Monitoring of Network Traffic based on Queuing Theory

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ABSTRACT

Network traffic monitoring is an important way for network performance analysis and monitor. The present article explores how to build the basic model of network traffic analysis based on Queuing Theory. In the present work, two queuing models (M/M/1): ((C+1)/FCFS) and (M/M/2): ((C+1)/FCFS) have been applied to determine the forecast way for the stable congestion rate of the network traffic. Using this, we can obtain the network traffic forecasting ways and the stable congestion rate formula. Combining the general network traffic monitor parameters, we can realize the estimation and monitor process for the network traffic rationally.

Keywords: Network traffic, Queuing Theory, stable congestion rate

1. INTRODUCTION

Network traffic monitoring is an important way for network performance analysis and monitor. The present analysis seeks to explore how to build the basic model of network traffic analysis based on Queuing Theory [1]. Using this, we can obtain the network traffic forecasting ways and the stable congestion rate formula, combining the general network traffic monitor parameters. Consequently we can realize the estimation and monitor process for the network traffic rationally.

Queuing Theory, also called random service theory, is a branch of Operation Research in the field of Applied Mathematics. It is a subject which analyze the random regulation of queuing phenomenon, and builds up the mathematical model by analyzing the date of the network. Through the prediction of the system, we can reveal the regulation about the queuing probability and choose the optimal method for the system.

Adopting Queuing Theory to estimate the network traffic, it becomes the important ways of network performance prediction, analysis and estimation and, through this way, we can imitate the true network, it is useful and reliable for organizing, monitoring and defending the network.

2. THE MATHEMATICAL MODEL OF THE QUEUING THEORY

In network communication, from sending, transferring to receiving data and the proceeding of the data coding, decoding and sending to the higher layer, in all these process, we can find a simple queuing model. According to the Queuing Theory, this correspond procedure can be abstracted as Queuing theory model [2], like fig. 1. Considering this kind of simple data transmitting system satisfies the queue model [3].

Figure 1: The abstract model of communication process
From the above fig. 1,
- $\lambda'$: Sending rate of the sender.
- $T_N$: Transportation delay time.
- $\lambda$: Arriving speed of the data packets
- $N_q$: Quantity of data packets stored in the buffer (temporary storage).
- $\gamma$: Packets rate which have mistake in sending from receiver i.e., lost rate of the receiver.
- $T_t$: Service time of data packets in the server

where $T_s = T_d + T_b + T_c$
- $T_d$: Decoding time
- $T_b$: Dispatching time
- $T_c$: Calculating time or evaluating time or handling time.

3. Model-1: The Queuing model with one server (M/M/1):((C+1)/FCFS)

The system of differential difference equation is

$$\frac{d}{dt} \{P_n(t)\} = -\lambda_n P_n(t) - \mu_n P_n(t) + \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t), \text{ for } n \geq 1$$

And

$$\frac{d}{dt} \{P_0(t)\} = -\lambda_0 P_0(t) + \mu_1 P_1(t), \text{ for } n=0$$

In model M/M/1, we let

$$\lambda_n = \lambda, \text{ and } \mu_n = \mu$$

Where $\lambda$ and $\mu$ are constants.

Then eqs.(1) and (2) reduces to

$$\frac{d}{dt} \{P_n(t)\} = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\lambda + \mu) P_n(t), \text{ for } n \geq 1$$

And

$$\frac{d}{dt} \{P_0(t)\} = -\lambda P_0(t) + \mu P_1(t), \text{ for } n=0$$

Here, $\lambda$ is considered as the arrival rate while $\mu$ as the service rate.

In the steady state equation

$$L(t) P_n(t) = P_n$$

And

$$L(t) \frac{d}{dt} \{P_n(t)\} = 0$$

Hence, from eqs. (3) and (4) when $t \to \infty$ we get

$$0 = \lambda P_{n-1} + \mu P_{n+1} - (\lambda + \mu) P_n \text{ for } n \geq 1$$

And

$$0 = -\lambda P_0 + \mu P_1$$

This implies

$$P_1 = \frac{\lambda}{\mu} P_0$$

From eq.(5) when $n=1$, we get

$$(\lambda + \mu) P_1 = \lambda P_0 + \mu P_2 \text{ (3)}$$

Therefore,

$$P_2 = \frac{(\lambda)^2}{\mu} P_0 \text{ (4)}$$
In general, \( P_n = \left( \frac{\lambda}{\mu} \right)^n P_0 \)

or, \( P_n = \rho^n P_0 \) where \( \rho = \frac{\lambda}{\mu} \)

Here, \( \rho \) is called server utilization factor or traffic intensity.

We know, \( \sum_{n=0}^{\infty} P_n = 1 \)

Also, \( P_n = \rho^n P_0 \)

Therefore, \( \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{\infty} \rho^n P_0 \)

or, \( 1 = P_0 \sum_{n=0}^{\infty} \rho^n \)

Consequently, \( P_0 = 1 - \rho \), where \( \rho < 1 \)

Hence, \( P_n = \rho^n (1 - \rho) \), \( n=0, 1, 2, \ldots \).

Suppose, \( L \) stands for the length of the queue under the steady state condition. It includes the average volume of all the data packets which enter the processing module and store in the buffer.

\[
L = \sum_{n=0}^{\infty} nP_n = \sum_{n=1}^{\infty} n\rho^n (1 - \rho)
\]

\[
= (1 - \rho) \sum_{n=1}^{\infty} n\rho^n
\]

Hence, \( L = \frac{\rho}{1 - \rho} \) \hspace{1cm} (7)

Also \( L = \frac{\lambda}{\mu - \lambda} \) Since, \( \rho = \frac{\lambda}{\mu} \)

If \( N_q \) denotes the average volume of the buffers data packets then

\[
N_q = L - \rho = \frac{\rho^2}{1 - \rho} \hspace{1cm} (8)
\]

Also

\[
N_q = \frac{\lambda^2}{\mu(\mu - \lambda)}
\]

Using the Little’s law we have

\[
\rho = \lambda T_s \quad \text{and} \quad \lambda = \lambda'
\] \hspace{1cm} (9)

Using eq.(9), eq.(8) reduces to

\[
N_q = \frac{\rho^2}{1 - \rho}
\]

This implies \( (1 - \lambda T_s)N_q = (\lambda T_s)^2 \)

Or, \( (\lambda T_s)^2 + \lambda T_s N_q - N_q = 0 \), \hspace{1cm} \( (\lambda = \lambda') \)

(10)

The above equation eq.(10) provides the relation between following parameters:

\[ T_s = \text{Service time} \]

\[ \lambda' = \text{Sending rate} \]

\( N_q = \text{Quantity of data packets stored in the buffer} \)

If we know any two variables, it is easy to gain the numerical value of the third one.

So, these three variables are key parameters for measuring the performance of the transmission system.

4. QUEUING THEORY AND THE NETWORK TRAFFIC MONITOR

4.1. Forecasting the network traffic using Queuing Theory

The network traffic is very common [5]. The system will be in worse condition, when the traffic becomes under extreme situation, in which leads to the network congestion [6]. There are a great deal of research about monitoring the congestion at present, besides, the documents which make use of Queuing Theory to research the traffic rate appear more and more. For forecasting the traffic rate, we often test the data disposal function of the router used in the network.

Considering a router’s arrival rate of data flow in groups is \( \lambda \), and the average time which the routers use to dispose each group is \( \frac{1}{\mu} \), the buffer of the routers is \( C \), if a certain group arrives, the waiting length of the queue in groups has already reached, so the group has to be lost. When the arriving time of group timeouts, the group has to resend. Suppose, the group’s average...
waiting time is \( \frac{1}{\mu} \). We identify \( P_i(t) \) to be the arrival probability of the queue length for the routers group at the moment of \( t \), supposing the queue length is \( i \):

\[
P(t) = (P_0(t), P_1(t), \ldots, P_{n-1}(t)), i = 0, 1, \ldots, C+1.
\]

Then the queuing system of the router’s date groups satisfies simple Markov Process [7], according to Markov Process, we can find the diversion strength of matrix of model 1 as follow:

\[
Q = \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & \cdots & 0 & 0 \\
\mu & (\lambda + \mu + \gamma) & \lambda + \gamma & 0 & \cdots & 0 & 0 \\
0 & \mu & (\lambda + \mu + 2\gamma) & \lambda + 2\gamma & \cdots & 0 & 0 \\
0 & 0 & \mu & (\lambda + \mu + 3\gamma) & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & - (\lambda + \mu + C\gamma) & \lambda + C\gamma \\
0 & 0 & 0 & 0 & \cdots & \mu & -\mu
\end{pmatrix}
\]

4.2 Network Congestion Rate

Network congestion rate is changing all the time [8]. The instantaneous congestion rate and the stable congestion rate are often used to analysis the network traffic in network monitor. The instantaneous rate \( A_c(t) \) is the congestion rate at the moment of \( t \). The \( A_c(t) \) can be obtained by solving the system length of the queue’s probability distributing, which is called \( P_{c+1}(t) \).

Let, \( P_k(t) (k=0,1, \ldots, C+1) \) to be the arrival probability of the queue length for the routers group at the moment of \( t \) by considering the queue length is \( k \).

Then, the queuing system of the router’s date groups satisfies simple Markov Process. According to Markov Process, \( P_k(t) \) satisfies the following system of differential difference equations.

Let,

\[
P_i(t) = \text{Prob} \{ k \text{ number of data packets present in the system in time } t \}
\]

and \( P_{k+1}(t) = \text{Prob} \{ k \text{ number of data packets present in the system in time } (t+\Delta t) \} \)

Case 1:

For \( k \geq 1 \)

\[
P_{k+1}(t + \Delta t) = \text{Prob} \{ k \text{ number of data packets present in the system at time } t \} \times \text{Prob} \{ \\
\text{no data packet arrival in time } (\Delta t) \} \times \text{Prob} \{ \text{no data packet departure in time } \Delta t \}
\]

\[
+ \text{Prob} \{ (k-1) \text{ number of data packets present in the system at time } t \} \times
\]

\[
\text{Prob} \{ 1 \text{ data packet arrival in time } (\Delta t) \} \times
\]

\[
\text{Prob} \{ \text{no data packet departure in time } \Delta t \} +
\]

\[
\text{Prob} \{ (k+1) \text{ number of data packets present in the system at time } t \} \times
\]

\[
\text{Prob} \{ \text{no data packet arrival in time } (\Delta t) \} \times
\]

\[
\text{Prob} \{ 1 \text{ data packet departure in time } \Delta t \} \} + \ldots
\]

\[
\Rightarrow P_k(t + \Delta t) = P_k(t)\{1 - \lambda_k \Delta t + o(\Delta t)\} \{1 - \mu_k \Delta t + o(\Delta t)\}
\]

\[
+ P_{k-1}(t)\{\lambda_{k-1} \Delta t + o(\Delta t)\} \{1 - \mu_{k-1} \Delta t + o(\Delta t)\}
\]

\[
+ P_{k+1}(t)\{1 - \lambda_{k+1} \Delta t + o(\Delta t)\} \{\mu_{k+1} \Delta t + o(\Delta t)\} +
\]

\[
\text{o}(\Delta t)
\]

\[
\Rightarrow P(t+\Delta t) - P(t) = - (\lambda_k + \mu_k) P_k(t) \Delta t + \lambda_{k+1} P_{k+1}(t) +
\]

\[
\mu_{k+1} P_{k+1}(t)
\]

Dividing both sides by \( \Delta t \) and taking limit as \( \Delta t \to 0 \)

\[
\Rightarrow \frac{d}{dt} \{P_k(t)\} = - (\lambda_k + \mu_k) P_k(t) + \lambda_{k+1} P_{k+1}(t) +
\]

\[
\mu_{k+1} P_{k+1}(t)
\]

\[
\mu_{k+1} P_{k+1}(t), \text{ since } \lim_{t \to \infty} \frac{o(\Delta t)}{\Delta t} = 0
\]
Here, in state \( k \), data packets arrival is \( \lambda_k = \lambda + k\gamma \) 

i.e. \( \hat{\lambda}_k = \lambda + k\gamma \)

Also, in state \( k \), data packet departure is \( \mu \) 

i.e. \( \mu_k = \mu \)

Hence, eq.(11) reduces to

\[
\frac{d}{dt}P_0(t) = -\lambda P_0(t) + \mu P_1(t) + \gamma P_1(t) + \gamma P_2(t)
\]

where \( k = 1,2,\ldots,C \)

**Case 2:**

For \( k = 0 \), we have

\[
P_0(t+\Delta t) = \text{Prob} \{ \text{no data packet present in the system at time } (t+\Delta t) \}
\]

- \( = \text{Prob} \{ \text{no data packet present in time } t \} \times \text{Prob} \{ \text{one data packet present in time } t \} \times \text{Prob} \{ \text{no data packet departure in time } \Delta t \} \times \text{Prob} \{ \text{one data packet departure in time } \Delta t \}
\]

\[
P_0(t) \left[ \mu + \gamma \right] P_0(t) + \left[ \lambda + (k-1)\gamma \right] P_{k-1}(t) + \mu P_k(t) + \gamma P_{k+1}(t)
\]

(12)

Dividing both sides by \( t \Delta t \) and taking limit as \( 0 \to 0 \) we get

\[
\frac{d}{dt} \left( P_{c+1}(t) \right) = P_c(t) \hat{\lambda}_c - \mu C + P_{c+1}(t)
\]

**Case 3:**

For \( k = C+1 \), we have

\[
P_{C+1}(t+\Delta t) = \text{Prob} \{ (C+1) \text{ no. of data packets present in the system at time } (t+\Delta t) \}
\]

- \( = \text{Prob} \{ C \text{ no. of data packets present in time } t \} \times \text{Prob} \{ \text{C no. of data packets present in time } t \} \times \text{Prob} \{ \text{no data packet departure in time } \Delta t \} \times \text{Prob} \{ \text{one data packet departure in time } \Delta t \}
\]

\[
= \left[ P_c(t) \left[ \mu + \gamma \right] + P(t) \left[ \lambda + \alpha \right] \right] \Gamma + P_0(t) (1 - \mu C + P_{C+1}(t))
\]

\[
= P_{C+1}(t+\Delta t) - P_{C+1}(t) + \mu C + P_{C+1}(t) \Delta t + o(\Delta t)
\]

By solving this differential equation system, we get the instantaneous congestion rate \( A_0(t) \) as

\[
A_0(t) = \frac{\lambda}{\mu + \lambda} (1 - e^{-\left(\mu + \lambda\right)t})
\]

The instantaneous congestion rate can not be used to measure the stable operating condition of the system, so we must obtain the stable congestion rate of the system. The so-called stable congestion rate means, it will not change with the time changing, when the system works in a stable operating condition. The definition of the stable congestion rate is

\[
A_C = \lim_{t \to \infty} A_C(t)
\]

(13)

Considering, \( P = \lim_{t \to \infty} P(t) \) as the distributing of the stable length of the queue and \( C \) as the buffer of the router, the stable congestion rate can be obtained in two ways: firstly, we obtain the instantaneous congestion rate, then find its limit. According to its definition, it can be obtained with the distributing of the length of the queue. Secondly, according to the Markov Process, we know that the distributing of the stable length of queue
can be obtained through system of steady state equations.

From eq.(12), eq.(13) and eq.(14), we have the system of differential difference equations as follows

\[
\frac{d}{dt} \{P_k(t)\} = (\lambda + k\gamma + \mu)P_k(t) + \{\lambda + (k-1)\gamma\}P_{k-1}(t) + \mu P_{k+1}(t)
\]  

(15)

for \(k = 1, 2, 3, \ldots, C\)

\[
\frac{d}{dt} \{P_0(t)\} = -\lambda P_0(t) + \mu P_1(t) \quad \text{for } k = 0
\]  

(16)

\[
\frac{d}{dt} \{P_{C+1}(t)\} = (\lambda + C\gamma)P_C(t) - \mu P_{C+1}(t) \quad \text{for } k = C+1
\]  

(17)

According to some properties of Markov process, we know that \(P_i(t)\) \((i=0,1,2,\ldots,C+1)\) satisfies the above differential equation.

Here, \(P(t) = [P_0(t), P_1(t), \ldots, P_{C+1}(t)]\]

\[
P_0(0) = 1, P_i(0) = 0, P_{C+1}(0) = 0
\]

\(P(0) = [P_0(0), P_1(0), \ldots, P_{C+1}(0)]\)

For steady state equation,

\[
\lim_{t \to \infty} P_k(t) = P_k \quad \text{and} \quad \lim_{t \to \infty} \frac{dP_k(t)}{dt} = 0
\]

Under steady state condition, eqs.(15),(16) and (17) transform to following balance equations.

\[
(\lambda + k\gamma + \mu)P_k(t) + \{\lambda + (k-1)\gamma\}P_{k-1}(t) + \mu P_{k+1}(t)
\]

for \(k = 1, 2, 3, \ldots, C\)

(18)

\[
0 = -\lambda P_0(t) + \mu P_1(t) \quad \text{for } k = 0
\]

(19)

\[
0 = (\lambda + C\gamma)P_C(t) - \mu P_{C+1}(t) \quad \text{for } k = C+1
\]

(20)

The above system of steady state equations can be written in matrix form as

\[
PQ = 0
\]

and

\[
\sum_{i=0}^{C+1} P_i = 1
\]

where \(P = (P_0, P_1, \ldots, P_{C+1})\)

\[
Q = \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & \cdots & 0 & 0 \\
\mu & -\lambda & \mu + \gamma & 0 & \cdots & 0 & 0 \\
0 & \mu & -\lambda & \mu + 2\gamma & \cdots & 0 & 0 \\
0 & 0 & \mu & -\lambda & \mu + 3\gamma & \cdots & 0 & 0 \\
0 & 0 & 0 & \mu & -\lambda & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -\lambda + \mu + C\gamma & \lambda + C\gamma & \mu & -\mu
\end{pmatrix}
\]

For \(C = 0\),

From eq.(19), we have

\[
\lambda P_0 = \mu P_1
\]

(21)

Also, \(P_0 + P_1 = 1\)

(22)

Solving (21) and (22) we get

\[
P_1 = \frac{\lambda}{\lambda + \mu}
\]

Hence,

\[
A_0 = P_1 = \frac{\lambda}{\lambda + \mu}
\]

(23)

For \(C = 1\)

\[
0 = -\lambda P_0 + \mu P_1
\]

(24)

\[
0 = -\mu P_1 + \lambda P_0 + \mu P_2
\]

(25)

\[
0 = (\lambda + \gamma)P_1 - \mu P_2
\]

(26)

Also, \(P_0 + P_1 + P_2 = 1\)

From eq.(23), we get

\[
\frac{P_0}{\mu} = \frac{P_1}{\lambda} = k
\]

Therefore, \(P_0 = k\mu, P_1 = \lambda k\)

From eq.(25), we have

\[
P_2 = \left(\frac{\lambda + \gamma}{\mu}\right)\lambda k, \quad \text{since, } P_1 = \lambda k
\]

(27)
Using eq.(26), we obtain
\[ k[\mu + \lambda + \frac{\lambda + \gamma}{\mu}] = 1 \]
\[ k = \frac{\mu}{\lambda(\lambda + \gamma) + \mu(\lambda + \mu)} \]
From eq.(27) yields
\[ P_2 = \left(\frac{\lambda + \gamma}{\mu}\right)k, \frac{\mu}{\lambda(\lambda + \gamma) + \mu(\lambda + \mu)} \]
Hence,
\[ A_2 = P_2 = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)}{\mu^2(\lambda + \mu) + \mu\lambda(\lambda + \gamma) + \lambda(\lambda + \gamma)(\lambda + 2\gamma)} \]
For C=2, yielding
\[ 0 = -(\lambda + \mu + \gamma)P_1 + \lambda P_0 + \mu P_2 \]
\[ 0 = -(\lambda + 2\gamma)P_1 + \lambda P_0 + \mu P_2 \]
\[ A_1 = P_2 = \frac{\lambda(\lambda + \gamma)}{\lambda(\lambda + \gamma) + \mu(\lambda + \mu)} \]
\[ A_1 = P_2 = \frac{\lambda(\lambda + \gamma)}{\lambda(\lambda + \gamma) + \mu(\lambda + \mu)} \]
\[ 0 = \lambda P_0 + \mu P_1 \]
\[ 0 = \lambda P_0 + \mu P_1 \]
\[ 0 = (\lambda + 2\gamma)P_2 - \mu P_3 \]
\[ 0 = (\lambda + 2\gamma)P_2 - \mu P_3 \]
Also, \( P_0 + P_1 + P_2 + P_3 = 1 \)
\[ P_0 = k\mu \quad \text{and} \quad P_1 = k\lambda \]
From eq.(28), we obtain
\[ P_0 = k\mu \quad \text{and} \quad P_1 = k\lambda \]
From eq.(31), we have
\[ P_3 = \left(\frac{\lambda + 2\gamma}{\mu}\right)P_2 \]
\[ = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)}{\mu^2}k \]
From eq.(32), we have
\[ P_4 = \left(\frac{\lambda + 3\gamma}{\mu}\right)P_3 \]
\[ = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)(\lambda + 3\gamma)}{\mu^3}k \]
Also, \( P_0 + P_1 + P_2 + P_3 + P_4 = 1 \)
\[ k = \frac{\lambda^3}{\mu(\lambda + \mu) + \lambda(\lambda + \gamma)\mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)\mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)(\lambda + 3\gamma)} \]
Hence,
\[ A_4 = P_4 = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)(\lambda + 3\gamma)}{\mu(\lambda + \mu) + \lambda(\lambda + \gamma)\mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)\mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)(\lambda + 3\gamma)} \]
On the analogy of this, we conclude that, the stable congestion rate is

\[ \lambda_c = P_{C0} = \frac{\mu}{(\lambda + \mu + C\gamma)A_{C-1} - (\lambda + (C-1)\gamma)(1-A_{C-2}) + \mu}, \text{ for } C \geq 2 \]

5. THE QUEUING MODEL WITH ADDITIONAL ONE SERVER (M/M/2) : ((C+1)/FCFS)

In this model, number of servers or channels is two and these are arranged in parallel. Here, arrival distribution is Poisson distribution with mean rate \( \lambda \) per unit time. The service time is exponential with mean rate \( \mu \) per unit time. Each server is identical i.e. each server gives identically service with mean rate \( \mu \) per unit time. The overall service rate can be obtained in two situations. If there are \( n \) numbers of data packets are present in the system.

**Case-1**

For \( n < 2 \)

There will be no queue. Therefore (2-\( n \)) server will remain idle and the combined service rate will be

\[ \mu_n = n\mu , \quad 1 \leq n < 2 \]

**Case-2**

For \( n \geq 2 \)

Then, all the servers will be busy. So, maximum \( (n-2) \) \((c+1)\) number of data packets present in the queue.

The combined service rate will be

\[ \mu_n = 2\mu , \quad n \geq 2 \]

Hence, combining **Case-1** and **Case-2**, we have

\[ \lambda_n = \lambda \quad \text{for all } n \geq 0 \]

\[ \mu_n = n\mu \quad \text{for } 1 \leq n < 2 \]

\[ \mu_n = 2\mu , \quad n \geq 2 \]

\[ \mu_0 = 0 , \quad n = 0 \]

\[ \mu_1 = \mu , \quad n = 1 \]

The steady state equations are

\[ \lambda P_0 = \mu P_1 \quad \text{for } n=0 \]

\[(\lambda + \gamma + \mu)P_1 = \lambda P_0 + 2\mu P_2 \quad \text{for } n = 1 \]

\[\{\lambda + (n-1)\gamma\}P_{n-1} + 2\mu P_{n+1} = (\lambda + n\gamma)P_n + 2\mu P_n \quad \text{for } 1 \leq n \leq C \]

\[(\lambda + C\gamma)P_C = 2\mu P_{C+1} \quad \text{for } k = C + 1 \]

The above system of steady state balance equations can be written in matrix form as

\[ PQ = 0 \]
\[\sum_{i=0}^{C+1} P_i = 1\]

where \( P = (P_0, P_1, \ldots, P_{C+1}) \)

and

\[ Q = \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & \cdots & 0 & 0 \\
\mu & -(\lambda + \mu + \gamma) & \lambda + \gamma & 0 & \cdots & 0 & 0 \\
0 & 2\mu & -(\lambda + 2\mu + 2\gamma) & \lambda + 2\gamma & \cdots & 0 & 0 \\
0 & 0 & 2\mu & -(\lambda + 2\mu + 3\gamma) & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & -(\lambda + 2\mu + C\gamma) & \lambda + C\gamma \\
0 & 0 & 0 & 0 & \cdots & 2\mu & -2\mu
\end{pmatrix}\]

From eq.(49), we obtain

\[ P_2 = \frac{(\lambda + \gamma)}{2\mu} \lambda k \quad \text{since,} \quad P_0 = \mu k, P_1 = \lambda k \]

From eq.(51), we get

\[ P_3 = \frac{(\lambda + 2\gamma)}{2\mu} P_2 \]

\[ \Rightarrow P_3 = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)}{4\mu^2} k, \quad \text{Using the value of} \ P_2 \]

Since, \( P_0 + P_1 + P_2 + P_3 = 1 \)

or, \( k[\lambda + \mu + \frac{(\lambda + \gamma)}{2\mu} \lambda + \frac{(\lambda + 2\gamma)(\lambda + \gamma)}{4\mu^2} \lambda] = 1 \)

\[ \Rightarrow k = \frac{4\mu^2}{4\mu^2(\lambda + \mu) + (\lambda + \gamma)2\lambda \mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)} \]

Hence

\[ A_2 = P_3 = \frac{\lambda(\lambda + \gamma)(\lambda + 2\gamma)}{4\mu^2(\lambda + \mu) + (\lambda + \gamma)2\lambda \mu + \lambda(\lambda + \gamma)(\lambda + 2\gamma)} \]

For \( C = 3 \)

\[ \lambda P_0 = \mu P_1 \]

\[ (\lambda + \gamma + \mu)P_0 = \lambda P_0 + 2\mu P_2 \]

\[ (\lambda + 2\gamma + 2\mu)P_2 = (\lambda + \gamma)P_1 + 2\mu P_3 \]

\[ (\lambda + 3\gamma + 2\mu)P_3 = (\lambda + 2\gamma)P_2 + 2\mu P_4 \]

\[ (\lambda + 3\gamma)P_1 = 2\mu P_3 \]

From eq.(53), we have

\[ P_0 = \mu k \quad \text{and} \quad P_1 = \lambda k \]

From eq.(54)

\[ 2\mu P_2 = (\lambda + \gamma + \mu)P_1 - \lambda P_0 \]

\[ \Rightarrow P_2 = \frac{(\lambda + \gamma + \mu)}{2\mu} \lambda k - \frac{\lambda \mu k}{2\mu} \]

\[ = \frac{(\lambda + \gamma)}{2\mu} \lambda k \]

From eq.(55), we get

\[ 2\mu P_3 = (\lambda + 2\gamma + 2\mu)P_2 - (\lambda + \gamma)P_1 \]

\[ \Rightarrow 2\mu P_3 = \frac{(\lambda + 2\gamma + 2\mu)(\lambda + \gamma)\lambda k}{2\mu} - (\lambda + \gamma)\lambda k \]
\[
(\lambda+\gamma)\lambda k \left[\frac{\lambda+2\gamma+2\mu}{2\mu}\right] - 1
\]

\[
= \frac{\lambda k (\lambda+\gamma)(\lambda+2\gamma)}{2\mu}
\]

\[
\Rightarrow P_3 = \frac{\lambda k (\lambda+\gamma)(\lambda+2\gamma)}{4\mu^2}
\]

From eq. (57)

\[
P_4 = \frac{(\lambda+3\gamma)}{2\mu} P_3
\]

\[
= \frac{\lambda k (\lambda+\gamma)(\lambda+2\gamma)(\lambda+3\gamma)}{8\mu^3}
\]

Also, \( P_0 + P_1 + P_2 + P_3 + P_4 = 1 \)

\[
= k(\lambda+\mu) + \frac{\lambda (\lambda+\gamma)(\lambda+2\gamma)}{2\mu} + \frac{\lambda (\lambda+\gamma)(\lambda+2\gamma)}{4\mu} + \frac{\lambda (\lambda+\gamma)(\lambda+2\gamma)}{8\mu}
\]

\[
= k - \frac{8\mu}{8\mu(\lambda+\gamma)+4\mu(\lambda+\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)}
\]

Hence,

\[
A=P_4 = \frac{\lambda (\lambda+\gamma)(\lambda+2\gamma)(\lambda+3\gamma)}{8\mu(\lambda+\mu)+4\mu(\lambda+\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)+2\mu(\lambda+\gamma)(\lambda+2\gamma)}
\]

On the analogy of this, we conclude that, the stable congestion rate is

\[
A = P_{ca} = 1 - \frac{2\mu}{(\lambda+2\mu+C)\lambda A_1 - \{\lambda+(C-1)\} \lambda (0-A_2) A_2 + 2\mu}, \quad \text{for} \ C \geq 2
\]

6. CONCLUSION

This research paper cites the analysis of the network traffic model through Queuing Theory. In the present work two queuing models (M/M/1): ((C+1)/FCFS) and (M/M/2):((C+1)/FCFS) have been applied. These two models are used to determine the forecast way for the stable congestion rate of the network traffic. Using the Queuing Theory models, it is convenient and simple way for calculating and monitoring the network traffic properly in the network communication system. We can monitor the network efficiently, in the view of the normal, optimal and or even for the high overhead network management, by monitoring and analyzing the network traffic rate.

Finally, we can say that network traffic rate can have an important role in the network communication system.

REFERENCES


