journal, Volume 177 , Issue 20 (October 2007), Pages 4271-4294.

[19] M. Ghatee, S. M. Hashemi, M. Zarepisheh and E. Khorram, Preemptive priority-based algorithms for fuzzy minimal cost flow problem: An application in hazardous materials transportation, Computers & Industrial Engineering, vol. 5, Issue 1, 2009, pp. 341-354.

[20] M. Ghatee, S. M Hashemi, Optimal network design and storage management in petroleum distribution network under uncertainty, Engineering Applications of Artificial Intelligence, Volume 22, Issues 4-5, June 2009, Pages 796-807.

[21] M. Ghatee, S. M Hashemi, Some concepts of the fuzzy multicommodity flow problem and their application in fuzzy network design, Mathematical and Computer Modelling, Volume 49, Issues 5-6, March 2009, Pages 1030-1043.

[22] M. Ghatee, S. M Hashemi, Application of fuzzy minimum cost flow problems to network design under uncertainty, Fuzzy Sets and Systems, Volume 160, Issue 22, 16 November 2009, Pages 3263-3289.

[23] M. Ghatee, S. M Hashemi, Traffic assignment model with fuzzy level of travel demand: An efficient algorithm based on quasi-Logit formulas, European Journal of Operational Research, Volume 194, Issue 2, 16 April 2009, Pages 432-451.

[24] M. Ghatee, S. M. Hashemi, B. Hashemi, M. Dehghan, The solution and duality of imprecise network problems, Computers & Mathematics with Applications, Volume 55, Issue 12, June 2008, Pages 2767-2790.

[25] M. Ghatee, S. M Hashemi, Generalized minimal cost flow problem in fuzzy nature: An application in bus network planning problem, Applied Mathematical Modelling, Volume 32, Issue 12, December 2008, Pages 2490-2508.

[26] J. J. Buckley and T. Feuring, Evolutionary algorithm solution to fuzzy problems: Fuzzy linear programming, Fuzzy Sets and Systems vol. 109, 2000, pp. 35-53.

[27] B. Asady and A. Zendehnam, Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling, 31, (2007) 2589-2598.

[28] C. B. Chen and C. M. Klein, A simple approach to ranking a group of aggregated fuzzy utilities, IEEE Transactions on Systems, Man and Cybernetics, B. vol. SMC-27, 1997, pp.26-35.

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[29] S. H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, vol. 17, 1985, pp. 113-129.

[30] F. Choobineh and H. Li, An index for ordering fuzzy numbers, Fuzzy Sets and Systems, vol. 54,1993, pp. 287-294.

[31] P. Fortemps and M. Roubens, Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems, vol. 82, 1996, pp. 319-330.

[32] R. Yager, A procedure for ordering fuzzy subsets of the unit interval, Information Science, vol. 24, 1981, pp. 143-161.

[33] U. W. Kulisch and W. L. Mirankar, Computer Arithmetic in Theory and Practice, Academic Press, New York, 1981.

[34] R. E. Moore, Interval Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1966.

[35] M. Hukuhara, Integration des applications measurable dont la valeur est un compact convexe, Funkc. Ekvacioj. Vol. 10, 1967, pp. 205-223.

[36] C. V. Negoita and D. A. Ralescu, Applications of Fuzzy Sets to Systems Analysis, Wiley, New York, 1975.